CHAPTER 5

Linear Functions

Chapter Focus

- Translate among different representations of linear functions.
- Find and interpret slopes and intercepts of linear equations that model real-world problems
- Solve real-world problems involving linear equations.

Take *Flight*

You can use linear functions to describe patterns and relationships in flight times.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. coefficient
2. coordinate plane
3. transformation
4. perpendicular

A. a change in the size or position of a figure
B. forming right angles
C. a two-dimensional system formed by the intersection of a horizontal number line and a vertical number line
D. an ordered pair of numbers that gives the location of a point
E. a number that is multiplied by a variable

Ordered Pairs

Graph each point on the same coordinate plane.

5. $A(2, 5)$
6. $B(-1, -3)$
7. $C(-5, 2)$
8. $D(4, -4)$
9. $E(-2, 0)$
10. $F(0, 3)$
11. $G(8, 7)$
12. $H(-8, -7)$

Solve for a Variable

Solve each equation for the indicated variable.

13. $2x + y = 8$; y
14. $5y = 5x - 10$; y
15. $2y = 6x - 8$; y
16. $10x + 25 = 5y$; y

Evaluate Expressions

Evaluate each expression for the given value of the variable.

17. $4g - 3$; $g = -2$
18. $8p - 12$; $p = 4$
19. $4x + 8$; $x = -2$
20. $-5t - 15$; $t = 1$

Connect Words and Algebra

21. The value of a stock begins at \$0.05 and increases by \$0.01 each month. Write an equation representing the value of the stock v in any month m.
22. Write a situation that could be modeled by the equation $b = 100 - s$.

Rates and Unit Rates

Find each unit rate.

23. 322 miles on 14 gallons of gas
24. \$14.25 for 3 pounds of deli meat
25. 32 grams of fat in 4 servings
26. 120 pictures on 5 rolls of film

CHAPTER 5

Study Guide: Preview

Where You've Been

Previously, you

- wrote equations in function notation.
- graphed functions.
- identified the domain and range of functions.
- identified independent and dependent variables.

In This Chapter

You will study

- writing and graphing linear functions.
- identifying and interpreting the components of linear graphs, including the x-intercept, y-intercept, and slope.
- graphing and analyzing families of functions.

Where You're Going

You can use the skills in this chapter

- to solve systems of linear equations in Chapter 6.
- to identify rates of change in linear data in biology and economics.
- to make calculations and comparisons in your personal finances.

Key Vocabulary/Vocabulario

constant of variation	constante de variación
direct variation	variación directa
family of functions	familia de funciones
linear function	función lineal
parallel lines	líneas paralelas
perpendicular lines	líneas perpendiculares
slope	pendiente
transformation	transformación
x-intercept	intersección con el eje x
y-intercept	intersección con el eje y

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What shape do you think is formed when a **linear function** is graphed on a coordinate plane?

2. The meaning of *intercept* is similar to the meaning of *intersection*. What do you think an **x-intercept** might be?

3. **Slope** is a word used in everyday life, as well as in mathematics. What is your understanding of the word *slope*?

4. A family is a group of related people. Use this concept to define **family of functions**.

CHAPTER 5

Reading and Writing Math

Study Strategy: Use Multiple Representations

Representing a math concept in more than one way can help you understand it more clearly. As you read the explanations and example problems in your text, note the use of tables, lists, graphs, diagrams, and symbols, as well as words to explain a concept.

From Lesson 4-4:

In this example from Chapter 4, the given function is described using an equation, a table, ordered pairs, and a graph.

Graphing Functions

Graph each function.

