3-7 Solving Absolute-Value Inequalities

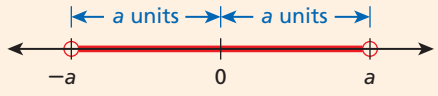
Objective
Solve inequalities in one variable involving absolute-value expressions.

Why learn this?
You can solve an absolute-value inequality to determine the safe range for the pressure of a fire extinguisher. (See Example 3.)

When an inequality contains an absolute-value expression, it can be rewritten as a compound inequality. The inequality $|x| < 5$ describes all real numbers whose distance from 0 is less than 5 units. The solutions are all numbers between -5 and 5, so $|x| < 5$ can be rewritten as $-5 < x < 5$ or as $x > -5$ AND $x < 5$.

Know it! Note

Absolute-Value Inequalities Involving <

WORDS	NUMBERS
The inequality $\|x\| < a$ (when $a > 0$) asks, "What values of x have an absolute value less than a?" The solutions are numbers between $-a$ and a.	$\|x\| < 5$ $-5 < x < 5$ $x > -5$ AND $x < 5$
GRAPH	**ALGEBRA**
	$\|x\| < a$ (when $a > 0$) $-a < x < a$ $x > -a$ AND $x < a$

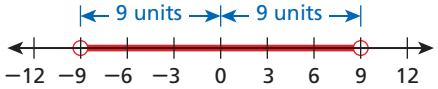
The same properties are true for inequalities that use the symbol $\leq$.

EXAMPLE 1 **Solving Absolute-Value Inequalities Involving <**

Solve each inequality and graph the solutions.

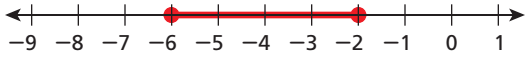
A $|x| + 3 < 12$

$|x| + 3 < 12$
$-3 \quad -3$
$|x| < 9$ — *Since 3 is added to $|x|$, subtract 3 from both sides to undo the addition.*
$x > -9$ AND $x < 9$ — *Write as a compound inequality.*



B $|x + 4| \leq 2$

$x + 4 \geq -2$ AND $x + 4 \leq 2$ — *Write as a compound inequality.*
$-4 \quad -4 \qquad -4 \quad -4$ — *Solve each inequality.*
$x \geq -6$ AND $x \leq -2$ — *Write as a compound inequality.*



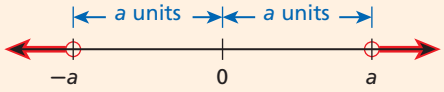
Helpful Hint
Just as you do when solving absolute-value equations, you first isolate the absolute-value expression when solving absolute-value inequalities.

Solve each inequality and graph the solutions.

1a. $2|x| \leq 6$ **1b.** $|x + 3| - 4.5 \leq 7.5$

The inequality $|x| > 5$ describes all real numbers whose distance from 0 is greater than 5 units. The solutions are all numbers less than -5 or greater than 5. The inequality $|x| > 5$ can be rewritten as the compound inequality $x < -5$ OR $x > 5$.

Absolute-Value Inequalities Involving >

WORDS	NUMBERS
The inequality $\|x\| > a$ (when $a > 0$) asks, "What values of x have an absolute value greater than a?" The solutions are numbers less than $-a$ or greater than a.	$\|x\| > 5$ $x < -5$ OR $x > 5$
GRAPH	**ALGEBRA**
	$\|x\| > a$ (when $a > 0$) $x < -a$ OR $x > a$

The same properties are true for inequalities that use the symbol $\geq$.

EXAMPLE 2 Solving Absolute-Value Inequalities Involving >

Solve each inequality and graph the solutions.

A $|x| - 20 > -13$

$|x| - 20 > -13$

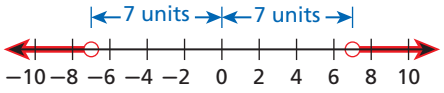
$+20 \quad +20$

$|x| > 7$

Since 20 is subtracted from $|x|$, add 20 to both sides to undo the subtraction.

$x < -7$ OR $x > 7$

Write as a compound inequality.



B $|x - 8| + 5 \geq 11$

$|x - 8| + 5 \geq 11$

$-5 \quad -5$

$|x - 8| \geq 6$

Since 5 is added to $|x - 8|$, subtract 5 from both sides to undo the addition.

