8-3 Factoring $x^2 + bx + c$

Objective
Factor quadratic trinomials of the form $x^2 + bx + c$.

Why learn this?
Factoring polynomials will help you find the dimensions of rectangular shapes, such as a fountain. (See Exercise 71.)

In Chapter 7, you learned how to multiply two binomials using the Distributive Property or the FOIL method. In this lesson, you will learn how to factor a trinomial into two binomials.

Notice that when you multiply $(x + 2)(x + 5)$, the constant term in the trinomial is the product of the constants in the binomials.

$$(x + 2)(x + 5) = x^2 + 7x + 10$$

You can use this fact to factor a trinomial into its binomial factors. Look for two numbers that are factors of the constant term in the trinomial. Write two binomials with those numbers, and then multiply to see if you are correct.

EXAMPLE 1 **Factoring Trinomials by Guess and Check**

Factor $x^2 + 19x + 60$ by guess and check.

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(x + \square)(x + \square)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

Remember!
When you multiply two binomials, multiply:
First terms
Outer terms
Inner terms
Last terms

The constant term in the trinomial is 60.

$(x + 1)(x + 60) = x^2 + 61x + 60$ ✗ *Try factors of 60 for the constant terms in the binomials.*

$(x + 2)(x + 30) = x^2 + 32x + 60$ ✗

$(x + 3)(x + 20) = x^2 + 23x + 60$ ✗

$(x + 4)(x + 15) = x^2 + 19x + 60$ ✓

The factors of $x^2 + 19x + 60$ are $(x + 4)$ and $(x + 15)$.

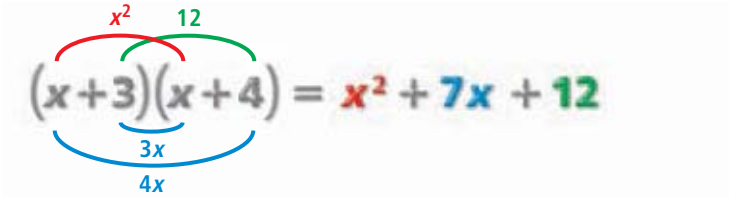
$x^2 + 19x + 60 = (x + 4)(x + 15)$

CHECK IT OUT!

Factor each trinomial by guess and check.

1a. $x^2 + 10x + 24$ **1b.** $x^2 + 7x + 12$

The guess and check method is usually not the most efficient method of factoring a trinomial. Look at the product of $(x + 3)$ and $(x + 4)$.



The coefficient of the middle term is the sum of 3 and 4. The third term is the product of 3 and 4.

Know it! Note

Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b. If no such integers exist, the trinomial is not factorable.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.

Factors of 18	Sum	
1 and 18	19	✗
2 and 9	11	✗
3 and 6	9	✓

$x^2 + 9x + 18 = (x + 3)(x + 6)$

When c is positive, its factors have the same sign. The sign of b tells you whether the factors are positive or negative. When b is positive, the factors are positive, and when b is negative, the factors are negative.

EXAMPLE 2 Factoring $x^2 + bx + c$ When c Is Positive

Factor each trinomial. Check your answer.

A $x^2 + 6x + 8$

$(x + \square)(x + \square)$ *$b = 6$ and $c = 8$; look for factors of 8 whose sum is 6.*

Factors of 8	Sum	
1 and 8	9	✗
2 and 4	6	✓

The factors needed are 2 and 4.

$(x + 2)(x + 4)$

Check $(x + 2)(x + 4) = x^2 + 4x + 2x + 8$ *Use the FOIL method.*

$= x^2 + 6x + 8$ ✓ *The product is the original polynomial.*

B $x^2 + 5x + 6$

$(x + \square)(x + \square)$ *$b = 5$ and $c = 6$; look for factors of 6 whose sum is 5.*

Factors of 6	Sum	
1 and 6	7	✗
2 and 3	5	✓

The factors needed are 2 and 3.