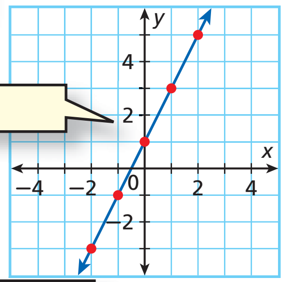
A $2x + 1 = y$ — **Equation**

Step 1 Choose several values of x and generate ordered pairs. — **Table**

x	$2x + 1 = y$	(x, y)
−3	$2(-3) + 1 = -5$	$(-3, -5)$
−2	$2(-2) + 1 = -3$	$(-2, -3)$
−1	$2(-1) + 1 = -1$	$(-1, -1)$
0	$2(0) + 1 = 1$	$(0, 1)$
1	$2(1) + 1 = 3$	$(1, 3)$
2	$2(2) + 1 = 5$	$(2, 5)$
3	$2(3) + 1 = 7$	$(3, 7)$

Ordered Pairs

Step 2 Plot enough points to see a pattern. — **Graph**



Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.

Try This

1. If an employee earns $8.00 an hour, $y = 8x$ gives the total pay y the employee will earn for working x hours. For this equation, make a table of ordered pairs and a graph. Explain the relationships between the equation, the table, and the graph. How does each one describe the situation?

2. What situations might make one representation more useful than another?

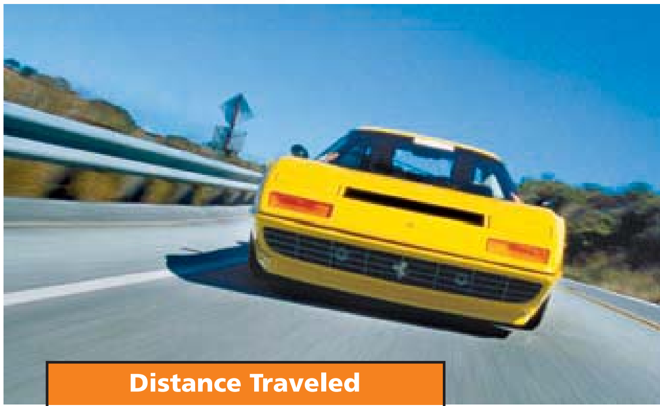
5-1 Identifying Linear Functions

Objectives
Identify linear functions and linear equations.

Graph linear functions that represent real-world situations and give their domain and range.

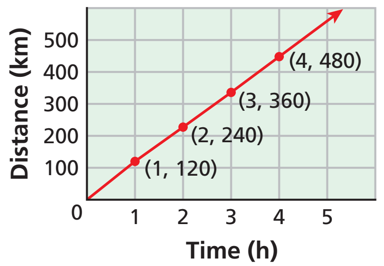
Vocabulary
linear function
linear equation

Why learn this?
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.



Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, $y = 120x$ gives the number of kilometers y that the car would travel in x hours. Solutions are shown in the graph.

Distance Traveled

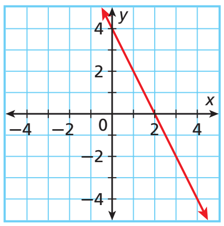


The graph represents a function because each domain value (x-value) is paired with exactly one range value (y-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.

EXAMPLE 1 Identifying a Linear Function by Its Graph

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

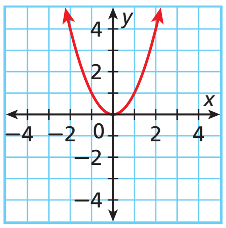
A



Each domain value is paired with exactly one range value. The graph forms a line.

linear function

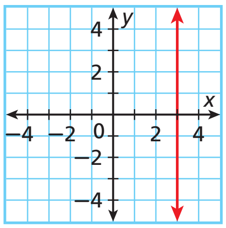
B



Each domain value is paired with exactly one range value. The graph is not a line.

not a linear function

C

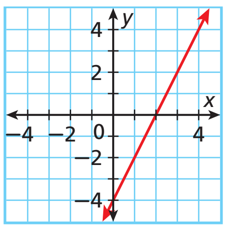


The only domain value, 3, is paired with many different range values.

