Systems of Equations and Inequalities

6A Systems of Linear Equations

CHAPTER

- Lab Solve Linear Equations by Using a Spreadsheet
- 6-1 Solving Systems by Graphing
- Lab Model Systems of Linear Equations
- 6-2 Solving Systems by Substitution
- 6-3 Solving Systems by Elimination
- 6-4 Solving Special Systems

6B Linear Inequalities

- 6-5 Solving Linear Inequalities
- 6-6 Solving Systems of Linear Inequalities
- Lab Solve Systems of Linear Inequalities



 Solve real-world problems involving systems of linear equations and inequalities.

Where's the Money?

You can solve a system of equations to determine how many basketball game tickets you can buy at different price levels.

> go.hrw.com Chapter Project Online KEYWORD: MA7 ChProj



🞯 Vocabulary

Match each term on the left with a definition on the right.

- inequality
 a pair of numbers (*x*, *y*) that represent the coordinates of a point
 - **B.** a statement that two quantities are not equal
 - **C.** the *y*-value of the point at which the graph of an equation crosses the *y*-axis
- **5.** solution of an equation

3. ordered pair

4. slope

- **D.** a value of the variable that makes the equation true
 - **E.** the ratio of the vertical change to the horizontal change for a nonvertical line
 - **F.** an equation whose graph is a straight line

🞯 Graph Linear Functions

Graph each function.

6. $y = \frac{3}{4}x + 1$	7. $y = -3x + 5$	8. $y = x - 6$
9. $x + y = 4$	10. $y = -\frac{2}{3}x + 4$	11. $y = -5$

🞯 Solve Multi-Step Equations

Solve each equation.

12. $-7x - 18 = 3$	13. $12 = -3n + 6$	14. $\frac{1}{2}d + 30 = 32$
15. $-2p + 9 = -3$	16. $33 = 5y + 8$	17. $-3 + 3x = 27$

Solve for a Variable

Solve each equation for *y*.

18. $7x + y = 4$	19. $y + 2 = -4x$	20. $8 = x - y$
21. $x + 2 = y - 5$	22. $2y - 3 = 12x$	23. $y + \frac{3}{4}x = 4$

Second Expressions

Evaluate each expression for the given value of the variable.

24. $t - 5$ for $t = 7$	25. $9 - 2a$ for $a = 4$	26. $\frac{1}{2}x - 2$ for $x = 14$
27. $n + 15$ for $n = 37$	28. $9c + 4$ for $c = \frac{1}{3}$	29. $16 + 3d$ for $d = 5$

Solve and Graph Inequalities

Solve and graph each inequality.

30. $b - 9 \ge 1$ 31. $-2x < 10$ 32. $3y \le -3$ 33. $\frac{1}{2}y \le 1$	30. $b - 9 \ge 1$	31. $-2x < 10$	32. $3y \le -3$	33. $\frac{1}{2}y \le 5$
---	--------------------------	-----------------------	------------------------	---------------------------------

CHAPTER

Study Guide: Preview

Where You've Been

Previously, you

- solved one-step and multi-step equations.
- solved one-step and multistep inequalities.
- graphed linear equations on a coordinate plane.

Key Vocabulary/Vocabulario

consistent system	sistema consistente
dependent system	sistema dependiente
inconsistent system	sistema inconsistente
independent system	sistema independiente
linear inequality	desigualdad lineal
solution of a linear inequality	solución de una desigualdad lineal
system of linear equations	sistema de ecuaciones lineales

In This Chapter

You will study

- how to find a solution that satisfies two linear equations.
- how to find solutions that satisfy two linear inequalities.
- how to graph one or more linear inequalities on a coordinate plane.

Where You're Going

You can use the skills in this chapter

- to determine which purchases are better deals.
- in other classes, such as Economics and Chemistry.
- to solve linear equations that involve three or more variables in future math classes.

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

- The word *system* means "a group." How do you think a system of linear equations is different from a linear equation?
- **2.** A **consistent system** has *at least one* solution. How many solutions do you think an **inconsistent system** has?
- **3.** A **dependent system** has infinitely many solutions. Which vocabulary term above means a system with *exactly one* solution?
- **4.** In Chapter 5, you saw that a solution of a linear equation was an ordered pair that made the equation true. Modify this to define solution of a linear inequality.