$(x + 2)(x + 3)$

Check $(x + 2)(x + 3) = x^2 + 3x + 2x + 6$ *Use the FOIL method.*

$= x^2 + 5x + 6$ ✓ *The product is the original polynomial.*

Factor each trinomial. Check your answer.

C $x^2 - 10x + 16$

$(x + \square)(x + \square)$

$b = -10$ and $c = 16$; look for factors of 16 whose sum is -10.

Factors of 16	Sum	
-1 and -16	-17	✗
-2 and -8	-10	✓

The factors needed are -2 and -8.

$(x - 2)(x - 8)$

Check $(x - 2)(x - 8) = x^2 - 8x - 2x + 16$ *Use the FOIL method.*

$= x^2 - 10x + 16$ ✓ *The product is the original polynomial.*

CHECK IT OUT! **Factor each trinomial. Check your answer.**

2a. $x^2 + 8x + 12$ **2b.** $x^2 - 5x + 6$

2c. $x^2 + 13x + 42$ **2d.** $x^2 - 13x + 40$

When c is negative, its factors have opposite signs. The sign of b tells you which factor is positive and which is negative. The factor with the greater absolute value has the same sign as b.

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor each trinomial.

A $x^2 + 7x - 18$

$(x + \square)(x + \square)$

$b = 7$ and $c = -18$; look for factors of -18 whose sum is 7. The factor with the greater absolute value is positive.

Factors of -18	Sum	
-1 and 18	17	✗
-2 and 9	7	✓

The factors needed are -2 and 9.

$(x - 2)(x + 9)$

B $x^2 - 5x - 24$

$(x + \square)(x + \square)$

$b = -5$ and $c = -24$; look for factors of -24 whose sum is -5. The factor with the greater absolute value is negative.

Factors of -24	Sum	
1 and -24	-23	✗
2 and -12	-10	✗
3 and -8	-5	✓

The factors needed are 3 and -8.

$(x + 3)(x - 8)$

Helpful Hint

If you have trouble remembering the rules for which factor is positive and which is negative, you can try all the factor pairs and check their sums.

CHECK IT OUT! **Factor each trinomial. Check your answer.**

3a. $x^2 + 2x - 15$ **3b.** $x^2 - 6x + 8$ **3c.** $x^2 - 8x - 20$

A polynomial and the factored form of the polynomial are equivalent expressions. When you evaluate these two expressions for the same value of the variable, the results are the same.

EXAMPLE 4 Evaluating Polynomials

Factor $n^2 + 11n + 24$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

$n^2 + 11n + 24$

$(n + \square)(n + \square)$ — *b = 11 and c = 24; look for factors of 24 whose sum is 11.*

Factors of 24	Sum	
1 and 24	25	✗
2 and 12	14	✗
3 and 8	11	✓

The factors needed are 3 and 8.

$(n + 3)(n + 8)$

Evaluate the original polynomial and the factored form for $n = 0, 1, 2, 3$, and 4.

n	$n^2 + 11n + 24$
0	$0^2 + 11(0) + 24 = 24$
1	$1^2 + 11(1) + 24 = 36$
2	$2^2 + 11(2) + 24 = 50$
3	$3^2 + 11(3) + 24 = 66$
4	$4^2 + 11(4) + 24 = 84$

n	$(n + 3)(n + 8)$
0	$(0 + 3)(0 + 8) = 24$
1	$(1 + 3)(1 + 8) = 36$
2	$(2 + 3)(2 + 8) = 50$
3	$(3 + 3)(3 + 8) = 66$
4	$(4 + 3)(4 + 8) = 84$

The original polynomial and the factored form have the same value for the given values of n.

CHECK IT OUT!