$x - 8 \leq -6$ OR $x - 8 \geq 6$

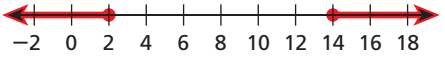
Write as a compound inequality.

$+8 \quad +8 \qquad +8 \quad +8$

$x \leq 2$ OR $x \geq 14$

Solve each inequality.

Write as a compound inequality.



Solve each inequality and graph the solutions.

2a. $|x| + 10 \geq 12$ **2b.** $\left|x + 2\frac{1}{2}\right| + \frac{1}{2} \geq 4$

EXAMPLE 3 Safety Application

Some fire extinguishers contain pressurized water. The water pressure should be 162.5 psi (pounds per square inch), but it is acceptable for the pressure to differ from this value by at most 12.5 psi. Write and solve an absolute-value inequality to find the range of acceptable pressures. Graph the solutions.

Let p represent the actual water pressure of a fire extinguisher.

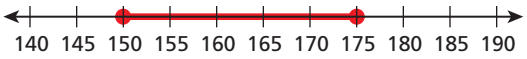
The difference between p and the ideal pressure	is at most	12.5 psi.
$\lvert p - 162.5 \rvert$	$\leq$	12.5

$$\lvert p - 162.5 \rvert \leq 12.5$$

$$p - 162.5 \geq -12.5 \text{ AND } p - 162.5 \leq 12.5 \quad \textit{Solve the two inequalities.}$$

$$+162.5 \quad +162.5 \qquad +162.5 \quad +162.5$$

$$p \geq 150 \text{ AND } p \leq 175$$



The range of acceptable pressures is $150 \leq p \leq 175$.

CHECK IT OUT!

3. A dry-chemical fire extinguisher should be pressurized to 125 psi, but it is acceptable for the pressure to differ from this value by at most 75 psi. Write and solve an absolute-value inequality to find the range of acceptable pressures. Graph the solutions.

When solving an absolute-value inequality, you may get a statement that is true for all values of the variable. In this case, all real numbers are solutions of the original inequality. If you get a false statement when solving an absolute-value inequality, the original inequality has no solutions.

EXAMPLE 4 Special Cases of Absolute-Value Inequalities

Solve each inequality.

A $\lvert x - 6 \rvert + 7 > 2$

$$\lvert x - 6 \rvert + 7 > 2$$

$$-7 \quad -7 \qquad \textit{Subtract 7 from both sides.}$$

$$\lvert x - 6 \rvert > -5 \qquad \textit{Absolute-value expressions are always nonnegative.}$$

Therefore, the statement is true for all values of x.

All real numbers are solutions.

B $\lvert x + 12 \rvert - 5 \leq -6$

$$\lvert x + 12 \rvert - 5 \leq -6$$

$$+5 \quad +5 \qquad \textit{Add 5 to both sides.}$$

$$\lvert x + 12 \rvert \leq -1 \qquad \textit{Absolute-value expressions are always nonnegative.}$$

Therefore, the statement is false for all values of x.

The inequality has no solutions.

Remember!

An absolute value represents a distance, and distance cannot be less than 0.

CHECK IT OUT!

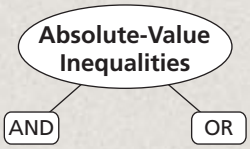
Solve each inequality.

4a. $\lvert x \rvert - 9 \geq -11$

4b. $4\lvert x - 3.5 \rvert \leq -8$

THINK AND DISCUSS

1. Describe how the solutions of $7|x| \leq 21$ are different from the solutions of $7|x| < 21$.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the indicated type of absolute-value inequality and then solve.



Know it! Note

3-7 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA11 3-7
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1 p. 212

1. $|x| - 5 \leq -2$
2. $|x + 1| - 7.8 < 6.2$
3. $|3x| + 2 < 8$
4. $4|x| \leq 20$
5. $|x - 5| + 1 < 2$
6. $\left|x + \frac{1}{2}\right| - \frac{1}{2} \leq 3\frac{1}{2}$

SEE EXAMPLE 2 p. 213

7. $|x| - 6 > 16$
8. $|x| + 2.9 > 8.6$
9. $2|x| \geq 8$
10. $|x + 2| > 7$
11. $|x - 3| + 2 \geq 4$
12. $|x + 5| - 4\frac{1}{2} \geq 7\frac{1}{2}$

SEE EXAMPLE 3 p. 214

13. **Nutrition** A nutritionist recommends that an adult male consume 55 grams of fat per day. It is acceptable for the fat intake to differ from this amount by at most 25 grams. Write and solve an absolute-value inequality to find the range of fat intake that is acceptable. Graph the solutions.

