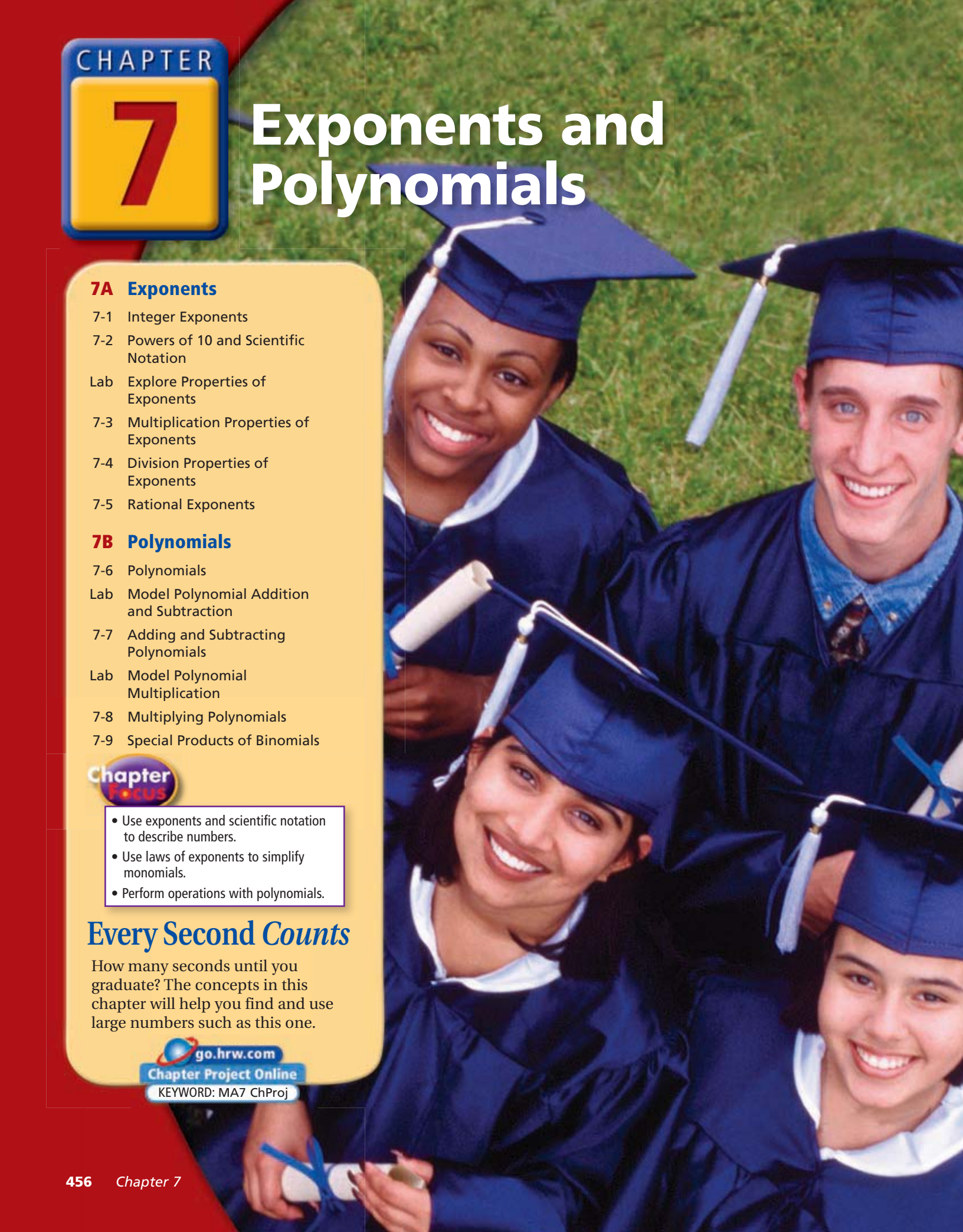
CHAPTER 7

Exponents and Polynomials



Chapter Focus

- Use exponents and scientific notation to describe numbers.
- Use laws of exponents to simplify monomials.
- Perform operations with polynomials.

