CHAPTER 9

Quadratic Functions and Equations

Chapter Focus

- Graph quadratic functions.
- Solve quadratic equations.
- Use quadratic functions and equations to solve real-world problems.

FREE *Falling*

Physicists use quadratic equations to describe the motion of falling objects, such as water over a waterfall.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. factoring
2. quadratic
3. trinomial
4. x-intercept

A. the process of writing a number or an algebraic expression as a product

B. the x-coordinate(s) of the point(s) where a graph intersects the x-axis

C. a polynomial with three terms

D. a polynomial with degree 2

E. the first number of an ordered pair of numbers that describes the location of a point on the coordinate plane

Graph Functions

Graph each function for the given domain.

5. $y = -2x + 8$; D: $\{-4, -2, 0, 2, 4\}$
6. $y = (x + 1)^2$; D: $\{-3, -2, -1, 0, 1\}$
7. $y = x^2 + 3$; D: $\{-2, -1, 0, 1, 2\}$
8. $y = 2x^2$; D: all real numbers

Multiply Binomials

Find each product.

9. $(m + 2)(m + 5)$
10. $(y - 7)(y + 2)$
11. $(2a + 4)(5a + 6)$
12. $(x + 1)(x + 1)$
13. $(t + 5)(t + 5)$
14. $(3n - 8)(3n - 8)$

Factor Trinomials

Factor each polynomial completely.

15. $x^2 - 2x + 1$
16. $x^2 - x - 2$
17. $x^2 - 6x + 5$
18. $x^2 - x - 12$
19. $x^2 - 9x + 18$
20. $x^2 - 7x - 18$

Squares and Square Roots

Find each square root.

21. $\sqrt{36}$
22. $\sqrt{121}$
23. $-\sqrt{64}$
24. $\sqrt{16}\sqrt{81}$
25. $\sqrt{\frac{9}{25}}$
26. $-\sqrt{6(24)}$

Solve Multi-Step Equations

Solve each equation.

27. $3m + 5 = 11$
28. $3t + 4 = 10$
29. $5n + 13 = 28$
30. $2(k - 4) + k = 7$
31. $10 = \frac{r}{3} + 8$
32. $2(y - 6) = 8.6$

CHAPTER 9 Study Guide: Preview

Where You've Been

Previously, you

- identified and graphed linear functions.
- transformed linear functions.
- solved linear equations.
- factored quadratic polynomials, including perfect-square trinomials.

In This Chapter

You will study

- identifying and graphing quadratic functions.
- transforming quadratic equations.
- solving quadratic equations.
- using factoring to graph quadratic functions and solve quadratic equations.

Where You're Going

You can use the skills in this chapter

- to determine the maximum height of a ball thrown into the air.
- to graph higher-degree polynomials in future math classes, including Algebra 2.
- to solve problems about the height of launched or thrown objects in Physics.

Key Vocabulary/Vocabulario

axis of symmetry	eje de simetría
completing the square	completar el cuadrado
maximum	máximo
minimum	mínimo
parabola	parábola
quadratic equation	ecuación cuadrática
quadratic function	función cuadrática
vertex	vértice
zero of a function	cero de una función

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The value of a function is determined by its rule. The rule is an algebraic expression. What is true about the algebraic expression that determines a **quadratic function**?
2. The shape of a **parabola** is similar to the shape of an open parachute. Predict the shape of a *parabola*.
3. A **minimum** is a point on the graph of a curve with the least y-coordinate. How might a **maximum** be described?
4. An axis is an imaginary line. Use this information and your understanding of symmetry to define the term **axis of symmetry**.