SEE EXAMPLE 4 p. 214

Solve each inequality.

14. $|x| + 8 \leq 2$
15. $|x + 3| < -5$
16. $|x + 4| \geq -8$
17. $|x - 5| + \frac{1}{3} > -1$
18. $|3x| + 7 > 2$
19. $|x - 7| + 3.5 \leq 2$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–25	1
26–31	2
32	3
33–38	4

Extra Practice

Skills Practice p. S9
Application Practice p. S30

Solve each inequality and graph the solutions.

20. $|x| + 6 \leq 10$
21. $|x - 3| < 1$
22. $|x - 2| - 8 \leq -3$
23. $|5x| < 15$
24. $|x - 2.4| + 4 \leq 6.4$
25. $4 + |x + 3| < 7$
26. $|x - 1| > 2$
27. $6|x| \geq 60$
28. $|x - 4| + 3 > 8$
29. $2|x + 2| \geq 16$
30. $3 + |x - 4| > 4$
31. $\left|x - \frac{1}{2}\right| + 9 > 10\frac{1}{2}$
32. The thermostat for a sauna is set to 175 °F, but the actual temperature of the sauna may vary by as much as 12 °F. Write and solve an absolute-value inequality to find the range of possible temperatures. Graph the solutions.

Solve each inequality.

33. $12 + |x| \leq 10$
34. $\left|x + \frac{3}{5}\right| - 2 > -4$
35. $|x + 1| + 5 \geq 4$
36. $|4x| - 3 < -6$
37. $3|x - 4| \leq -9$
38. $|2x| + 9 \geq 9$

Tell whether each statement is sometimes, always, or never true. Explain.

39. The value of $|x + 1|$ is greater than -5.

40. The value of $|x - 7|$ is less than 0.

41. An absolute-value inequality has all real numbers as solutions.

Write and solve an absolute-value inequality for each expression. Graph the solutions on a number line.

42. All numbers whose absolute value is less than or equal to 15

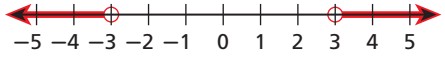
43. All numbers less than or equal to 3 units from 2 on the number line

44. All numbers at least 2 units from 8 on the number line

Write an absolute-value inequality for each graph.

45.

46.



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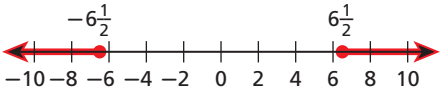

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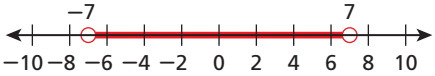
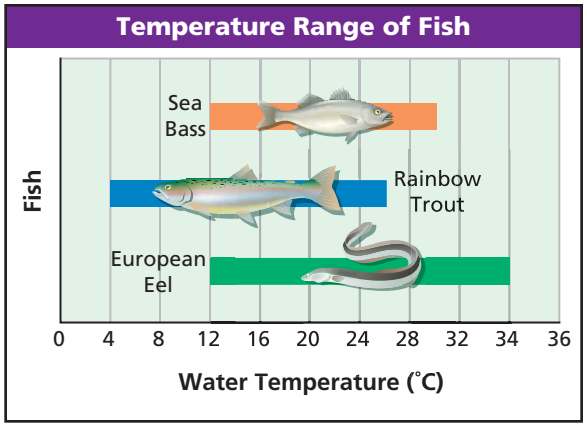


49. Multi-Step The frequency of a sound wave determines its pitch. The human ear can detect a wide range of frequencies, from 20 Hz (very low notes) to 20,000 Hz (very high notes).

a. What frequency is at the middle of the range?

b. Write an absolute-value inequality for the range of frequencies the human ear can detect.

50. Biology The diagram shows the temperature range at which several fish species can survive. For each species, write an absolute-value inequality that gives the range of temperatures at which it can survive.



51. Entertainment On a game show, a contestant must guess a secret two-digit number. The secret number is 23. Write an absolute-value inequality that shows that the contestant's guess is more than 12 numbers away from the secret number.