Writing Strategy: Write a Convincing **Argument/Explanation**

The Write About It icon ≽ appears throughout the book. These icons identify questions that require you to write a complete argument or explanation. Writing a convincing argument or explanation shows that you have a solid understanding of a concept.

To be effective, an argument or explanation should include

- evidence, work, or facts.
- a complete response that will answer or explain.

From Lesson 2-10

23. Write About It Lewis invested \$1000 at 3% simple interest for 4 years. Lisa invested \$1000 at 4% simple interest for 3 years. Explain why Lewis and Lisa earned the same amount of interest.

Identify what you need to answer or explain.

Explain why Lewis and Lisa earned the same amount of interest.



itep 3

Give evidence, work, or facts that are needed to answer the question.

Use the formula for simple interest to find the amount of interest earned: I = Prt. **Lisa:** P = 1000, r = 0.04, t = 3

Lewis: P = 1000, r = 0.03, t = 4

- I = Prt = 1000(0.03)(4) = 120 I = Prt = 1000(0.04)(3) = 120
- I = 1000(0.12) = \$120

I = 1000(0.12) = \$120

Write a complete response that answers or explains.

Lewis and Lisa both invested the same amount of money, \$1000. They earned the same amount of interest because 0.04×3 and 0.03×4 both equal 0.12. They both earned $0.12 \times \$1000$, or \$120.

Try This

Write a convincing argument or explanation.

- **1.** What is the least whole number that is a solution of 12x + 15.4 > 118.92? Explain.
- **2.** Which equation has an error? Explain the error.
 - **A.** $4(6 \cdot 5) = (4)6 \cdot (4)5$ **B.** $4(6 \cdot 5) = (4 \cdot 6)5$



Solve Linear Equations by Using a Spreadsheet

You can use a spreadsheet to answer "What if...?" questions. By changing one or more values, you can quickly model different scenarios.

Company Z Profit

Number of

A

DVD Players Cost (\$)

= 400 + 20*A2

1

Use with Lesson 6-1



Company Z makes DVD players. The company's costs are \$400 per week plus \$20 per DVD player. Each DVD player sells for \$45. How many DVD players must company Z sell in one week to make a profit?



Let *n* represent the number of DVD players company Z sells in one week.

c = 400 + 20n	The total cost is \$400 plus \$20 times the number of DVD players made.
s = 45n	The total sales income is \$45 times the number of DVD players sold.
p = s - c	The total profit is the sales income minus the total cost.

- Set up your spreadsheet with columns for number of DVD players, total cost, total income, and profit.
- **2** Under Number of DVD Players, enter 1 in cell A2.
- 3 Use the equations above to enter the formulas for total cost, total sales, and total profit in row 2.
 - In cell B2, enter the formula for total cost.
 - In cell C2, enter the formula for total sales income.
 - In cell D2, enter the formula for total profit.
- Fill columns A, B, C, and D by selecting cells A1 through D1, clicking the small box at the bottom right corner of cell D2, and dragging the box down through several rows.
- Find the point where the profit is \$0. This is known as the breakeven point, where total cost and total income are the same.

Company Z Profit B A Number of Total Total DVD Players Cost (\$) Income (\$) Profit (\$ Breakeven point 15 700 675 720 16 720 17 740 765 Profit begins.

B

420

Total

Income (\$)

= 45*A2

Profit (\$)

375

= C2 - B2

45

Total

Company Z must sell 17 DVD players to make a profit. The profit is \$25.

Try This

For Exercises 1 and 2, use the spreadsheet from the activity.

- **1.** If company Z sells 10 DVD players, will they make a profit? Explain. What if they sell 16?
- 2. Company Z makes a profit of \$225 dollars. How many DVD players did they sell?

For Exercise 3, make a spreadsheet.

3. Company Y's costs are \$400 per week plus \$20 per DVD player. They want the breakeven point to occur with sales of 8 DVD players. What should the sales price be?

6-1

Solving Systems by Graphing

Objectives

Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

Vocabulary

system of linear equations solution of a system of linear equations

Why learn this?

You can compare costs by graphing a system of linear equations. (See Example 3.)

Sometimes there are different charges for the same service or product at different places. For example, Bowlo-Rama charges \$2.50 per game plus \$2 for shoe rental while Bowling Pinz charges \$2 per game plus \$4 for shoe rental. A *system of linear equations* can be used to compare these charges.



