CHAPTER 8

Factoring Polynomials

Chapter Focus

- Factor polynomials.
- Apply factoring techniques to solve problems involving area and volume.

High Fliers

You can use polynomials to model area. When given the area of a kite as a polynomial, you can factor to find the kite's dimensions.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. binomial

2. composite number

3. factor

4. multiple

5. prime number

A. a whole number greater than 1 that has more than two positive factors

B. a polynomial with two terms

C. the product of any number and a whole number

D. a number that is written as the product of its prime factors

E. a whole number greater than 1 that has exactly two positive factors, itself and 1

F. a number that is multiplied by another number to get a product

Multiples

Write the first four multiples of each number.

6. 3 **7.** 4 **8.** 8 **9.** 15

Factors

Tell whether the second number is a factor of the first number.

10. 20, 5 **11.** 50, 6 **12.** 120, 8 **13.** 245, 7

Prime and Composite Numbers

Tell whether each number is prime or composite. If the number is composite, write it as the product of two numbers.

14. 2 **15.** 7 **16.** 10 **17.** 38

18. 115 **19.** 147 **20.** 151 **21.** 93

Multiply Monomials and Polynomials

Multiply.

22. $2(x + 5)$ **23.** $3h(h + 1)$ **24.** $xy(x^2 - xy^3)$ **25.** $6m(m^2 - 4m - 1)$

Multiply Binomials

Find each product.

26. $(x + 3)(x + 8)$ **27.** $(b - 7)(b + 1)$

28. $(2p - 5)(p - 1)$ **29.** $(3n + 4)(2n + 3)$

CHAPTER 8

Study Guide: Preview

Where You've Been

Previously, you

- used properties of exponents to evaluate and simplify expressions.
- added and subtracted polynomials by combining like terms.
- multiplied polynomials.

In This Chapter

You will study

- greatest common factors.
- how to factor polynomials.
- how to factor special products.
- how to choose a factoring method.

Where You're Going

You can use the skills in this chapter

- in Geometry to solve area problems.
- in Physics to solve quadratic equations.
- in the real world to calculate dimensions in landscaping, construction, or design work.

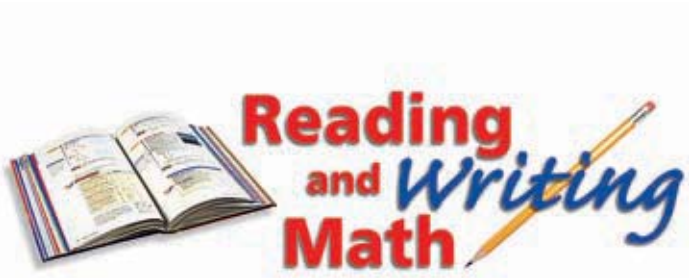
Key Vocabulary/Vocabulario

greatest common factor	máximo común divisor
prime factorization	factorización prima

Vocabulary Connections

To become familiar with the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *factor* refers to a number or polynomial that is multiplied by another number or polynomial to form a product. What do you think the word *factor* means when it is used as a verb (action word)?
2. List some words that end with the suffixes *-ize* or *-ization*. What does the ending *-ization* seem to mean? What do you think *factorization* means?
3. The words *prime, primer, primary*, and *primitive* all come from the same root word. What are the meanings of these words? How can their meanings help you understand what a *prime factor* is?
4. What is a prime number? How might the **prime factorization** of a number differ from another factorization?
5. What does the word *common* mean? How can you use this meaning to understand the term **greatest common factor**?



CHAPTER 8

Reading Strategy: Read a Lesson for Understanding

To help you learn new concepts, you should read each lesson with a purpose. As you read a lesson, make notes. Include the main ideas of the lesson and any questions you have. In class, listen for explanations of the vocabulary, clarification of the examples, and answers to your questions.

Reading Tips

Objectives
Evaluate and multiply by powers of 10.

Convert between standard notation and scientific notation.

The objectives tell you the main idea of the lesson.

If a power of 10 has a negative integer exponent, does that make the number negative?
How do I enter numbers written in scientific notation into my calculator?

Write down questions you have as you read the lesson.

EXAMPLE 1 Evaluating Powers of 10

Find the value of each power of 10.

A 10^{-3}

Start with 1 and move the decimal point three places to the left.