52. This problem will help prepare you for the Multi-Step Test Prep on page 218.

The manager of a band recommends that the band sell its CDs for $8.75. The band decides to sell the CDs for p dollars.

a. Write an absolute-value expression that tells how far the band's price is from the recommended price.

b. The band wants the price of its CD to be no more than $1.25 from the recommended price. Write an absolute-value inequality that gives the range of possible prices for the CD.

c. Solve the inequality. Write the solution as a compound inequality.

53. Critical Thinking For which values of k does the inequality $|x| + 1 < k$ have no solutions? Explain.

54. Write About It Describe how to use an absolute-value inequality to find all the values on a number line that are within 5 units of -6.

TEST PREP

55. What is the solution of the inequality $3 + |x + 4| < 6$?

Ⓐ $-13 < x < 5$ Ⓒ $-6 < x < -2$

Ⓑ $-7 < x < -1$ Ⓓ $1 < x < 7$

56. A thermometer gives temperature readings that may be inaccurate by at most 2 °F. The actual temperature is 75 °F. Which absolute-value inequality describes the range of temperatures that may be shown on the thermometer?

Ⓕ $|x - 75| \leq 2$ Ⓖ $|x + 75| \leq 2$ Ⓗ $|x - 75| \geq 2$ Ⓙ $|x + 75| \geq 2$

57. The inequality $|w - 156| \leq 3$ describes the weights of members of a wrestling team. Which statement is NOT true?

Ⓐ All of the team members weigh no more than 159 pounds.

Ⓑ A team member may weigh 152 pounds.

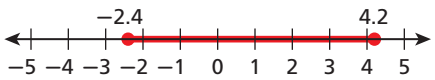
Ⓒ Every member of the team is at most 3 pounds away from 156 pounds.

Ⓓ There are no team members who weigh 160 pounds.

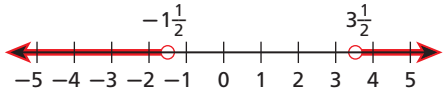
CHALLENGE AND EXTEND

Write an absolute-value inequality for each graph.

58.



59.



60. Critical Thinking Fill in the missing reasons to justify each step in solving $|2x - 6| + 5 \leq 7$.

Statements	Reasons
1. $\|2x - 6\| + 5 \leq 7$	Given
2. $\|2x - 6\| \leq 2$	?
3. $2x - 6 \geq -2$ AND $2x - 6 \leq 2$	Definition of absolute value
4. $2x \geq 4$ AND $2x \leq 8$	?
5. $x \geq 2$ AND $x \leq 4$	?

SPIRAL REVIEW

Solve each proportion. Check your answer. *(Lesson 2-7)*

61. $\frac{x+1}{4} = \frac{5}{8}$ **62.** $\frac{2}{15} = \frac{6}{y-5}$ **63.** $\frac{12}{m+2} = \frac{8}{3}$ **64.** $\frac{7+g}{10} = \frac{6}{8}$

Describe the solutions of each inequality in words. *(Lesson 3-1)*

65. $16 > 8m$ **66.** $c + 4 < 11$ **67.** $-4 \leq x + 2$ **68.** $0 \geq x + 7$

Solve each compound inequality and graph the solutions. *(Lesson 3-6)*

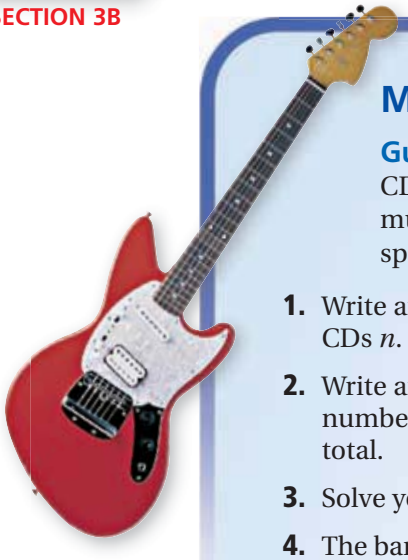
69. $-3 < x - 3 < 1$ **70.** $-3 \leq 2x + 1 \leq 9$

71. $x - 2 < -1$ OR $x - 2 > 2$ **72.** $x + 4 \leq 3$ OR $x + 4 \geq 6$

CHAPTER 3

MULTI-STEP TEST PREP

SECTION 3B



Multi-Step and Compound Inequalities

Guitar Picks Cullen and his band are interested in recording a CD of their music. The recording studio charges $450 to record the music and then charges $5 for each CD. The band is required to spend at least $1000 for the total of the recording and CD charges.