4. Factor $n^2 - 7n + 10$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

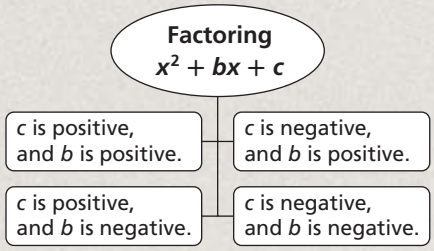
THINK AND DISCUSS

1. Explain in your own words how to factor $x^2 + 9x + 14$. Show how to check your answer.

2. Explain how you can determine the signs of the factors of c when factoring a trinomial of the form $x^2 + bx + c$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of a trinomial with the given properties and factor it.



8-3 Exercises

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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 560

Factor each trinomial by guess and check.

1. $x^2 + 13x + 36$
2. $x^2 + 11x + 24$
3. $x^2 + 14x + 40$

Factor each trinomial. Check your answer.

SEE EXAMPLE 2 p. 561

4. $x^2 + 4x + 3$
5. $x^2 + 10x + 16$
6. $x^2 + 15x + 44$
7. $x^2 - 7x + 6$
8. $x^2 - 9x + 14$
9. $x^2 - 11x + 24$

SEE EXAMPLE 3 p. 562

10. $x^2 - 6x - 7$
11. $x^2 + 6x - 27$
12. $x^2 + x - 30$
13. $x^2 - x - 2$
14. $x^2 - 3x - 18$
15. $x^2 - 4x - 45$

SEE EXAMPLE 4 p. 563

16. Factor $n^2 + 6n - 7$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–25	2
26–31	3
32	4

Extra Practice

Skills Practice p. S18
Application Practice p. S35

Factor each trinomial by guess and check.

17. $x^2 + 13x + 30$
18. $x^2 + 11x + 28$
19. $x^2 + 16x + 48$

Factor each trinomial. Check your answer.

20. $x^2 + 12x + 11$
21. $x^2 + 16x + 28$
22. $x^2 + 15x + 36$
23. $x^2 - 6x + 5$
24. $x^2 - 9x + 18$
25. $x^2 - 12x + 32$
26. $x^2 + x - 12$
27. $x^2 + 4x - 21$
28. $x^2 + 9x - 36$
29. $x^2 - 12x - 13$
30. $x^2 - 10x - 24$
31. $x^2 - 2x - 35$

32. Factor $n^2 - 12n - 45$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

Match each trinomial with its correct factorization.

33. $x^2 + 3x - 10$
34. $x^2 - 7x + 10$
35. $x^2 - 9x - 10$
36. $x^2 + 11x + 10$

A. $(x - 2)(x - 5)$
B. $(x + 1)(x + 10)$
C. $(x - 2)(x + 5)$
D. $(x + 1)(x - 10)$

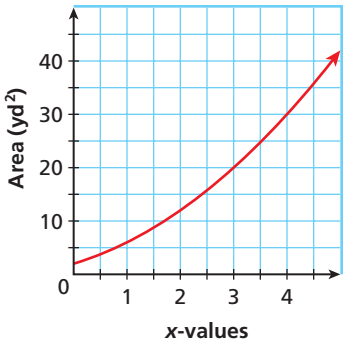
37. **Write About It** Compare multiplying binomials with factoring polynomials into binomial factors.

Factor each trinomial. Check your answer.

38. $x^2 + x - 20$
39. $x^2 - 11x + 18$
40. $x^2 - 4x - 21$
41. $x^2 + 10x + 9$
42. $x^2 - 12x + 32$
43. $x^2 + 13x + 42$
44. $x^2 - 7x + 12$
45. $x^2 + 11x + 18$
46. $x^2 - 6x - 27$
47. $x^2 + 5x - 24$
48. $x^2 - 10x + 21$
49. $x^2 + 4x - 45$

50. Factor $n^2 + 11n + 28$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

51. Estimation The graph shows the areas of rectangles with dimensions $(x + 1)$ yards and $(x + 2)$ yards. Estimate the value of x for a rectangle with area 9 square yards.