0. 0 0 1

0.001

Work through the examples and write down any questions you have.

CHECK IT OUT!

Practice what you've learned in the Check It Out sections.

Try This

Read Lesson 8-1 prior to your next class. Then answer the questions below.

1. What are the lesson objectives?
2. What vocabulary, formulas, and symbols are new?
3. Which examples, if any, are unclear?
4. What questions do you have about the lesson?

8-1 Factors and Greatest Common Factors

Objectives
Write the prime factorization of numbers.

Find the GCF of monomials.

Vocabulary
prime factorization
greatest common factor

Who uses this?

Web site designers who sell electronic greeting cards can use the greatest common factor of numbers to design their Web sites. (See Example 4.)

The numbers that are multiplied to find a product are called *factors* of that product. A number is divisible by its factors.

You can use the factors of a number to write the number as a product. The number 12 can be factored several ways.

Factorizations of 12

$1 \cdot 12 \quad 2 \cdot 6 \quad 3 \cdot 4 \quad 1 \cdot 4 \cdot 3 \quad 2 \cdot 2 \cdot 3$

Remember!

A prime number is a whole number that has exactly two positive factors, itself and 1. The number 1 is not prime because it only has one positive factor.

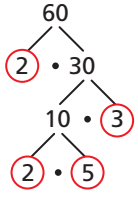
The order of the factors does not change the product, but there is only one example above that cannot be factored further. The circled factorization is the **prime factorization** because all the factors are prime numbers. The prime factors can be written in any order, and, except for changes in the order, there is only one way to write the prime factorization of a number.

EXAMPLE 1 Writing Prime Factorizations

Write the prime factorization of 60.

Method 1 Factor tree

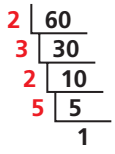
Choose any two factors of 60 to begin. Keep finding factors until each branch ends in a prime factor.



$60 = 2 \cdot 2 \cdot 5 \cdot 3$

Method 2 Ladder diagram

Choose a prime factor of 60 to begin. Keep dividing by prime factors until the quotient is 1.



$60 = 2 \cdot 3 \cdot 2 \cdot 5$

The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$ or $2^2 \cdot 3 \cdot 5$.

CHECK IT OUT!

Write the prime factorization of each number.

1a. 40 **1b.** 33 **1c.** 49 **1d.** 19

Factors that are shared by two or more whole numbers are called *common factors*. The greatest of these common factors is called the **greatest common factor**, or GCF.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 32: 1, 2, 4, 8, 16, 32
Common factors: 1, 2, 4
The greatest of the common factors is 4.

EXAMPLE 2 Finding the GCF of Numbers

Find the GCF of each pair of numbers.

A **24 and 60**

Method 1 List the factors.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 — *List all the factors.*
factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 — *Circle the GCF.*

The GCF of 24 and 60 is 12.

B **18 and 27**

Method 2 Use prime factorization.

$18 = 2 \cdot 3 \cdot 3$ — *Write the prime factorization of each number.*
$27 = 3 \cdot 3 \cdot 3$ — *Align the common factors.*
$3 \cdot 3 = 9$

The GCF of 18 and 27 is 9.

Find the GCF of each pair of numbers.

2a. 12 and 16 **2b.** 15 and 25

You can also find the GCF of monomials that include variables. To find the GCF of monomials, write the prime factorization of each coefficient and write all powers of variables as products. Then find the product of the common factors.

EXAMPLE 3 Finding the GCF of Monomials

Find the GCF of each pair of monomials.

A **$3x^3$ and $6x^2$**

$3x^3 = 3 \cdot x \cdot x \cdot x$ — *Write the prime factorization of each coefficient and write powers as products.*
$6x^2 = 2 \cdot 3 \cdot x \cdot x$ — *Align the common factors.*
$3 \cdot x \cdot x = 3x^2$ — *Find the product of the common factors.*

The GCF of $3x^3$ and $6x^2$ is $3x^2$.

B **$4x^2$ and $5y^3$**

$4x^2 = 2 \cdot 2 \cdot x \cdot x$ — *Write the prime factorization of each coefficient and write powers as products.*
$5y^3 = 5 \cdot y \cdot y \cdot y$ — *Align the common factors.*
There are no common factors other than 1.

The GCF of $4x^2$ and $5y^3$ is 1.