Every Second *Counts*

How many seconds until you graduate? The concepts in this chapter will help you find and use large numbers such as this one.

go.hrw.com
Chapter Project Online
KEYWORD: MA7 ChProj

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. Associative Property	**A.** a number that is raised to a power
2. coefficient	**B.** a number that is multiplied by a variable
3. Commutative Property	**C.** a property of addition and multiplication that states you can add or multiply numbers in any order
4. exponent	**D.** the number of times a base is used as a factor
5. like terms	**E.** terms that contain the same variables raised to the same powers
	F. a property of addition and multiplication that states you can group the numbers in any order

Exponents

Write each expression using a base and an exponent.

6. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **7.** $5 \cdot 5$ **8.** $(-10)(-10)(-10)(-10)$

9. $x \cdot x \cdot x$ **10.** $k \cdot k \cdot k \cdot k \cdot k$ **11.** 9

Evaluate Powers

Evaluate each expression.

12. 3^4 **13.** -12^2 **14.** 5^3

15. 2^5 **16.** 4^3 **17.** $(-1)^6$

Multiply Decimals

Multiply.

18. 0.006×10 **19.** 25.25×100 **20.** 2.4×6.5

Combine Like Terms

Simplify each expression.

21. $6 + 3p + 14 + 9p$ **22.** $8y - 4x + 2y + 7x - x$

23. $(12 + 3w - 5) + 6w - 3 - 5w$ **24.** $6n - 14 + 5n$

Squares and Square Roots

Tell whether each number is a perfect square. If so, identify its positive square root.

25. 42 **26.** 81 **27.** 36 **28.** 50

29. 100 **30.** 4 **31.** 1 **32.** 12

CHAPTER 7

Study Guide: Preview

Where You've Been

Previously, you

- wrote and evaluated exponential expressions.
- simplified algebraic expressions by combining like terms.

In This Chapter

You will study

- properties of exponents.
- powers of 10 and scientific notation.
- how to add, subtract, and multiply polynomials by using properties of exponents and combining like terms.

Where You're Going

You can use the skills in this chapter

- to model area, perimeter, and volume in geometry.
- to express very small or very large quantities in science classes such as Chemistry, Physics, and Biology.
- in the real world to model business profits and population growth or decline.

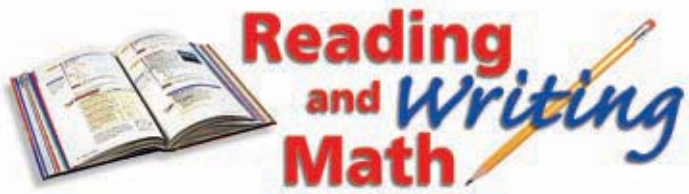
Key Vocabulary/Vocabulario

binomial	binomio
degree of a monomial	grado de un monomio
degree of a polynomial	grado de un polinomio
leading coefficient	coeficiente principal
monomial	monomio
perfect-square trinomial	trinomio cuadrado perfecto
polynomial	polinomio
scientific notation	notación científica
standard form of a polynomial	forma estándar de un polinomio
trinomial	trinomio

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. Very large and very small numbers are often encountered in the sciences. If *notation* means a method of writing something, what might **scientific notation** mean?
2. A **polynomial** written in standard form may have more than one algebraic term. What do you think the **leading coefficient** of a polynomial is?
3. A simple definition of **monomial** is "an expression with exactly one term." If the prefix *mono-* means "one" and the prefix *bi-* means "two," define the word **binomial**.
4. What words do you know that begin with the prefix *tri-*? What do they all have in common? Define the word **trinomial** based on the prefix *tri-* and the information given in Problem 3.



CHAPTER 7

Reading Strategy: Read and Understand the Problem

Follow this strategy when solving word problems.

- Read the problem through once.
- Identify exactly what the problem asks you to do.
- Read the problem again, slowly and carefully, to break it into parts.
- Highlight or underline the key information.
- Make a plan to solve the problem.