1. Write an equation for the cost c of the CDs based on the number of CDs n.
2. Write an inequality that can be used to determine the minimum number of CDs that must be burned at this studio to meet the $1000 total.
3. Solve your inequality from Problem 2.
4. The band orders the minimum number of CDs found in Problem 3. They want to sell the CDs and make at least as much money as they spent for the recording studio and making the CDs. Write an inequality that can be solved to determine the minimum amount the band should charge for their CDs.
5. Solve your inequality from Problem 4.
6. If the band has 30 more CDs made than the minimum number found in Problem 3 and charges the minimum price found in Problem 5, will they make a profit? If so, how much profit will the band make?

READY TO GO ON?

Quiz for Lessons 3-4 Through 3-7

3-4 Solving Two-Step and Multi-Step Inequalities

Solve each inequality and graph the solutions.

1. $2x + 3 < 9$ **2.** $3t - 2 > 10$ **3.** $7 \geq 1 - 6r$

Solve each inequality.

4. $2(x - 3) > -1$ **5.** $\frac{1}{3}a + \frac{1}{2} > \frac{2}{3}$

6. $15 < 5(m - 7)$ **7.** $2 + (-6) > 0.8p$

8. The average of Mindy's two test scores must be at least 92 to make an A in the class. Mindy got an 88 on her first test. What scores can she get on her second test to make an A in the class?

3-5 Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.

9. $5x < 3x + 8$ **10.** $6p - 3 > 9p$ **11.** $r - 8 \geq 3r - 12$

Solve each inequality.

12. $3(y + 6) > 2(y + 4)$ **13.** $4(5 - g) \geq g$

14. $4x < 4(x - 1)$ **15.** $3(1 - x) \geq -3(x + 2)$

16. Phillip has \$100 in the bank and deposits \$18 per month. Gil has \$145 in the bank and deposits \$15 per month. For how many months will Gil have a larger bank balance than Phillip?

3-6 Solving Compound Inequalities

Solve each compound inequality and graph the solutions.

17. $-2 \leq x + 3 < 9$ **18.** $m + 2 < -1$ OR $m - 2 > 6$

19. $-3 \geq x - 1 > 2$ **20.** $-2 > r + 2$ OR $r + 4 < 5$

21. It is recommended that a certain medicine be stored in temperatures above 32 °F and below 70 °F. Write a compound inequality to show the acceptable storage temperatures for this medicine.

3-7 Solving Absolute-Value Inequalities

Solve each inequality and graph the solutions.

22. $|x| + 9 \leq 12$ **23.** $|x + 7| - 15 < 6$ **24.** $4.5|x| \geq 31.5$

Solve each inequality.

25. $|x - 2| \leq 14$ **26.** $|x| - 9.2 < -5.7$

27. $\frac{1}{2} + 2|x| > -4$ **28.** $7 + |3x| > 13$

29. Eli attended a concert. The decibel level of the music averaged 110 decibels but varied by 22 decibels from the average. Write and solve an absolute-value inequality to find the decibel range. Graph the solutions.

CHAPTER 3

Study Guide: Review

Vocabulary

compound inequality 204
inequality 170
intersection 205
solution of an inequality 170
union 206

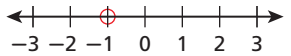
Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a mathematical statement that two quantities are not equal.
2. The numbers that are solutions to either inequality of a compound inequality is the ___?___.
3. A statement formed by combining two simple inequalities with the words AND or OR is a(n) ___?___.
4. The numbers that are solutions to both inequalities of a compound inequality is the ___?___.
5. Any value that makes the inequality true is a(n) ___?___.

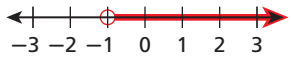
3-1 Graphing and Writing Inequalities *(pp. 170–175)*

EXAMPLES

■ **Graph the inequality $y > -1$.**

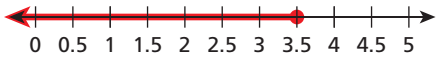


Draw an empty circle at −1.