A **system of linear equations** is a set of two or more linear equations containing two or more variables. A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

EXAMPLE 1 Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A
$$(4, 1); \begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$$

$$\frac{x + 2y = 6}{4 + 2(1)} \begin{cases} x + 2y = 6 \\ 6 \\ 4 + 2 \end{cases}$$

$$\frac{x - y = 3}{4 - 1} \end{cases}$$

Substitute 4 for x and 1 for y in each equation in the system.

The ordered pair (4, 1) makes both equations true.

(4, 1) is a solution of the system.

1a. (1, 3); $\begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$

3x - 2y = 5 3(-1) - 2(2) = 5 -3 - 4 = 5 $-7 = 5 \times$

Substitute –1 for x and 2 for y in each equation in the system.

The ordered pair (-1, 2) makes one equation true, but not the other. (-1, 2) is not a solution of the system.



Tell whether the ordered pair is a solution of the given system.

1b.
$$(2, -1); \begin{cases} x - 2y = 4 \\ 3x + y = 6 \end{cases}$$

Helpful Hint

If an ordered pair does not satisfy the first equation in the system, there is no need to check the other equations. All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

$$\begin{cases} y = 2x - 1\\ y = -x + 5 \end{cases}$$



The point (2, 3) is where the two lines intersect and is a solution of both equations, so (2, 3) is the solution of the system.

EXAMPLE

Helpful Hint Sometimes it is

difficult to tell

your answer by substituting it into both equations.

exactly where the lines cross when you

solve by graphing. It is good to confirm

2 Solving a System of Linear Equations by Graphing

Solve each system by graphing. Check your answer.



E

Graph the system.

The solution appears to be at (1, -2).



Check Substitute (1, -2) into the system.

<i>y</i> =	= x - 3	<i>y</i> =	= - x - 1
-2	1 – 3	-2	−1 − 1
-2	-2 🗸	-2	-2 🗸

The solution is (1, -2).

$$\begin{cases} x + y = 0\\ y = -\frac{1}{2}x + 1\\ x + y = 0\\ -\frac{x}{y} = -\frac{x}{-x} \end{cases}$$

Rewrite the first equation in slope-intercept form.

Graph using a calculator and then use the intersection command.





The solution is (-2, 2).

Solve each system by graphing. Check your answer. **2b.** $\begin{cases} y = \frac{1}{3}x - 3\\ 2x + y = 4 \end{cases}$

2a.
$$\begin{cases} y = -2x - 1 \\ y = x + 5 \end{cases}$$

398 Chapter 6 Systems of Equations and Inequalities

CHECK



Problem-Solving Application



Bowl-o-Rama charges \$2.50 per game plus \$2 for shoe rental, and Bowling Pinz charges \$2 per game plus \$4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?

1 Understand the Problem

The answer will be the number of games played for which the total cost is the same at both bowling alleys. List the important information:

- Bowl-o-Rama \$2.50 • Game price:
- Shoe-rental fee: Bowl-o-Rama \$2
- **Bowling Pinz: \$2**
- **Bowling Pinz: \$4**

Make a Plan

Write a system of equations, one equation to represent the price at each company. Let *x* be the number of games played and *y* be the total cost.

	Tota	l co	ost	is	price	per	game	tir	nes	ga	imes	plu	shoe	rent	tal.
Bowl-o-Ra	ama	y		=		2.5			•		x	+		2	
Bowling P	inz	y		=		2			•		x	+		4	

3 Solve

Graph y = 2.5x + 2 and y = 2x + 4. The lines appear to intersect at (4, 12). So, the cost at both places will be the same for 4 games bowled and that cost will be \$12.

Look Back

Check (4, 12) using both equations. Cost of bowling 4 games at Bowl-o-Rama: $2.5(4) + 2 = 10 + 2 = 12 \checkmark$ Cost of bowling 4 games at Bowling Pinz: $2(4) + 4 = 8 + 4 = 12 \checkmark$





3. Video club A charges \$10 for membership and \$3 per movie rental. Video club B charges \$15 for membership and \$2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost?

THINK AND DISCUSS

- 1. Explain how to use a graph to solve a system of linear equations.
- 2. Explain how to check a solution of a system of linear equations.



3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.