Reading and Writing Math

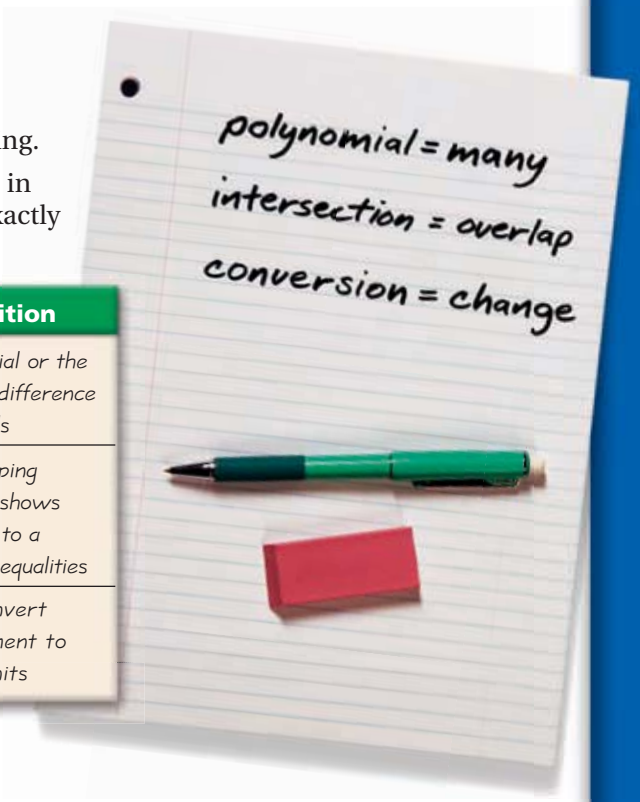
CHAPTER 9

Study Strategy: Learn Vocabulary

Mathematics has a vocabulary all its own. Many new terms appear on the pages of your textbook. Learn these new terms as they are introduced. They will give you the necessary tools to understand new concepts.

Some tips to learning new vocabulary include:

- Look at the **context** in which a new word appears.
- Use **prefixes** or **suffixes** to figure out the word's meaning.
- Relate the new term to familiar **everyday words.** Keep in mind that a word's mathematical meaning may not exactly match its everyday meaning.



Vocabulary Word	Study Tip	Definition
Polynomial	*The prefix "poly-" means many.*	*One monomial or the sum or the difference of monomials*
Intersection	*Relate it to the meaning of the "intersection of two roads".*	*The overlapping region that shows the solution to a system of inequalities*
Conversion Factor	*Relate it to the word "convert", which means change or alter.*	*Used to convert a measurement to different units*

Try This

Complete the chart.

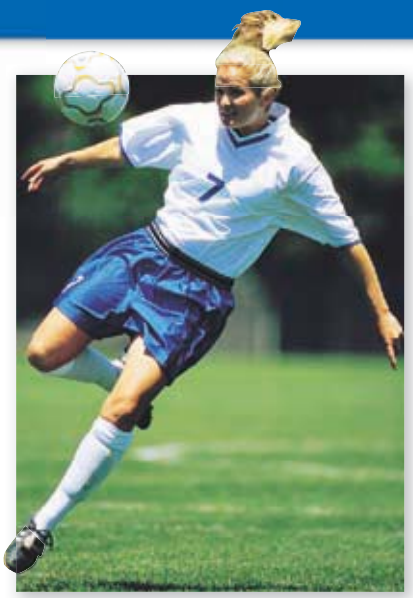
	Vocabulary Word	Study Tips	Definition
1.	Trinomial		
2.	Independent system		
3.	Variable		

Use the context of each sentence to define the underlined word. Then relate the word to everyday words.

4. If two linear equations in a system have the same graph, the graphs are called coincident lines, or simply the same line.

5. In the formula $d = rt$, d is isolated.

9-1 Identifying Quadratic Functions



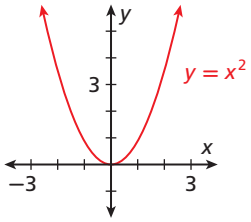
Objectives
Identify quadratic functions and determine whether they have a minimum or maximum.

Graph a quadratic function and give its domain and range.

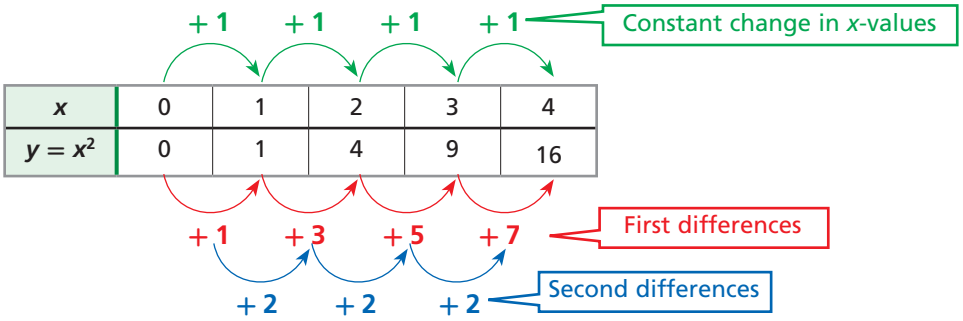
Vocabulary
quadratic function
parabola
vertex
minimum
maximum

Why learn this?
The height of a soccer ball after it is kicked into the air can be described by a quadratic function. (See Exercise 51.)