Shade all the numbers greater than −1.

■ **Write the inequality shown by the graph.**

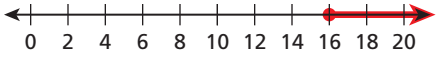


$n \leq 3.5$ *Use the variable n. The arrow points left, so use either < or ≤. The solid circle means 3.5 is a solution, so use ≤.*

■ **Write an inequality for the situation and graph the solutions.**

Applicants for a driver's permit must be at least 16 years old.

age	must be at least	16 years
a	$\geq$	16



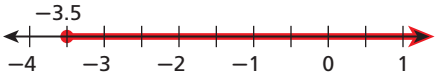
EXERCISES

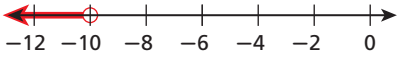
Graph each inequality.

6. $x > -3$
7. $p \leq 4$
8. $-1 > t$
9. $r \geq 9.5$
10. $2(3 - 5) < k$
11. $w < 3$

Write the inequality shown by each graph.

12. 

13. 

14. 

Define a variable and write an inequality for each situation. Graph the solutions.

15. The temperature must be at least 72 °F.
16. No more than 12 students were present.
17. It takes less than 30 minutes to complete the lab activity.

3-2 Solving Inequalities by Adding or Subtracting *(pp. 176–181)*

EXAMPLES

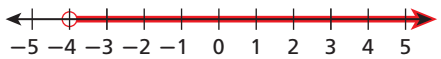
Solve each inequality and graph the solutions.

■ $x + 6 > 2$

$x + 6 > 2$ *Since 6 is added to x, subtract 6 from both sides.*

$-6 \quad -6$

$x > -4$

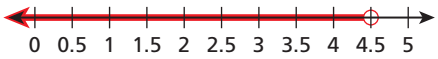


■ $n - 1.3 < 3.2$

$n - 1.3 < 3.2$ *Since 1.3 is subtracted from x, add 1.3 to both sides.*

$+1.3 \quad +1.3$

$n < 4.5$



EXERCISES

Solve each inequality and graph the solutions.

18. $t + 3 < 10$

19. $k - 7 \leq -5$

20. $-1 < m + 4$

21. $x + 2.3 \geq 6.8$

22. $w - 3 < 6.5$

23. $4 > a - 1$

24. $h - \frac{1}{4} < \frac{3}{4}$

25. $5 > 7 + v$

26. Tammy wants to run at least 10 miles per week. So far this week, she ran 4.5 miles. Write and solve an inequality to determine how many more miles Tammy must run this week to reach her goal.

27. Rob has a gift card for $50. So far, he has selected a shirt that costs $32. Write and solve an inequality to determine the additional amount Rob could spend without exceeding the gift card limit.

3-3 Solving Inequalities by Multiplying or Dividing *(pp. 182–187)*

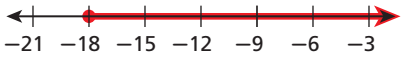
EXAMPLES

■ **Solve $\frac{p}{-3} \leq 6$ and graph the solutions.**

$\frac{p}{-3} \leq 6$ *Since p is divided by −3, multiply both sides by −3.*

$-3 \cdot \frac{p}{-3} \geq -3 \cdot 6$

$p \geq -18$ *Change ≤ to ≥.*



■ **Pizzas cost $5.50 each. What are the possible numbers of pizzas that can be purchased with $30?**

Let n represent the number of pizzas that can be purchased.

$5.50	times	number of pizzas	is at most	$30.
5.50	•	n	≤	30

$5.50n \leq 30$

$\frac{5.50n}{5.50} \leq \frac{30}{5.50}$ *Since n is multiplied by 5.50, divide both sides by 5.50.*

$n \leq 5\frac{5}{11}$

Only a whole number of pizzas can be purchased, so 0, 1, 2, 3, 4, or 5 pizzas can be purchased.

EXERCISES

Solve each inequality and graph the solutions.

28. $3a \leq 15$

29. $-18 < 6t$

30. $\frac{p}{4} > 2$

31. $\frac{2}{5}x \leq -10$

32. $-3n < -18$

33. $\frac{g}{-2} > 6$

34. $-2k < 14$

35. $-3 > \frac{1}{3}r$

36. $27 < -9h$

37. $-0.4g > -1$

38. Notebooks cost $1.39 each. What are the possible numbers of notebooks that can be purchased with $10?

39. A senior class is selling lanyards as a fundraiser. The profit for each lanyard is $0.75. Write and solve an inequality to determine the number of lanyards the class must sell to make a profit of at least $250.