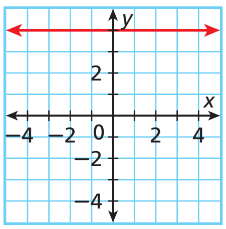
not a function

CHECK IT OUT! **Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?**

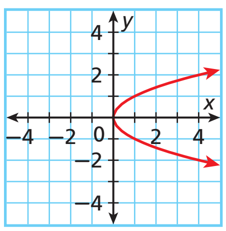
1a.



1b.



1c.



You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in x corresponds to a constant change in y.

Caution!

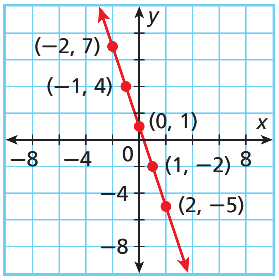
If you find a constant change in the y-values, check for a constant change in the x-values. Both need to be constant for the function to be linear.

x	y
−2	7
−1	4
0	1
1	−2
2	−5

+1 +1 +1 +1 (in x)
−3 −3 −3 −3 (in y)

In this table, a constant change of +1 in x corresponds to a constant change of −3 in y. These points satisfy a linear function.

The points from this table lie on a line.

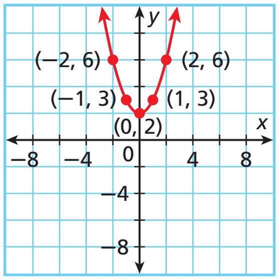


x	y
−2	6
−1	3
0	2
1	3
2	6

+1 +1 +1 +1 (in x)
−3 −1 +1 +3 (in y)

In this table, a constant change of +1 in x does *not* correspond to a constant change in y. These points do *not* satisfy a linear function.

The points from this table do not lie on a line.



EXAMPLE 2 Identifying a Linear Function by Using Ordered Pairs

Tell whether each set of ordered pairs satisfies a linear function. Explain.

A $\{(2, 4), (5, 3), (8, 2), (11, 1)\}$

x	y
2	4
5	3
8	2
11	1

+3 +3 +3 (in x)
−1 −1 −1 (in y)

Write the ordered pairs in a table. Look for a pattern.

A constant change of +3 in x corresponds to a constant change of −1 in y.

These points satisfy a linear function.

B $\{(-10, 10), (-5, 4), (0, 2), (5, 0)\}$

x	y
−10	10
−5	4
0	2
5	0

+5 +5 +5 (in x)
−6 −2 −2 (in y)

Write the ordered pairs in a table. Look for a pattern.

A constant change of +5 in x corresponds to different changes in y.

These points do not satisfy a linear function.

CHECK IT OUT!

2. Tell whether the set of ordered pairs $\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}$ satisfies a linear function. Explain.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation*. A **linear equation** is any equation that can be written in the *standard form* shown below.

Know it! Note

Standard Form of a Linear Equation

$Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0

Notice that when a linear equation is written in standard form

- x and y both have exponents of 1.
- x and y are not multiplied together.
- x and y do not appear in denominators, exponents, or radical signs.

Linear		Not Linear	
$3x + 2y = 10$	*Standard form*	$3xy + x = 1$	*x and y are multiplied.*
$y - 2 = 3x$	*Can be written as $3x - y = -2$*	$x^3 + y = -1$	*x has an exponent other than 1.*
$-y = 5x$	*Can be written as $5x + y = 0$*	$x + \frac{6}{y} = 12$	*y is in a denominator.*

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

EXAMPLE 3

Graphing Linear Functions

Tell whether each function is linear. If so, graph the function.

A $y = x + 3$

$$\begin{aligned} y &= x + 3 && \text{Write the equation in standard form.} \\ -x & \quad -x && \text{Subtraction Property of Equality} \\ y - x &= 3 \\ -x + y &= 3 && \text{The equation is in standard form } (A = -1, B = 1, C = 3). \end{aligned}$$

Remember!