The function $y = x^2$ is shown in the graph. Notice that the graph is not linear. This function is a *quadratic function*. A **quadratic function** is any function that can be written in the standard form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The function $y = x^2$ can be written as $y = 1x^2 + 0x + 0$,where $a = 1$, $b = 0$, and $c = 0$.



In Lesson 5-1, you identified linear functions by finding that a constant change in x corresponded to a constant change in y. The differences between y-values for a **constant change in x-values** are called ***first differences***.



Notice that the quadratic function $y = x^2$ does not have constant first differences. It has constant ***second differences***. This is true for all quadratic functions.

EXAMPLE 1 Identifying Quadratic Functions

Tell whether each function is quadratic. Explain.

A

x	y
−4	8
−2	2
0	0
2	2
4	8

+2, +2, +2, +2 (between x-values) | −6, −2, +2, +6 (between y-values) | +4, +4

Since you are given a table of ordered pairs with a constant change in x-values, see if the second differences are constant.

Find the first differences, then find the second differences.

The function is quadratic. The second differences are constant.

B $y = -3x + 20$ *Since you are given an equation, use $y = ax^2 + bx + c$.*

This is not a quadratic function because the value of a is 0.

Caution!
Be sure there is a constant change in x-values before you try to find first or second differences.

Helpful Hint

In a quadratic function, only a cannot equal 0. It is okay for the values of b and c to be 0.

Tell whether each function is quadratic. Explain.

C $y + 3x^2 = -4$

$-3x^2 \quad -3x^2$

$y = -3x^2 - 4$

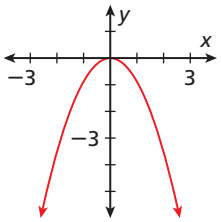
Try to write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract $3x^2$ from both sides.

This is a quadratic function because it can be written in the form $y = ax^2 + bx + c$ where $a = -3$, $b = 0$, and $c = -4$.

CHECK IT OUT! **Tell whether each function is quadratic. Explain.**

1a. $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$ **1b.** $y + x = 2x^2$

The graph of a quadratic function is a curve called a **parabola**. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.



EXAMPLE 2 Graphing Quadratic Functions by Using a Table of Values

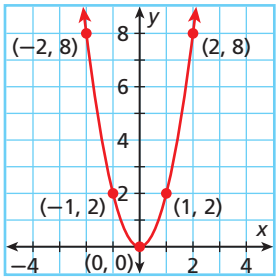
Use a table of values to graph each quadratic function.

A $y = 2x^2$

x	$y = 2x^2$
−2	8
−1	2
0	0
1	2
2	8

Make a table of values. Choose values of x and use them to find values of y.

Graph the points. Then connect the points with a smooth curve.

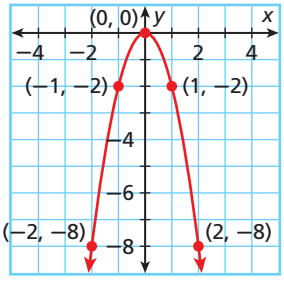


B $y = -2x^2$

x	$y = -2x^2$
−2	−8
−1	−2
0	0
1	−2
2	−8

Make a table of values. Choose values of x and use them to find values of y.

Graph the points. Then connect the points with a smooth curve.



CHECK IT OUT! **Use a table of values to graph each quadratic function.**

2a. $y = x^2 + 2$ **2b.** $y = -3x^2 + 1$

As shown in the graphs in Examples 2A and 2B, some parabolas open upward and some open downward. Notice that the only difference between the two equations is the value of a. When a quadratic function is written in the form $y = ax^2 + bx + c$, the value of a determines the direction a parabola opens.

- A parabola opens **upward** when $a > 0$.
- A parabola opens **downward** when $a < 0$.

EXAMPLE 3 Identifying the Direction of a Parabola

Tell whether the graph of each quadratic function opens upward or downward. Explain.

A $y = 4x^2$

$y = 4x^2$

$a = 4$ *Identify the value of a.*

Since $a > 0$, the parabola opens **upward**.

B $2x^2 + y = 5$

$2x^2 + y = 5$

$-2x^2 \quad -2x^2$ *Write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract $2x^2$ from both sides.*

$y = -2x^2 + 5$

$a = -2$ *Identify the value of a.*

Since $a < 0$, the parabola opens **downward**.