Helpful Hint

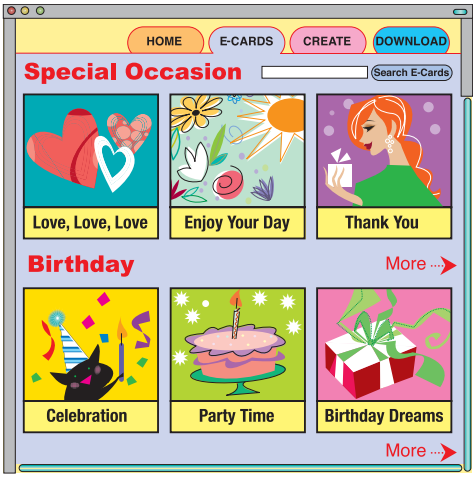
If two terms contain the same variable raised to different powers, the GCF will contain that variable raised to the lower power.

Find the GCF of each pair of monomials.

3a. $18g^2$ and $27g^3$ **3b.** $16a^6$ and $9b$ **3c.** $8x$ and $7v^2$

EXAMPLE 4 *Technology Application*

Garrison is creating a Web page that offers electronic greeting cards. He has 24 special occasion designs and 42 birthday designs. The cards will be displayed with the same number of designs in each row. Special occasion and birthday designs will not appear in the same row. How many rows will there be if Garrison puts the greatest possible number of designs in each row?



The 24 special occasion designs and 42 birthday designs must be divided into groups of equal size. The number of designs in each row must be a common factor of 24 and 42.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

Find the common factors of 24 and 42.

The GCF of 24 and 42 is 6.

The greatest possible number of designs in each row is 6. Find the number of rows of each group of designs when there are 6 designs in each row.

$$\frac{24 \text{ special occasion designs}}{6 \text{ designs per row}} = 4 \text{ rows}$$

$$\frac{42 \text{ birthday designs}}{6 \text{ designs per row}} = 7 \text{ rows}$$

When the greatest possible number of designs is in each row, there are 11 rows in total.

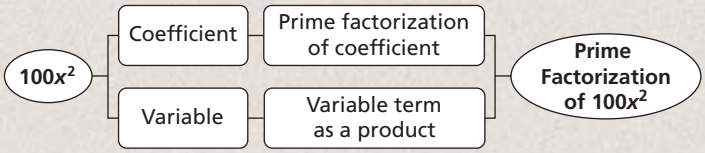
CHECK IT OUT!

4. Adrianne is shopping for a CD storage unit. She has 36 CDs by pop music artists and 48 CDs by country music artists. She wants to put the same number of CDs on each shelf without putting pop music and country music CDs on the same shelf. If Adrianne puts the greatest possible number of CDs on each shelf, how many shelves does her storage unit need?

THINK AND DISCUSS

1. Describe two ways you can find the prime factorization of a number.

2. GET ORGANIZED Copy and complete the graphic organizer. Show how to write the prime factorization of $100x^2$ by filling in each box.



Know it! Note

8-1 Exercises

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GUIDED PRACTICE

1. Vocabulary Define the term *greatest common factor* in your own words.

SEE EXAMPLE 1 p. 544

Write the prime factorization of each number.

2. 20 **3.** 36 **4.** 27 **5.** 54

6. 96 **7.** 7 **8.** 100 **9.** 75

SEE EXAMPLE 2 p. 545

Find the GCF of each pair of numbers.

10. 12 and 60 **11.** 14 and 49 **12.** 55 and 121

SEE EXAMPLE 3 p. 545

Find the GCF of each pair of monomials.

13. $6x^2$ and $5x^2$ **14.** $15y^3$ and $-20y$ **15.** $13q^4$ and $2p^2$

SEE EXAMPLE 4 p. 546

16. Samantha is making beaded necklaces using 54 glass beads and 18 clay beads. She wants each necklace to have the same number of beads, but each necklace will have only one type of bead. If she puts the greatest possible number of beads on each necklace, how many necklaces can she make?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–24	1
25–27	2
28–30	3
31	4

Extra Practice

Skills Practice p. S18
Application Practice p. S35

Write the prime factorization of each number.

17. 18 **18.** 64 **19.** 12 **20.** 150

21. 17 **22.** 226 **23.** 49 **24.** 63

Find the GCF of each pair of numbers.