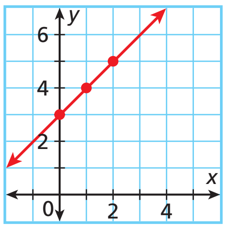
- $y - x = y + (-x)$
- $y + (-x) = -x + y$
- $-x = -1x$
- $y = 1y$

The equation can be written in standard form, so the function is linear.

To graph, choose three values of x, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

x	$y = x + 3$	(x, y)
0	$y = 0 + 3 = 3$	$(0, 3)$
1	$y = 1 + 3 = 4$	$(1, 4)$
2	$y = 2 + 3 = 5$	$(2, 5)$

Plot the points and connect them with a straight line.



B $y = x^2$

This is not linear, because x has an exponent other than 1.

CHECK IT OUT!

Tell whether each function is linear. If so, graph the function.

3a. $y = 5x - 9$ **3b.** $y = 12$ **3c.** $y = 2^x$

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

EXAMPLE 4 *Career Application*

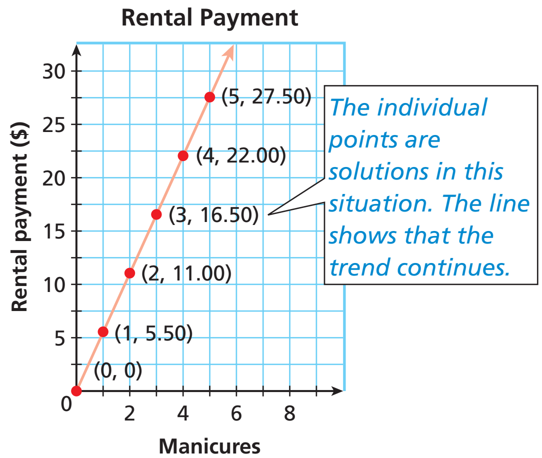
Sue rents a manicure station in a salon and pays the salon owner $5.50 for each manicure she gives. The amount Sue pays each day is given by $f(x) = 5.50x$, where x is the number of manicures. Graph this function and give its domain and range.

Choose several values of x and make a table of ordered pairs.

x	$f(x) = 5.50x$
0	$f(0) = 5.50(0) = 0$
1	$f(1) = 5.50(1) = 5.50$
2	$f(2) = 5.50(2) = 11.00$
3	$f(3) = 5.50(3) = 16.50$
4	$f(4) = 5.50(4) = 22.00$
5	$f(5) = 5.50(5) = 27.50$

The number of manicures must be a whole number, so the domain is $\{0, 1, 2, 3, \ldots\}$. The range is $\{0, 5.50, 11.00, 16.50, \ldots\}$.

Graph the ordered pairs.



Remember!

$f(x) = y$, so in Example 4, graph the function values (dependent variable) on the y-axis.

CHECK IT OUT!

4. **What if...?** At another salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where x is the number of manicures. Graph this function and give its domain and range.

THINK AND DISCUSS

1. Suppose you are given five ordered pairs that satisfy a function. When you graph them, four lie on a straight line, but the fifth does not. Is the function linear? Why or why not?

2. In Example 4, why is every point on the line not a solution?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to use the information to identify a linear function. Include an example.

Know it! Note

Determining Whether a Function Is Linear

- From its graph
- From its equation
- From a list of ordered pairs

5-1 Exercises

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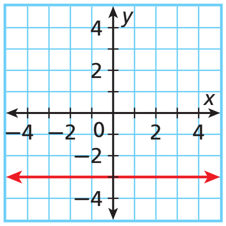
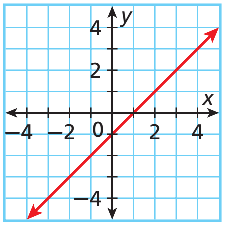
GUIDED PRACTICE

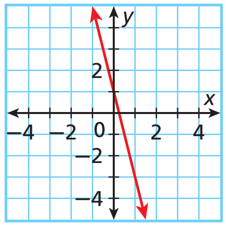
1. **Vocabulary** Is the *linear equation* $3x - 2 = y$ in standard form? Explain.