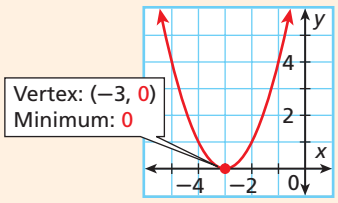
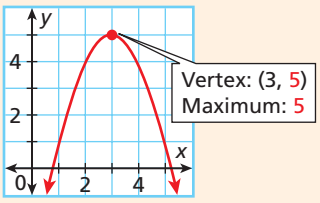
CHECK IT OUT! **Tell whether the graph of each quadratic function opens upward or downward. Explain.**

3a. $f(x) = -4x^2 - x + 1$ **3b.** $y - 5x^2 = 2x - 6$

The highest or lowest point on a parabola is the **vertex**. If a parabola opens upward, the vertex is the lowest point. If a parabola opens downward, the vertex is the highest point.

Know it! Note

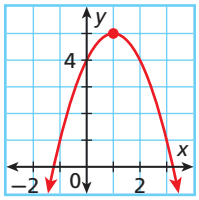
Minimum and Maximum Values

WORDS	If $a > 0$, the parabola opens upward, and the y-value of the vertex is the **minimum** value of the function.	If $a < 0$, the parabola opens downward, and the y-value of the vertex is the **maximum** value of the function.
GRAPHS	$y = x^2 + 6x + 9$ 	$y = -x^2 + 6x - 4$ 

EXAMPLE 4 Identifying the Vertex and the Minimum or Maximum

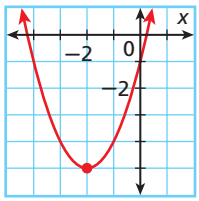
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A



The vertex is $(1, 5)$, and the maximum is 5.

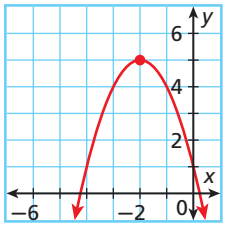
B



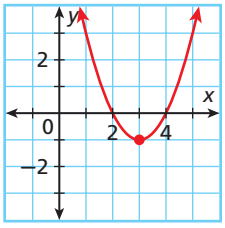
The vertex is $(-2, -5)$, and the minimum is -5.

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

4a.



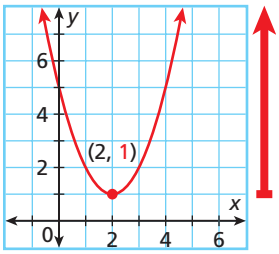
4b.



Caution!

You may not be able to see the entire graph, but that does not mean the graph stops. Remember that the arrows indicate that the graph continues.

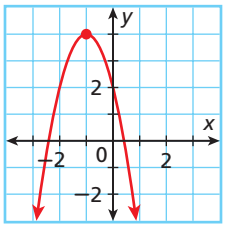
Unless a specific domain is given, you may assume that the domain of a quadratic function is all real numbers. You can find the range of a quadratic function by looking at its graph.



For the graph of $y = x^2 - 4x + 5$, the **range** begins at the minimum value of the function, where $y = 1$. All the y-values of the function are greater than or equal to 1. So the range is $y \geq 1$.

EXAMPLE 5 Finding Domain and Range

Find the domain and range.



Step 1 The graph opens downward, so identify the maximum.

The vertex is $(-1, 4)$, so the maximum is 4.

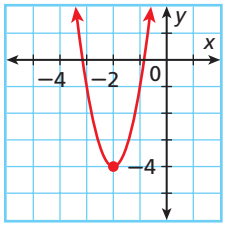
Step 2 Find the domain and range.

D: all real numbers

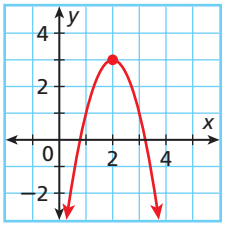
R: $y \leq 4$

Find the domain and range.

5a.



5b.

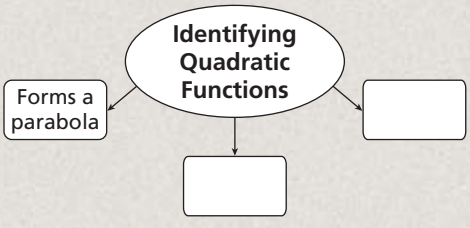


THINK AND DISCUSS

1. How can you identify a quadratic function from ordered pairs? from looking at the function rule?

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer below. In each box, describe a way of identifying quadratic functions.