From Lesson 6-6

29. **Multi-Step** Linda works at a pharmacy for $15 an hour. She also baby-sits for $10 an hour. Linda needs to earn at least $90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

Step 1	**Identify exactly what the problem asks you to do.**	• Show and describe the number of hours Linda can work at each job and earn at least $90 per week, without working more than 20 hours per week. • List two possible solutions of the system.
Step 2	**Break the problem into parts. Highlight or underline the key information.**	• Linda has two jobs. She makes **$15 per hour** at one job and **$10 per hour** at the other job. • She wants to earn **at least $90 per week.** • She does **not** want to work **more than 20 hours per week.**
Step 3	**Make a plan to solve the problem.**	• Write a system of inequalities. • Solve the system. • Identify two possible solutions of the system.

Try This

For the problem below,

a. identify exactly what the problem asks you to do.

b. break the problem into parts. Highlight or underline the key information.

c. make a plan to solve the problem.

1. The difference between the length and the width of a rectangle is 14 units. The area is 120 square units. Write and solve a system of equations to determine the length and the width of the rectangle. (*Hint:* The formula for the area of a rectangle is $A = \ell w$.)

7-1 Integer Exponents

Objectives

Evaluate expressions containing zero and integer exponents.

Simplify expressions containing zero and integer exponents.

Who uses this?

Manufacturers can use negative exponents to express very small measurements.

In 1930, the Model A Ford was one of the first cars to boast precise craftsmanship in mass production. The car's pistons had a diameter of $3\frac{7}{8}$ inches; this measurement could vary by at most 10^{-3} inch.

You have seen positive exponents. Recall that to simplify 3^2, use 3 as a factor 2 times: $3^2 = 3 \cdot 3 = 9$.

Remember!

Base x^4 Exponent

But what does it mean for an exponent to be negative or 0? You can use a table and look for a pattern to figure it out.

Power	5^5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}
Value	3125	625	125	25	5			

÷ 5 ÷ 5 ÷ 5 ÷ 5

When the exponent decreases by one, the value of the power is divided by 5. Continue the pattern of dividing by 5:

$$5^0 = \frac{5}{5} = 1 \qquad 5^{-1} = \frac{1}{5} = \frac{1}{5^1} \qquad 5^{-2} = \frac{1}{5} \div 5 = \frac{1}{25} = \frac{1}{5^2}$$

Know it! Note

Integer Exponents

WORDS	NUMBERS	ALGEBRA
Zero exponent—Any nonzero number raised to the zero power is 1.	$3^0 = 1$ $123^0 = 1$ $(-16)^0 = 1$ $\left(\frac{3}{7}\right)^0 = 1$	If $x \neq 0$, then $x^0 = 1$.
Negative exponent—A nonzero number raised to a negative exponent is equal to 1 divided by that number raised to the opposite (positive) exponent.	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$	If $x \neq 0$ and n is an integer, then $x^{-n} = \frac{1}{x^n}$.

Reading Math

2^{-4} is read "2 to the negative fourth power."

Notice the phrase "nonzero number" in the table above. This is because 0^0 and 0 raised to a negative power are both undefined. For example, if you use the pattern given above the table with a base of 0 instead of 5, you would get $0^0 = \frac{0}{0}$. Also, 0^{-6} would be $\frac{1}{0^6} = \frac{1}{0}$. Since division by 0 is undefined, neither value exists.

EXAMPLE 1 Manufacturing Application

The diameter for the Model A Ford piston could vary by at most 10^{-3} inch. Simplify this expression.

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1000}$$

10^{-3} inch is equal to $\frac{1}{1000}$ inch, or 0.001 inch.

1. A sand fly may have a wingspan up to 5^{-3} m. Simplify this expression.

EXAMPLE 2 Zero and Negative Exponents

Simplify.

A 2^{-3}

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$$

B 5^0

$5^0 = 1$ *Any nonzero number raised to the zero power is 1.*

C $(-3)^{-4}$

$$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-3)(-3)(-3)(-3)} = \frac{1}{81}$$

D -3^{-4}

$$-3^{-4} = -\frac{1}{3^4} = -\frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = -\frac{1}{81}$$

Caution!

In $(-3)^{-4}$, the base is negative because the negative sign is inside the parentheses.

In -3^{-4} the base (3) is positive.

Simplify.

2a. 10^{-4} **2b.** $(-2)^{-4}$ **2c.** $(-2)^{-5}$ **2d.** -2^{-5}

EXAMPLE 3 Evaluating Expressions with Zero and Negative Exponents

Evaluate each expression for the given value(s) of the variable(s).