SEE EXAMPLE 1
p. 300

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

2. 

3. 

4. 

SEE EXAMPLE 2
p. 301

Tell whether the given ordered pairs satisfy a linear function. Explain.

5.

x	5	4	3	2	1
y	0	2	4	6	8

6.

x	1	4	9	16	25
y	1	2	3	4	5

7. $\{(0, 5), (-2, 3), (-4, 1), (-6, -1), (-8, -3)\}$

8. $\{(2, -2), (-1, 0), (-4, 1), (-7, 3), (-10, 6)\}$

SEE EXAMPLE 3
p. 302

Tell whether each function is linear. If so, graph the function.

9. $2x + 3y = 5$ 10. $2y = 8$ 11. $\frac{x^2 + 3}{5} = y$ 12. $\frac{x}{5} = \frac{y}{3}$

SEE EXAMPLE 4
p. 303

13. **Transportation** A train travels at a constant speed of 75 mi/h. The function $f(x) = 75x$ gives the distance that the train travels in x hours. Graph this function and give its domain and range.

14. **Entertainment** A movie rental store charges a \$6.00 membership fee plus \$2.50 for each movie rented. The function $f(x) = 2.50x + 6$ gives the cost of renting x movies. Graph this function and give its domain and range.

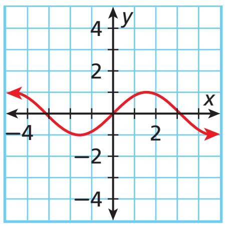
PRACTICE AND PROBLEM SOLVING

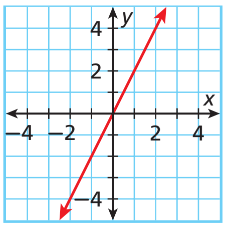
Independent Practice

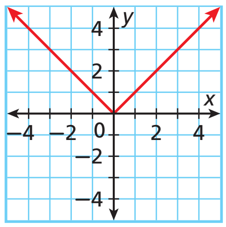
For Exercises	See Example
15–17	1
18–20	2
21–24	3
25	4

Extra Practice
Skills Practice p. S12
Application Practice p. S32

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

15. 

16. 

17. 

Tell whether the given ordered pairs satisfy a linear function. Explain.

18.

x	−3	0	3	6	9
y	−2	−1	0	2	4

19.

x	−1	0	1	2	3
y	−3	−2	−1	0	1

20. $\{(3, 4), (0, 2), (-3, 0), (-6, -2), (-9, -4)\}$

Tell whether each function is linear. If so, graph the function.

21. $y = 5$ **22.** $4y - 2x = 0$ **23.** $\frac{3}{x} + 4y = 10$ **24.** $5 + 3y = 8$

25. Transportation The gas tank in Tony's car holds 15 gallons, and the car can travel 25 miles for each gallon of gas. When Tony begins with a full tank of gas, the function $f(x) = -\frac{1}{25}x + 15$ gives the amount of gas $f(x)$ that will be left in the tank after traveling x miles (if he does not buy more gas). Graph this function and give its domain and range.

Tell whether the given ordered pairs satisfy a function. If so, is it a linear function?

26. $\{(2, 5), (2, 4), (2, 3), (2, 2), (2, 1)\}$ **27.** $\{(-8, 2), (-6, 0), (-4, -2), (-2, -4), (0, -6)\}$

28.

x	−10	−6	−2	2	4
y	0	0.25	0.50	0.75	1

29.

x	−5	−1	3	7	11
y	1	1	1	1	1

Tell whether each equation is linear. If so, write the equation in standard form and give the values of *A*, *B*, and *C*.