9-1 Exercises

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GUIDED PRACTICE

1. **Vocabulary** The y-value of the vertex of a parabola that opens upward is the ____?____ value of the function. (*maximum* or *minimum*)

SEE EXAMPLE 1 p. 610

Tell whether each function is quadratic. Explain.

2. $y + 6x = -14$

3. $2x^2 + y = 3x - 1$

4.

x	−4	−3	−2	−1	0
y	39	18	3	−6	−9

5. $\{(-10, 15), (-9, 17), (-8, 19), (-7, 21), (-6, 23)\}$

SEE EXAMPLE 2 p. 611

Use a table of values to graph each quadratic function.

6. $y = 4x^2$ 7. $y = \frac{1}{2}x^2$ 8. $y = -x^2 + 1$ 9. $y = -5x^2$

SEE EXAMPLE 3 p. 612

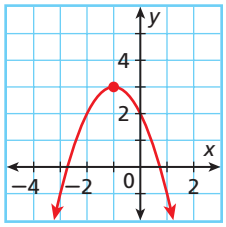
Tell whether the graph of each quadratic function opens upward or downward. Explain.

10. $y = -3x^2 + 4x$ 11. $y = 1 - 2x + 6x^2$ 12. $y + x^2 = -x - 2$
13. $y + 2 = x^2$ 14. $y - 2x^2 = -3$ 15. $y + 2 + 3x^2 = 1$

SEE EXAMPLE 4 p. 612

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

16.



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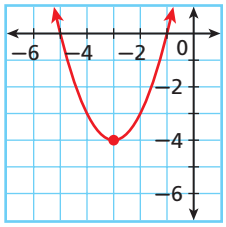
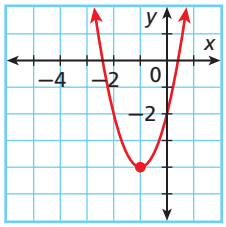


SEE EXAMPLE 5 p. 613

Find the domain and range.

18.



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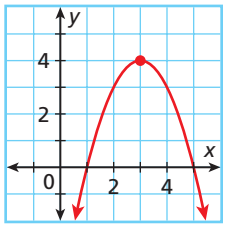

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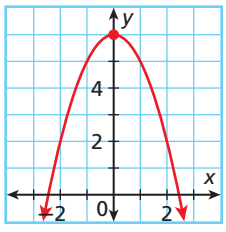

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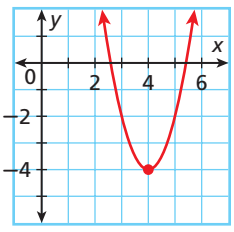


PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
22–25	1
26–29	2
30–32	3
33–34	4
35–38	5

Extra Practice

Skills Practice p. S20

Application Practice p. S36

Tell whether each function is quadratic. Explain.

22.

x	−2	−1	0	1	2
y	−1	0	4	9	15

23. $-3x^2 + x = y - 11$

24. $\{(0, -3), (1, -2), (2, 1), (3, 6), (4, 13)\}$

25. $y = \frac{2}{3}x - \frac{4}{9} + \frac{1}{6}x^2$

Use a table of values to graph each quadratic function.

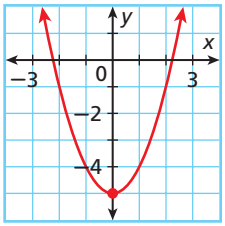
26. $y = x^2 - 5$ **27.** $y = -\frac{1}{2}x^2$ **28.** $y = -2x^2 + 2$ **29.** $y = 3x^2 - 2$

Tell whether the graph of each quadratic function opens upward or downward. Explain.

30. $y = 7x^2 - 4x$ **31.** $x - 3x^2 + y = 5$ **32.** $y = -\frac{2}{3}x^2$

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

33.



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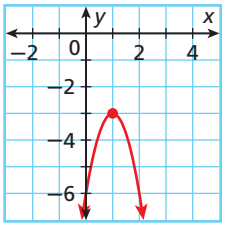

Find the domain and range.