3-4 Solving Two-Step and Multi-Step Inequalities *(pp. 190–195)*

EXAMPLES

Solve each inequality and graph the solutions.

■ $18 + 3t > -12$

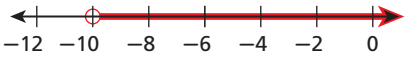
$18 + 3t > -12$ — *Since 18 is added to 3t, subtract 18 from both sides.*

$-18 \quad -18$

$3t > -30$

$\frac{3t}{3} > \frac{-30}{3}$ — *Since t is multiplied by 3, divide both sides by 3.*

$t > -10$



■ $3^2 - 5 \le 2(1 + x)$

$3^2 - 5 \le 2(1 + x)$ — *Simplify the left side using order of operations.*

$9 - 5 \le 2(1 + x)$

$4 \le 2(1 + x)$ — *Distribute 2 on the right side.*

$4 \le 2(1) + 2(x)$

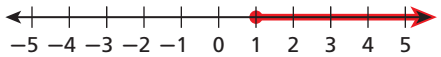
$4 \le 2 + 2x$ — *Since 2 is added to 2x, subtract 2 from both sides*

$-2 \quad -2$

$2 \le 2x$

$\frac{2}{2} \le \frac{2x}{2}$ — *Since x is multiplied by 2, divide both sides by 2.*

$1 \le x$



EXERCISES

Solve each inequality and graph the solutions.

40. $3x + 4 < 19$

41. $7 \le 2t - 5$

42. $\frac{m + 3}{2} > -4$

43. $2(x + 5) < 8$

44. $-4(2 - 5) > (-3)^2 - h$

45. $\frac{1}{5}x + \frac{1}{2} > \frac{4}{5}$

46. $0.5(b - 2) \le 4$

47. $\frac{1}{3}y - \frac{1}{2} > \frac{2}{3}$

48. $6 - 0.2n < 9$

49. Carl's Cable Company charges \$55 for monthly service plus \$4 for each pay-per-view movie. Teleview Cable Company charges \$110 per month with no fee for movies. For what number of movies is the cost of Carl's Cable Company less than the cost of Teleview?

3-5 Solving Inequalities with Variables on Both Sides *(pp. 196–202)*

EXAMPLES

■ **Solve $b + 16 < 3b$ and graph the solutions.**

$b + 16 > 3b$ — *Subtract b from both sides so that the coefficient of b is positive.*

$-b \quad -b$

$16 > 2b$

$\frac{16}{2} > \frac{2b}{2}$ — *Since b is multiplied by 2, divide both sides by 2.*

$8 > b$

■ **Solve the inequality $3(1 + k) > 4 + 3k$.**

$3 + 3k > 4 + 3k$ — *Distribute 3 on the left side.*

The same variable term ($3k$) appears on both sides.

For any number $3k$, adding 3 will never result in a greater number than adding 4.

There are no solutions.

EXERCISES

Solve each inequality and graph the solutions.

50. $5 + 2m < -3m$

51. $y \le 6 + 4y$

52. $4c - 7 > 9c + 8$

53. $-3(2 - q) \ge 6(q + 1)$

54. $2(5 - x) < 3x$

55. $3.5t - 1.8 < 1.6t + 3.9$

Solve each inequality.

56. $d - 2 < d - 4$

57. $2(1 - x) > -2(1 + x)$

58. $4(1 - p) < 4(2 + p)$

59. $3w + 1 > 3(w - 1)$

60. $5(4 - k) < 5k$

61. $3(c + 1) > 3c + 5$

62. Hanna has a savings account with a balance of \$210 and deposits \$16 per month. Faith has a savings account with a balance of \$175 and deposits \$20 per month. Write and solve an inequality to determine the number of months Hanna's account balance will be greater than Faith's account balance.