30. $2x - 8y = 16$ **31.** $y = 4x + 2$ **32.** $2x = \frac{y}{3} - 4$ **33.** $\frac{4}{x} = y$

34. $\frac{x+4}{2} = \frac{y-4}{3}$ **35.** $x = 7$ **36.** $xy = 6$ **37.** $3x - 5 + y = 2y - 4$

38. $y = -x + 2$ **39.** $5x = 2y - 3$ **40.** $2y = -6$ **41.** $y = \sqrt{x}$

Graph each linear function.

42. $y = 3x + 7$ **43.** $y = x + 25$ **44.** $y = 8 - x$ **45.** $y = 2x$

46. $-2y = -3x + 6$ **47.** $y - x = 4$ **48.** $y - 2x = -3$ **49.** $x = 5 + y$

50. Measurement One inch is equal to approximately 2.5 centimeters. Let x represent inches and y represent centimeters. Write an equation in standard form relating x and y. Give the values of A, B, and C.

51. Wages Molly earns $8.00 an hour at her job.

a. Let x represent the number of hours that Molly works. Write a function using x and $f(x)$ that describes Molly's pay for working x hours.

b. Graph this function and give its domain and range.

52. Write About It For $y = 2x - 1$, make a table of ordered pairs and a graph. Describe the relationships between the equation, the table, and the graph.

53. Critical Thinking Describe a real-world situation that can be represented by a linear function whose domain and range must be limited. Give your function and its domain and range.

MULTI-STEP TEST PREP

54. This problem will prepare you for the Multi-Step Test Prep on page 342.

a. Juan is running on a treadmill. The table shows the number of Calories Juan burns as a function of time. Explain how you can tell that this relationship is linear by using the table.

b. Create a graph of the data.

c. How can you tell from the graph that the relationship is linear?

Time (min)	Calories
3	27
6	54
9	81
12	108
15	135
18	162
21	189

55. **Physical Science** A ball was dropped from a height of 100 meters. Its height above the ground in meters at different times after its release is given in the table. Do these ordered pairs satisfy a linear function? Explain.

Time (s)	0	1	2	3
Height (m)	100	90.2	60.8	11.8

56. **Critical Thinking** Is the equation $x = 9$ a linear equation? Does it describe a linear function? Explain.

TEST PREP

57. Which is NOT a linear function?

Ⓐ $y = 8x$ Ⓑ $y = x + 8$ Ⓒ $y = \frac{8}{x}$ Ⓓ $y = 8 - x$

58. The speed of sound in 0 °C air is about 331 feet per second. Which function could be used to describe the distance in feet d that sound will travel in air in s seconds?

Ⓕ $d = s + 331$ Ⓖ $d = 331s$ Ⓗ $s = 331d$ Ⓙ $s = 331 - d$

59. **Extended Response** Write your own linear function. Show that it is a linear function in at least three different ways. Explain any connections you see between your three methods.

CHALLENGE AND EXTEND

60. What equation describes the x-axis? the y-axis? Do these equations represent linear functions?

Geometry **Copy and complete each table below. Then tell whether the table shows a linear relationship.**

61. Perimeter of a Square

Side Length	Perimeter
1	
2	
3	
4	

62. Area of a Square

Side Length	Area
1	
2	
3	
4	

63. Volume of a Cube

Side Length	Volume
1	
2	
3	
4	

SPIRAL REVIEW

Simplify each expression. *(Lesson 1-4)*

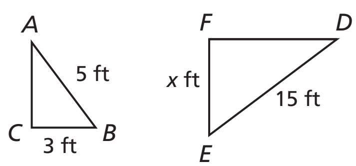
64. 8^2 65. $(-1)^3$ 66. $(-4)^4$ 67. $\left(\frac{1}{3}\right)^2$

Solve each equation. Check your answer. *(Lesson 2-4)*

68. $6m + 5 = 3m - 4$ 69. $2(t - 4) = 3 - (3t + 1)$ 70. $9y + 5 - 2y = 2y + 5 - y + 3$

Find the value of x in each diagram. *(Lesson 2-8)*

71. $\triangle ABC \sim \triangle DEF$



72. $ABCD \sim QRST$