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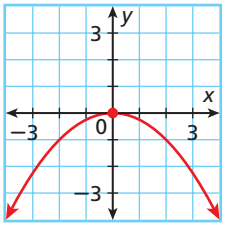

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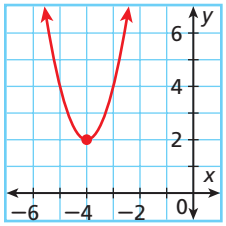
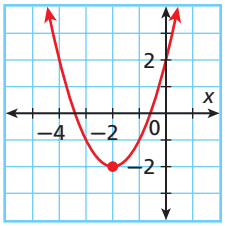


37.



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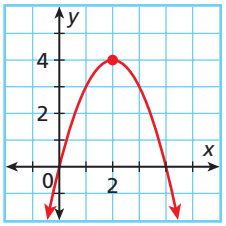


Tell whether each statement is sometimes, always, or never true.

39. The graph of a quadratic function is a straight line.

40. The range of a quadratic function is the set of all real numbers.

41. The highest power of the independent variable in a quadratic function is 2.

42. The graph of a quadratic function contains the point $(0, 0)$.

43. The vertex of a parabola occurs at the minimum value of the function.

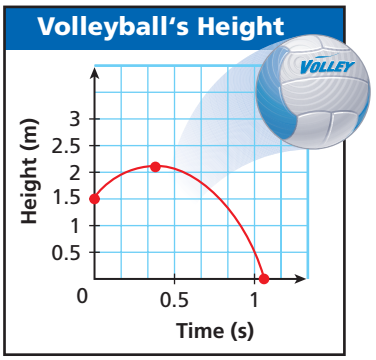
44. The graph of a quadratic function that has a minimum opens upward.

Tell whether each function is quadratic. If it is, write the function in standard form. If not, explain why not.

45. $y = 3x - 1$ **46.** $y = 2x^2 - 5 + 3x$ **47.** $y = (x + 1)^2$

48. $y = 5 - (x - 1)^2$ **49.** $y = 3x^2 - 9$ **50.** $y = (x + 1)^3 - x^2$

51. Estimation The graph shows the approximate height y in meters of a volleyball x seconds after it is served.



- **a.** Estimate the time it takes for the volleyball to reach its greatest height.
- **b.** Estimate the greatest height that the volleyball reaches.
- **c. Critical Thinking** If the domain of a quadratic function is all real numbers, why is the domain of this function limited to nonnegative numbers?

52. Sports The height in feet of a soccer ball x seconds after it is kicked into the air is modeled by the function $y = 48x - 16x^2$.

- **a.** Graph the function.
- **b.** In this situation, what values make sense for the domain?
- **c.** Does the soccer ball ever reach a height of 50 ft? How do you know?

Tell whether each function is linear, quadratic, or neither.

53. $y = \frac{1}{2}x - x^2$ **54.** $y = \frac{1}{2}x - 3$ **55.** $y + 3 = -x^2$ **56.** $y - 2x^2 = 0$

57. $y = \frac{1}{2}x(x^2)$ **58.** $y = \frac{3}{x^2}$ **59.** $y = \frac{3}{2}x$ **60.** $x^2 + 2x + 1 = y$

61. Marine Biology A scientist records the motion of a dolphin as it jumps from the water. The function $h(t) = -16t^2 + 32t$ models the dolphin's height in feet above the water after t seconds.

- **a.** Graph the function.
- **b.** What domain makes sense for this situation?
- **c.** What is the dolphin's maximum height above the water?
- **d.** How long is the dolphin out of the water?

62. Write About It Explain how to tell the difference between a linear function and a quadratic function when given each of the following:

- **a.** ordered pairs
- **b.** the function rule
- **c.** the graph

MULTI-STEP TEST PREP

63. This problem will prepare you for the Multi-Step Test Prep on page 640.

A rocket team is using simulation software to create and study water bottle rockets. The team begins by simulating the launch of a rocket without a parachute. The table gives data for one rocket design.

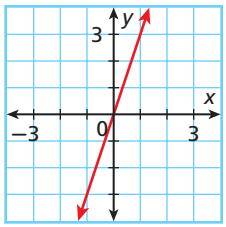
- **a.** Show that the data represent a quadratic function.
- **b.** Graph the function.
- **c.** The acceleration due to gravity is 9.8 m/s^2. How is this number related to the data for this water bottle rocket?