Solving a Linear System by Graphing





GUIDED PRACTICE

1. Vocabulary Describe a *solution of a system of linear equations*.

Tell whether the ordered pair is a solution of the given system.

2. $(2, -2); \begin{cases} 3x + y = 4 \\ x - 3y = -4 \end{cases}$	3. $(3, -1); \begin{cases} x - 2y = 5 \\ 2x - y = 7 \end{cases}$	4. $(-1, 5); \begin{cases} -x + y = 6\\ 2x + 3y = 13 \end{cases}$
--	---	--

Solve each system by graphing. Check your answer.

5. $\begin{cases} y = \frac{1}{2}x \\ y = -x + 3 \end{cases}$ 6. $\begin{cases} y = x - 2 \\ 2x + y = 1 \end{cases}$ 7. $\begin{cases} -2x - 1 = y \\ x + y = 3 \end{cases}$

8. To deliver mulch, Lawn and Garden charges \$30 per cubic yard of mulch plus a \$30 delivery fee. Yard Depot charges \$25 per cubic yard of mulch plus a \$55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be?

PRACTICE AND PROBLEM SOLVING

Independent PracticeForSeeExercisesExample9–11112–152163

Extra Practice Skills Practice p. S14 Application Practice p. S33 Tell whether the ordered pair is a solution of the given system.

9.
$$(1, -4);$$
 $\begin{cases} x - 2y = 8 \\ 4x - y = 8 \end{cases}$ **10.** $(-2, 1);$ $\begin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}$ **11.** $(5, 2);$ $\begin{cases} 2x + y = 12 \\ -3y - x = -11 \end{cases}$

Solve each system by graphing. Check your answer.

42	$y = \frac{1}{2}x + 2$	42	$\int y = x$		(-2x - 1 = y)	45	$\int x + y = 2$
12.	y = -x - 1	13. <	y = -x + 6	14.	$\begin{cases} x = -y + 3 \end{cases}$	15.	$\int y = x - 4$

- **16. Multi-Step** Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be?
- **17. School** The school band sells carnations on Valentine's Day for \$2 each. They buy the carnations from a florist for \$0.50 each, plus a \$16 delivery charge.
 - **a.** Write a system of equations to describe the situation.
 - **b.** Graph the system. What does the solution represent?
 - **c.** Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.



400 Chapter 6 Systems of Equations and Inequalities

Graphing Calculator Use a graphing calculator to graph and solve the systems of equations in Exercises 19–22. Round your answer to the nearest tenth.



Landscaping

Middleton Place Gardens, South Carolina, are the United States' oldest landscaped gardens. The gardens were established in 1741 and opened to the public in the 1920s. **19.** $\begin{cases} y = 4.7x + 2.1 \\ y = 1.6x - 5.4 \end{cases}$ **20.** $\begin{cases} 4.8x + 0.6y = 4 \\ y = -3.2x + 2.7 \end{cases}$ **21.** $\begin{cases} y = \frac{5}{4}x - \frac{2}{3} \\ \frac{8}{3}x + y = \frac{5}{9} \end{cases}$ **22.** $\begin{cases} y = 6.9x + 12.4 \\ y = -4.1x - 5.3 \end{cases}$

23. Landscaping The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether.

- **24. Fitness** Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for *x* minutes. In the afternoon, Rusty will jog for *x* minutes. How many minutes must he jog to burn at least as many Calories *y* in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.
- **25.** A tree that is 2 feet tall is growing at a rate of 1 foot per year. A 6-foot tall tree is growing at a rate of 0.5 foot per year. In how many years will the trees be the same height?
- **26.** Critical Thinking Write a real-world situation that could be represented by the system $\begin{cases} y = 3x + 10 \\ y = 5x + 20 \end{cases}$.
- **27. Write About It** When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?



28. Taxi company A charges \$4 plus \$0.50 per mile. Taxi company B charges \$5 plus \$0.25 per mile. Which system best represents this problem?

(A)
$$\begin{cases} y = 4x + 0.5 \\ y = 5x + 0.25 \end{cases}$$
(C) $\begin{cases} y = -4x + 0.5 \\ y = -5x + 0.25 \end{cases}$ (B) $\begin{cases} y = 0.5x + 4 \\ y = 0.25x + 5 \end{cases}$ (D) $\begin{cases} y = -0.5x + 4 \\ y = -0.25x + 5 \end{cases}$

29. Which system of equations represents the given graph?





30. Gridded Response Which value of *b* will make the system y = 2x + 2 and y = 2.5x + b intersect at the point (2, 6)?

CHALLENGE AND EXTEND

31. Entertainment If the pattern in the table continues, in what month will the number of sales of VCRs and DVD players be the same? What will that number be?