52. Geometry The area of a rectangle in square feet can be represented by $x^2 + 8x + 12$. The length is $(x + 6)$ ft. What is the width of the rectangle?

53. Remodeling A homeowner wants to enlarge a rectangular closet that has an area of $(x^2 + 3x + 2)$ ft^2. The length is $(x + 2)$ ft. After construction, the area will be $(x^2 + 8x + 15)$ ft^2 with a length of $(x + 3)$ ft.

a. Find the dimensions of the closet before construction.

b. Find the dimensions of the closet after construction.

c. By how many feet will the length and width increase after construction?

LINK

Art

The Dutch painter Theo van Doesburg (1883–1931) is most famous for his paintings composed of lines and rectangles, such as the one shown above.

Art **Write the polynomial modeled and then factor.**

54.

55.

56.

Copy and complete the table.

	$x^2 + bx + c$	Sign of c	Binomial Factors	Signs of Numbers in Binomials
	$x^2 + 4x + 3$	Positive	$(x + 1)(x + 3)$	Both positive
57.	$x^2 - 4x + 3$		$(x \square 1)(x \square 3)$	
58.	$x^2 + 2x - 3$		$(x \square 1)(x \square 3)$	
59.	$x^2 - 2x - 3$		$(x \square 1)(x \square 3)$	

60. Geometry A rectangle has area $x^2 + 6x + 8$. The length is $x + 4$. Find the width of the rectangle. Could the rectangle be a square? Explain why or why not.

MULTI-STEP TEST PREP

61. This problem will prepare you for the Multi-Step Test Prep on page 576.

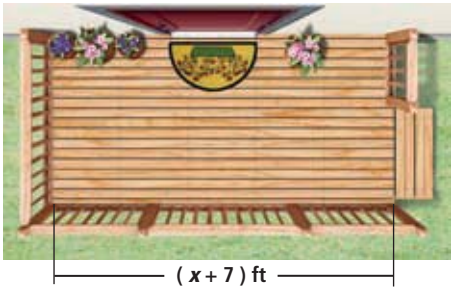
The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in feet per second, a is acceleration in feet per second squared, and t is time in seconds.

a. Janna has two toy race cars on a track. One starts with a velocity of 0 ft/s and accelerates at 2 ft/s^2. Write an equation for the distance the car travels in time t.

b. The second car travels at a constant speed of 4 ft/s. Write an equation for the distance the second car travels in time t. (*Hint:* When speed is constant, the acceleration is 0 ft/s^2.)

c. By setting the equations equal to each other you can determine when the cars have traveled the same distance: $t^2 = 4t$. This can be written as $t^2 - 4t = 0$. Factor the left side of the equation.

62. Construction The length of a rectangular porch is $(x + 7)$ ft. The area of the porch is $(x^2 + 9x + 14)$ ft^2. Find the width of the porch.



Tell whether each statement is true or false. If false, explain.

63. The third term in a factorable trinomial is equal to the product of the constants in its binomial factors.

64. The constants in the binomial factors of $x^2 + x - 2$ are both negative.

65. The correct factorization of $x^2 - 3x - 4$ is $(x + 4)(x - 1)$.

66. All trinomials of the form $x^2 + bx + c$ can be factored.

Fill in the missing part of each factorization.

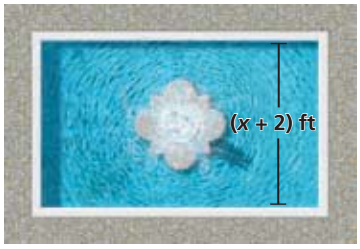
67. $x^2 - 6x + 8 = (x - 2)(x - \square)$

68. $x^2 - 2x - 8 = (x + 2)(x - \square)$

69. $x^2 + 2x - 8 = (x - 2)(x + \square)$

70. $x^2 + 6x + 8 = (x + 2)(x + \square)$

71. Construction The area of a rectangular fountain is $(x^2 + 12x + 20)$ ft^2. The width is $(x + 2)$ ft.



a. Find the length of the fountain.

b. A 2-foot walkway is built around the fountain. Find the dimensions of the outside border of the walkway.

c. Find the total area covered by the fountain and walkway.