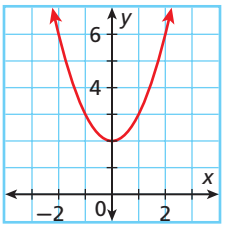
Time (s)	Height (m)
0	0
1	34.3
2	58.8
3	73.5
4	78.4
5	73.5
6	58.8
7	34.3
8	0

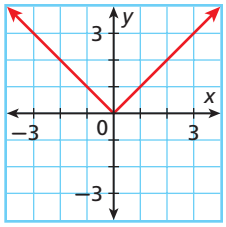
64. Critical Thinking Given the function $-3 - y = x^2 + x$, why is it incorrect to state that the parabola opens upward and has a minimum?

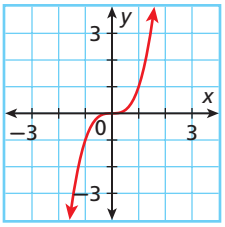
TEST PREP

65. Which of the following is the graph of a quadratic function?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

66. Which of the following quadratic functions has a maximum?

Ⓕ $2x^2 - y = 3x - 2$

Ⓖ $y = x^2 + 4x + 16$

Ⓗ $y - x^2 + 6 = 9x$

Ⓙ $y + 3x^2 = 9$

67. Short Response Is the function $f(x) = 5 - 2x^2 + 3x$ quadratic? Explain your answer by using two different methods of identification.

CHALLENGE AND EXTEND

68. Multi-Step A rectangular picture measuring 6 in. by 10 in. is surrounded by a frame with uniform width x. Write a quadratic function to show the combined area of the picture and frame.

69. Graphing Calculator Use a graphing calculator to find the domain and range of the quadratic functions $y = x^2 - 4$ and $y = -(x + 2)^2$.

SPIRAL REVIEW

Write each number as a power of the given base. *(Lesson 1-4)*

70. 10,000; base 10 **71.** 16; base −2 **72.** $\frac{8}{27}$; base $\frac{2}{3}$

73. A map shows a scale of 1 inch:3 miles. On the map, the distance from Lin's home to the park is $14\frac{1}{4}$ inches. What is the actual distance? *(Lesson 2-7)*

Write a function to describe the situation. Find the reasonable domain and range for the function. *(Lesson 4-3)*

74. Camp Wildwood has collected $400 in registration fees. It can enroll another 3 campers for $25 each.

75. Sal works between 30 and 35 hours per week. He earns $9 per hour.

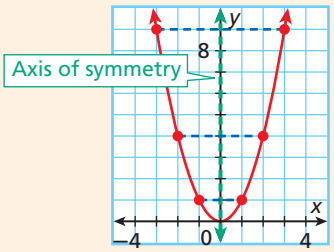
9-2 Algebra LAB

Use with Lesson 9-2

Explore the Axis of Symmetry

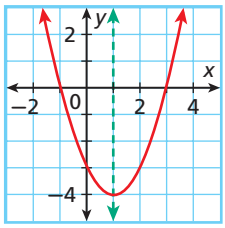
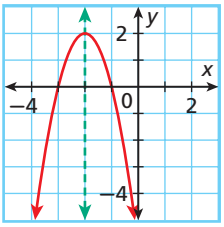
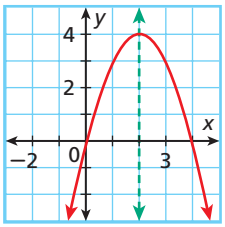
Every graph of a quadratic function is a parabola that is symmetric about a vertical line through its vertex called the *axis of symmetry*.

There is a relationship between a and b in the quadratic function and the equation of the axis of symmetry.



Activity

1 Copy and complete the table.

Function	$y = 1x^2 - 2x - 3$	$y = -2x^2 - 8x - 6$	$y = -1x^2 + 4x$
Graph			
a	1		
b	−2		
$\frac{b}{a}$			
Axis of Symmetry (from graph)	$x = 1$		

2 Compare the axis of symmetry with $\frac{b}{a}$ in your chart. What can you multiply $\frac{b}{a}$ by to get the number in the equation of the axis of symmetry? (*Hint:* Write and solve an equation to find the value.) Check your answer for each function.

3 Use your answer from Problem 2 to complete the equation of the axis of symmetry of a quadratic function. $x =$ ____?____

Try This

For the graph of each quadratic function, find the equation of the axis of symmetry.

1. $y = 2x^2 + 12x - 7$

2. $y = 4x^2 + 8x - 12$

3. $y = 5x^2 - 20x + 10$

4. $y = -3x^2 + 9x + 1$

5. $y = x^2 - 7$

6. $y = 3x^2 + x + 4$