A x^{-1} **for** $x = 2$

2^{-1} *Substitute 2 for x.*

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ *Use the definition* $x^{-n} = \frac{1}{x^n}$.

B a^0b^{-3} **for** $a = 8$ **and** $b = -2$

$8^0 \cdot (-2)^{-3}$ *Substitute 8 for a and –2 for b.*

$1 \cdot \frac{1}{(-2)^3}$ *Simplify expressions with exponents.*

$1 \cdot \frac{1}{(-2)(-2)(-2)}$ *Write the power in the denominator as a product.*

$1 \cdot \frac{1}{-8}$ *Simplify the power in the denominator.*

$-\frac{1}{8}$ *Simplify.*

Evaluate each expression for the given value(s) of the variable(s).

3a. p^{-3} for $p = 4$ **3b.** $8a^{-2}b^0$ for $a = -2$ and $b = 6$

What if you have an expression with a negative exponent in a denominator, such as $\frac{1}{x^{-8}}$?

$$x^{-n} = \frac{1}{x^n}, \text{ or } \frac{1}{x^n} = x^{-n} \qquad \textit{Definition of negative exponent}$$

$$\frac{1}{x^{-8}} = x^{-(-8)} \qquad \textit{Substitute } -8 \textit{ for } n.$$

$$= x^8 \qquad \textit{Simplify the exponent on the right side.}$$

So if a base with a negative exponent is in a denominator, it is equivalent to the same base with the opposite (positive) exponent in the numerator.

An expression that contains negative or zero exponents is not considered to be simplified. Expressions should be rewritten with only positive exponents.

EXAMPLE 4 Simplifying Expressions with Zero and Negative Exponents

Simplify.

A $3y^{-2}$

$$3y^{-2} = 3 \cdot y^{-2}$$
$$= 3 \cdot \frac{1}{y^2}$$
$$= \frac{3}{y^2}$$

B $\frac{-4}{k^{-4}}$

$$\frac{-4}{k^{-4}} = -4 \cdot \frac{1}{k^{-4}}$$
$$= -4 \cdot k^4$$
$$= -4k^4$$

C $\frac{x^{-3}}{a^0y^5}$

$$\frac{x^{-3}}{a^0y^5} = \frac{1}{x^3 \cdot 1 \cdot y^5} \qquad a^0 = 1 \text{ and } x^{-3} = \frac{1}{x^3}.$$
$$= \frac{1}{x^3y^5}$$

CHECK IT OUT! **Simplify.**

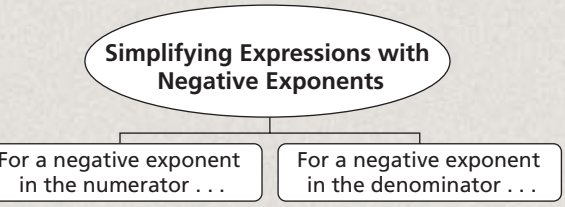
4a. $2r^0m^{-3}$ **4b.** $\frac{r^{-3}}{7}$ **4c.** $\frac{g^4}{h^{-6}}$

THINK AND DISCUSS

1. Complete each equation: $2b^{?} = \frac{2}{b^2}$, $\frac{s^{-3}}{k^{?}} = \frac{1}{s^3}$, $?^{-2} = \frac{1}{t^2}$

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to simplify, and give an example.



7-1 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 7-1
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 461

1. **Medicine** A typical virus is about 10^{-7} m in size. Simplify this expression.

SEE EXAMPLE 2 p. 461

Simplify.

2. 6^{-2} 3. 3^{0} 4. -5^{-2} 5. 3^{-3} 6. 1^{-8}
7. -8^{-3} 8. 10^{-2} 9. $(4.2)^{0}$ 10. $(-3)^{-3}$ 11. 4^{-2}

SEE EXAMPLE 3 p. 461

Evaluate each expression for the given value(s) of the variable(s).

12. b^{-2} for $b = -3$
13. $(2t)^{-4}$ for $t = 2$
14. $(m - 4)^{-5}$ for $m = 6$
15. $2x^{0}y^{-3}$ for $x = 7$ and $y = -4$

SEE EXAMPLE 4 p. 462

Simplify.