25. 36 and 63 **26.** 14 and 15 **27.** 30 and 40

Find the GCF of each pair of monomials.

28. $8a^2$ and 11 **29.** $9s$ and $63s^3$ **30.** $-64n^4$ and $24n^2$

31. José is making fruit-filled tart shells for a party. He has 72 raspberries and 108 blueberries. The tarts will each have the same number of berries. Raspberries and blueberries will not be in the same tart. If he puts the greatest possible number of fruits in each tart, how many tarts can he make?

Find the GCF of each pair of products.

32. $3 \cdot 5 \cdot t$ and $2 \cdot 2 \cdot 5 \cdot t \cdot t$ **33.** $-1 \cdot 2 \cdot 2 \cdot x \cdot x$ and $2 \cdot 2 \cdot 7 \cdot x \cdot x \cdot x$

34. $2 \cdot 2 \cdot 2 \cdot 11 \cdot x \cdot x \cdot x$ and $3 \cdot 11$ **35.** $2 \cdot 5 \cdot n \cdot n \cdot n$ and $-1 \cdot 2 \cdot 3 \cdot n$

36. Write About It The number 2 is even and is prime. Explain why all other prime numbers are odd numbers.

37. **Critical Thinking** The GCF of two numbers is 1. Explain whether this means the two numbers must be prime.

38. **Multi-Step** Angelo is making a rectangular floor for a clubhouse with an area of 84 square feet. The length of each side of the floor is a whole number of feet.

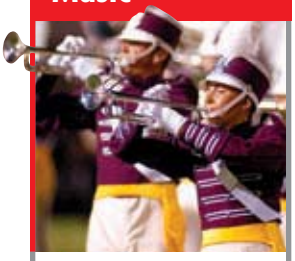
a. What are the possible lengths and widths for Angelo's clubhouse floor?

b. What is the minimum perimeter for the clubhouse floor?

c. What is the maximum perimeter for the clubhouse floor?

LINK

Music



DCI is a nonprofit organization that oversees drum and bugle corps performances and competitions for youths between the ages of 14 and 21.

39. **Music** The Cavaliers and the Blue Devils are two of the marching bands that are members of Drum Corps International (DCI). DCI bands are made up of percussionists, brass players, and color guard members who use flags and other props.

In 2004, there were 35 color guard members in the Cavaliers and 40 in the Blue Devils. The two color guards will march in rows with the same number of people in each row without mixing the guards together. If the greatest possible number of people are in each row, how many rows will there be?

For each set of numbers, determine which two numbers have a GCF greater than 1, and find that GCF.

40. 11, 12, 14

41. 8, 20, 63

42. 16, 21, 27

43. 32, 63, 105

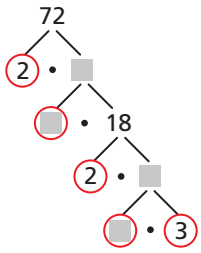
44. 25, 35, 54

45. 35, 54, 72

46. **Number Sense** The prime factorization of 24 is $2^3 \cdot 3$. Without using a diagram, write the prime factorization of 48. Explain your reasoning.

Fill in each diagram. Then write the prime factorization of the number.

47.



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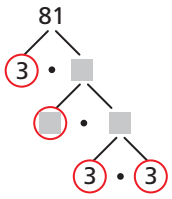

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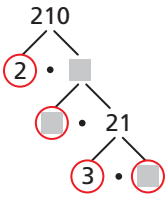

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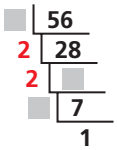

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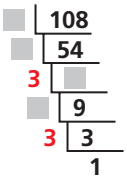

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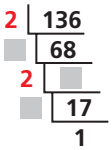

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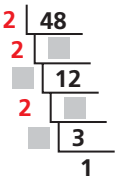

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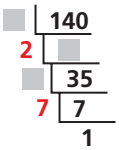

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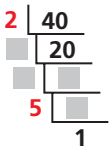


MULTI-STEP TEST PREP

56. This problem will prepare you for the Multi-Step Test Prep on page 576.

The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in ft/s, a is acceleration in ft/s^2, and t is time in seconds.

a. A toy car begins with a velocity of 2 ft/s and accelerates at 2 ft/s^2. Write an expression for the distance the toy car travels after t seconds.

b. What is the GCF of the terms in your expression from part **a**?