72. Critical Thinking Find all possible values of b so that $x^2 + bx + 6$ can be factored into binomial factors.

TEST PREP

73. Which is the correct factorization of $x^2 - 10x - 24$?

Ⓐ $(x - 4)(x - 6)$ Ⓒ $(x - 2)(x + 12)$

Ⓑ $(x + 4)(x - 6)$ Ⓓ $(x + 2)(x - 12)$

74. Which value of b would make $x^2 + bx - 20$ factorable?

Ⓕ 9 Ⓖ 12 Ⓗ 19 Ⓙ 21

75. Which value of b would NOT make $x^2 + bx - 36$ factorable?

Ⓐ 5 Ⓑ 9 Ⓒ 15 Ⓓ 16

76. Short Response What are the factors of $x^2 + 2x - 24$? Show and explain each step of factoring the polynomial.

CHALLENGE AND EXTEND

Factor each expression.

77. $x^4 + 18x^2 + 81$ **78.** $y^4 - 5y^2 - 24$ **79.** $d^4 + 22d^2 + 21$

80. $(u + v)^2 + 2(u + v) - 3$ **81.** $(de)^2 - (de) - 20$ **82.** $(m - n)^2 - 4(m - n) - 45$

83. Find all possible values of b such that, when $x^2 + bx + 28$ is factored, both constants in the binomials are positive.

84. Find all possible values of b such that, when $x^2 + bx + 32$ is factored, both constants in the binomials are negative.

85. The area of Beth's rectangular garden is $(x^2 + 13x + 42)$ ft^2. The width is $(x + 6)$ ft.

Item	Cost
Fertilizer	0.28 ($/ft^2)
Fencing	2.00 ($/ft)

a. What is the length of the garden?

b. Find the perimeter in terms of x.

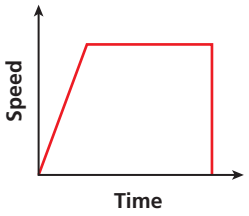
c. Find the cost to fence the garden when x is 5.

d. Find the cost of fertilizer when x is 5.

e. Find the total cost to fence and fertilize Beth's garden when x is 5.

SPIRAL REVIEW

86. Choose the situation that best describes the graph. *(Lesson 4-1)*



A. An object increases speed, stops, and then moves in reverse.

B. An object starts at rest, increases speed steadily, maintains constant speed, and then comes to an immediate stop.

C. An object increases speed quickly, then increases speed slowly, and then comes to an immediate stop.

Simplify. *(Lesson 7-3)*

87. x^3x^2 **88.** $m^8n^3m^{-12}$ **89.** $(t^4)^3$ **90.** $(-2xy^3)^5$

Factor each polynomial by grouping. *(Lesson 8-2)*

91. $x^3 + 2x^2 + 5x + 10$ **92.** $2n^3 - 8n^2 - 3n + 12$

93. $2p^4 - 4p^3 + 7p - 14$ **94.** $x^3 - 4x^2 + x - 4$

Career Path

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Jessica Rubino
Environmental Sciences major

Q: What math classes did you take in high school?
A: Algebra 1, Algebra 2, and Geometry

Q: What college math classes have you taken?
A: I took several computer modeling and programming classes as well as Statistics and Probability.

Q: How is math used in some of your projects?
A: Computer applications help me analyze data collected from a local waste disposal site. I used my mathematical knowledge to make recommendations on how to preserve surrounding water supplies.

Q: What plans do you have for the future?
A: I enjoy my studies in the area of water pollution. I would also like to research more efficient uses of natural energy resources.