16. $4m^{0}$ 17. $3k^{-4}$ 18. $\frac{7}{r^{-7}}$ 19. $\frac{x^{10}}{d^{-3}}$
20. $2x^{0}y^{-4}$ 21. $\frac{f^{-4}}{g^{-6}}$ 22. $\frac{c^{4}}{d^{-3}}$ 23. $p^{7}q^{-1}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
24	1
25–36	2
37–42	3
43–57	4

Extra Practice

Skills Practice p. S16
Application Practice p. S34

24. **Biology** One of the smallest bats is the northern blossom bat, which is found from Southeast Asia to Australia. This bat weighs about 2^{-1} ounce. Simplify this expression.

Simplify.

25. 8^{0} 26. 5^{-4} 27. 3^{-4} 28. -9^{-2}
29. -6^{-2} 30. 7^{-2} 31. $\left(\frac{2}{5}\right)^{0}$ 32. 13^{-2}
33. $(-3)^{-1}$ 34. $(-4)^{2}$ 35. $\left(\frac{1}{2}\right)^{-2}$ 36. -7^{-1}

Evaluate each expression for the given value(s) of the variable(s).

37. x^{-4} for $x = 4$
38. $\left(\frac{2}{3}v\right)^{-3}$ for $v = 9$
39. $(10 - d)^{0}$ for $d = 11$
40. $10m^{-1}n^{-5}$ for $m = 10$ and $n = -2$
41. $(3ab)^{-2}$ for $a = \frac{1}{2}$ and $b = 8$
42. $4w^{v}x^{v}$ for $w = 3$, $v = 0$, and $x = -5$

Simplify.

43. k^{-4} 44. $2z^{-8}$ 45. $\frac{1}{2b^{-3}}$ 46. $c^{-2}d$ 47. $-5x^{-3}$
48. $4x^{-6}y^{-2}$ 49. $\frac{2f^{0}}{7g^{-10}}$ 50. $\frac{r^{-5}}{s^{-1}}$ 51. $\frac{s^{5}}{t^{-12}}$ 52. $\frac{3w^{-5}}{x^{-6}}$
53. $b^{0}c^{0}$ 54. $\frac{2}{3}m^{-1}n^{5}$ 55. $\frac{q^{-2}r^{0}}{s^{0}}$ 56. $\frac{a^{-7}b^{2}}{c^{3}d^{-4}}$ 57. $\frac{h^{3}k^{-1}}{6m^{2}}$

Evaluate each expression for $x = 3$, $y = -1$, and $z = 2$.

58. z^{-5} **59.** $(x + y)^{-4}$ **60.** $(yz)^0$ **61.** $(xyz)^{-1}$

62. $(xy - 3)^{-2}$ **63.** x^{-y} **64.** $(yz)^{-x}$ **65.** xy^{-4}

66. **ERROR ANALYSIS** Look at the two equations below. Which is incorrect? Explain the error.

A $5x^{-3} = \frac{1}{5x^3}$ **B** $5x^{-3} = \frac{5}{x^3}$

Simplify.

67. a^3b^{-2} **68.** $c^{-4}d^3$ **69.** $v^0w^2y^{-1}$ **70.** $(a^2b^{-7})^0$ **71.** $-5y^{-6}$

72. $\frac{2a^{-5}}{b^{-6}}$ **73.** $\frac{2a^3}{b^{-1}}$ **74.** $\frac{m^2}{n^{-3}}$ **75.** $\frac{x^{-8}}{3y^{12}}$ **76.** $-\frac{20p^{-1}}{5q^{-3}}$

LINK Biology

When bleeding occurs, platelets (which appear green in the image above) help to form a clot to reduce blood loss. Calcium and vitamin K are also necessary for clot formation.

77. Biology Human blood contains red blood cells, white blood cells, and platelets. The table shows the sizes of these components. Simplify each expression.

Blood Components	
Part	**Size (m)**
Red blood cell	$125{,}000^{-1}$
White blood cell	$3(500)^{-2}$
Platelet	$3(1000)^{-2}$

Tell whether each statement is sometimes, always, or never true.