TEST PREP

57. Which set of numbers has a GCF greater than 6?

Ⓐ 18, 24, 36 Ⓑ 30, 35, 40 Ⓒ 11, 29, 37 Ⓓ 16, 24, 48

58. The slope of a line is the GCF of 48 and 12. The y-intercept is the GCF of the slope and 8. Which equation describes the line?

Ⓕ $y = 12x + 4$ Ⓖ $y = 6x + 2$ Ⓗ $y = 4x + 4$ Ⓙ $y = 3x + 1$

59. **Extended Response** Patricia is making a dog pen in her back yard. The pen will be rectangular and have an area of 24 square feet. Draw and label a diagram that shows all possible whole-number dimensions for the pen. Find the perimeter of each rectangle you drew. Which dimensions should Patricia use in order to spend the least amount of money on fencing materials? Explain your reasoning.

CHALLENGE AND EXTEND

Find the GCF of each set.

60. $4n^3, 16n^2, 8n$

61. $27y^3, 18y^2, 81y$

62. $100, 25s^5, 50s$

63. $2p^4r, 8p^3r^2, 16p^2r^3$

64. $2x^3y, 8x^2y^2, 17xy^3$

65. $8a^4b^3, 4a^3b^3, 12a^2b^3$

66. **Geometry** The area of a triangle is 10 in^2. What are the possible whole-number dimensions for the base and height of the triangle?

67. **Number Sense** The GCF of three different numbers is 7. The sum of the three numbers is 105. What are the three numbers?

68. **Critical Thinking** Find three different *composite* numbers whose GCF is 1. (*Hint:* A composite number has factors other than 1 and itself.)

SPIRAL REVIEW

Use dimensional analysis to convert each measure. *(Lesson 2-7)*

69. 11,232 s to h

70. 9 gal to pt

71. 18 in./day to mi/yr

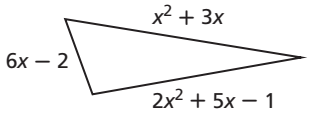
Find each value. Round to the nearest tenth if necessary. *(Lesson 2-9)*

72. 40% of 60

73. 250% of 16

74. What percent of 80 is 20?

75. Write a simplified polynomial expression for the perimeter of the triangle. *(Lesson 7-7)*



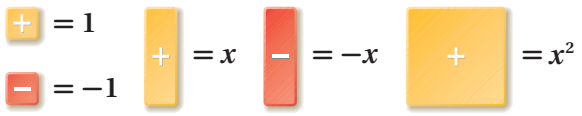
8-2 Algebra LAB

Model Factoring

You can use algebra tiles to write a polynomial as the product of its factors. This process is called *factoring*. Factoring is the reverse of multiplying.

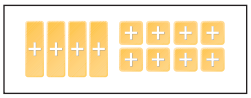
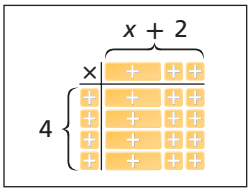
Use with Lesson 8-2

KEY

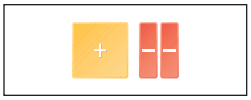
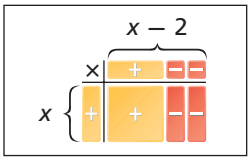


Activity

Use algebra tiles to factor $4x + 8$.

MODEL		ALGEBRA
	*Model 4x + 8.*	$4x + 8$
	*Arrange the tiles into a rectangle. The total area represents 4x + 8. The length and width represent the factors. The rectangle has a width of x + 2 and a length of 4.*	$4x + 8 = 4(x + 2)$

Use algebra tiles to factor $x^2 - 2x$.

MODEL		ALGEBRA
	*Model $x^2 - 2x$.*	$x^2 - 2x$
	*Arrange the tiles into a rectangle. The total area represents $x^2 - 2x$. The length and width represent the factors. The rectangle has a width of $x - 2$ and a length of x.*	$x^2 - 2x = x(x - 2)$

Try This

Use algebra tiles to factor each polynomial.

1. $3x + 9$
2. $2x + 8$
3. $4x - 12$
4. $3x - 12$
5. $2x^2 + 2x$
6. $x^2 + 4x$
7. $x^2 - 3x$
8. $2x^2 - 4x$