Total Number Sold					
Month	1	2	3	4	
VCRs	500	490	480	470	
DVD Players	250	265	280	295	

32. Long Distance Inc. charges a \$1.45 connection charge and \$0.03 per minute.

Far Away Calls charges a \$1.52 connection charge and \$0.02 per minute.

- a. For how many minutes will a call cost the same from both companies? What is that cost?
- **b.** When is it better to call using Long Distance Inc.? Far Away Calls? Explain.
- c. What if...? Long Distance Inc. raised its connection charge to \$1.50 and Far Away Calls decreased its connection charge by 2 cents. How will this affect the graphs? Now which company is better to use for calling long distance? Why?

SPIRAL REVIEW

Solve each equation. (Lesson 2-2)

33. $18 = \frac{3}{7}x$ **34.** $-\frac{x}{5} = 12$ **35.** -6y = -13.2 **36.** $\frac{2}{5} = \frac{y}{12}$

Describe the solutions of each inequality in words. (Lesson 3-1)

37. $3c < 15$ 38. $\frac{1}{3}x \ge 9$ 39. $5 + a$	> 11
---	------

Solve each inequality and graph the solutions. (Lesson 3-4) **40.** 4(2x+1) > 28 **41.** $3^3 + 9 \le -4c$

```
42. \frac{1}{8}x + \frac{3}{5} \le \frac{3}{8}
```

go.hrw.com

Career Resources Online KEYWORD: MA7 Career



Q: What math classes did you take in high school?

- A: Career Math, Algebra, and Geometry
- Q: What are you studying and what math classes have you taken?
- A: I am really interested in aviation. I am taking Statistics and Trigonometry. Next year I will take Calculus.

Q: How is math used in aviation?

A: I use math to interpret aeronautical charts. I also perform calculations involving wind movements, aircraft weight and balance, and fuel consumption. These skills are necessary for planning and executing safe air flights.

Q: What are your future plans?

A: I could work as a commercial or corporate pilot or even as a flight instructor. I could also work toward a bachelor's degree in aviation management, air traffic control, aviation electronics, aviation maintenance, or aviation computer science.



Model Systems of Linear Equations

You can use algebra tiles to model and solve some systems of linear equations.

Use with Lesson 6-2

KEY + = 1 - = -x - = -x	REMEMBER When two expressions are equal, you can substitute one for the other in any expression or equation.	go.hrw.com Lab Resources Online KEYWORD: MA7 Lab6		
Activity Use algebra tiles to model and sol	$\begin{cases} y = 2x - 3\\ x + y = 9 \end{cases}.$			
K		ALGEBKA		
+ + + + + + + + + + + + + + + + + + +	The first equation is solved for y. Model the second equation, x + y = 9, by substituting $2x - 3for y.$	x + y = 9 x + (2x - 3) = 9 3x - 3 = 9		
$\begin{array}{c} + + + + + + + + + + + + + + + + + + +$	Add 3 yellow tiles on both sides of the mat. This represents adding 3 to both sides of the equation. Remove zero pairs.	$3x - 3 = 9 = \frac{+3}{3x} = \frac{+3}{12}$		
+ + + + + + + + + + + + + + +	Divide each side into 3 equal groups. Align one x-tile with each group on the right side. One x-tile is equivalent to 4 yellow tiles. $x = 4$	$\frac{3x}{3} = \frac{12}{3}$ $x = 4$		
To solve for <i>y</i> , substitute 4 for <i>x</i> in one of the equations: $y = 2x - 3$ = 2(4) - 3				

The solution is (4, 5).

Try This

1.

Model and solve each system of equations.

$$\begin{cases} y = x + 3 \\ 2x + y = 6 \end{cases}$$
2.
$$\begin{cases} 2x + 3 = y \\ x + y = 6 \end{cases}$$
3.
$$\begin{cases} 2x + 3y = 1 \\ x = -1 - y \end{cases}$$
4.
$$\begin{cases} y = x + 1 \\ 2x - y = -5 \end{cases}$$

= 5