78. If n is a positive integer, then $x^{-n} = \frac{1}{x^n}$.

79. If x is positive, then x^{-n} is negative.

80. If n is zero, then x^{-n} is 1.

81. If n is a negative integer, then $x^{-n} = 1$.

82. If x is zero, then x^{-n} is 1.

83. If n is an integer, then $x^{-n} > 1$.

84. Critical Thinking Find the value of $2^3 \cdot 2^{-3}$. Then find the value of $3^2 \cdot 3^{-2}$. Make a conjecture about the value of $a^n \cdot a^{-n}$.

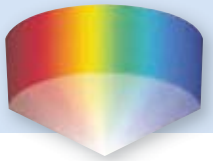
85. Write About It Explain in your own words why 2^{-3} is the same as $\frac{1}{2^3}$.

Find the missing value.

86. $\frac{1}{4} = 2^{\square}$ **87.** $9^{-2} = \frac{1}{\square}$ **88.** $\frac{1}{64} = \square^{-2}$ **89.** $\frac{\square}{3} = 3^{-1}$

90. $7^{-2} = \frac{1}{\square}$ **91.** $10^{\square} = \frac{1}{1000}$ **92.** $3 \cdot 4^{-2} = \frac{3}{\square}$ **93.** $2 \cdot \frac{1}{5} = 2 \cdot 5^{\square}$

MULTI-STEP TEST PREP



94. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. The product of the frequency f and the wavelength w of light in air is a constant v. Write an equation for this relationship.

b. Solve this equation for wavelength. Then write this equation as an equation with f raised to a negative exponent.

c. The units for frequency are hertz (Hz). One hertz is one cycle per second, which is often written as $\frac{1}{s}$. Rewrite this expression using a negative exponent.

TEST PREP

95. Which is NOT equivalent to the other three?

Ⓐ $\frac{1}{25}$ Ⓑ 5^{-2} Ⓒ 0.04 Ⓓ -25

96. Which is equal to 6^{-2}?

Ⓕ $6(-2)$ Ⓖ $(-6)(-6)$ Ⓗ $-\frac{1}{6 \cdot 6}$ Ⓙ $\frac{1}{6 \cdot 6}$

97. Simplify $\frac{a^3b^{-2}}{c^{-1}}$.

Ⓐ $\frac{a^3c}{b^2}$ Ⓑ $\frac{a^3b^2}{-c}$ Ⓒ $\frac{a^3}{-b^2c}$ Ⓓ $\frac{c}{a^3b^2}$

98. Gridded Response Simplify $\left[2^{-2} + (6 + 2)^0\right]$.

99. Short Response If a and b are real numbers and n is a positive integer, write a simplified expression for the product $a^{-n} \cdot b^0$ that contains only positive exponents. Explain your answer.

CHALLENGE AND EXTEND

100. Multi-Step Copy and complete the table of values below. Then graph the ordered pairs and describe the shape of the graph.

x	-4	-3	-2	-1	0	1	2	3	4
$y = 2^x$									

101. Multi-Step Copy and complete the table. Then write a rule for the values of 1^n and $(-1)^n$ when n is any negative integer.

n	-1	-2	-3	-4	-5
1^n					
$(-1)^n$					

SPIRAL REVIEW

Solve each equation. *(Lesson 2-3)*

102. $6x - 4 = 8$ **103.** $-9 = 3(p - 1)$ **104.** $\frac{y}{5} - 8 = -12$

105. $1.5h - 5 = 1$ **106.** $2w + 6 - 3w = -10$ **107.** $-12 = \frac{1}{2}n + 2 - n$

Identify the independent and dependent variables. Write a rule in function notation for each situation. *(Lesson 4-3)*

108. Pink roses cost $1.50 per stem.

109. For dog-sitting, Beth charges a $30 flat fee plus $10 a day.

Write the equation that describes each line in slope-intercept form. *(Lesson 5-7)*

110. slope $= 3$, y-intercept $= -4$ **111.** slope $= \frac{1}{3}$, y-intercept $= 5$

112. slope $= 0$, y-intercept $= \frac{2}{3}$ **113.** slope $= -4$, the point $(1, 5)$ is on the line.