3-6 Solving Compound Inequalities *(pp. 204–210)*

EXAMPLES

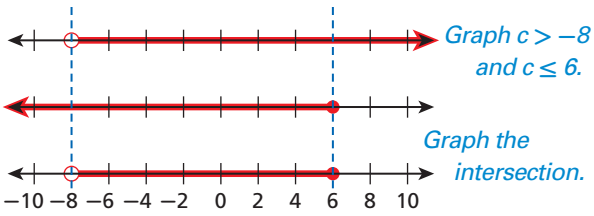
Solve each compound inequality and graph the solutions.

■ $-3 < c + 5 \leq 11$

$\underline{-5} \quad \underline{-5} \quad \underline{-5}$

$-8 < c \quad \leq 6$

Since 5 is added to c, subtract 5 from each part of the inequality.



Graph $c > -8$ and $c \leq 6$.

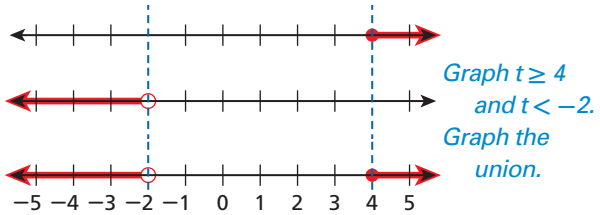
Graph the intersection.

■ $-2 + t \geq 2$ OR $t + 3 < 1$

$\underline{+2} \quad \underline{+2} \quad \underline{-3} \quad \underline{-3}$

$t \geq 4$ OR $t < -2$

Solve the simple inequalities.



Graph $t \geq 4$ and $t < -2$.

Graph the union.

EXERCISES

Solve each compound inequality and graph the solutions.

63. $-4 < t + 6 < 10$

64. $-8 < k - 2 \leq 5$

65. $-3 + r > 4$ OR $r + 1 < -1$

66. $2 > n + 3 > 5$

67. $12 \geq p + 7 > 5$

68. $3 < s + 9$ OR $1 > s - 4$

69. One day, the high temperature was 84 °F and the low temperature was 68 °F. Write a compound inequality to represent the day's temperatures.

70. The table shows formulas for the recommended heart rates during exercise for a person who is a years old. Write and solve a compound inequality to determine the heart rate range for a 16-year-old person.

Recommended Heart Rate Range	
Lower Limit	$0.5 \times (220 - a)$
Upper Limit	$0.9 \times (220 - a)$

3-7 Solving Absolute-Value Inequalities *(pp. 212–217)*

EXAMPLES

Solve each inequality and graph the solutions.

■ $|x| + 4 < 9$

$|x| + 4 < 9$

$\underline{-4} \quad \underline{-4}$

$|x| < 5$

Subtract 4 from both sides.

$x > -5$ AND $x < 5$

Write as a compound inequality.

■ $|x - 3| + 7 \geq 13$

$\underline{-7} \quad \underline{-7}$

$|x - 3| \geq 6$

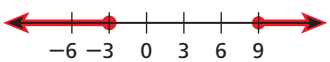
Subtract 7 from both sides.

$x - 3 \leq -6$ OR $x - 3 \geq 6$

$\underline{+3} \quad \underline{+3} \quad \underline{+3} \quad \underline{+3}$

$x \leq -3$ OR $x \geq 9$

Solve the two inequalities.



EXERCISES

Solve each inequality and graph the solutions.

71. $|x| - 7 \leq 15$

72. $|x + 4| > 8$

73. $6|x| \leq 24$

74. $|x + 9| + 11 < 20$

75. $3|x| \geq 9$

76. $4|2x| < 24$

Solve the inequality.

77. $|x| - 5.4 > 8.5$

78. $|5.2 + x| < 7.3$

79. $|x - 7| + 10 \geq 12$

80. $14|x| - 15 \geq 41$

81. $\left|x - \frac{1}{2}\right| + 4 \leq \frac{5}{2}$

82. $|x + 5.5| - 6.4 \leq 4.9$

83. The water depth for a pool is set to 6 ft, but the actual depth of the pool may vary by as much as 4 in. Write and solve an absolute-value inequality to find the range of possible water depths in inches. Graph the solutions.

CHAPTER 3

CHAPTER TEST

Describe the solutions of each inequality in words.

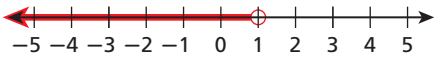
1. $-6 \le m$ **2.** $3t > 12$ **3.** $-x \ge 2$ **4.** $2 + b \le 10$

Graph each inequality.

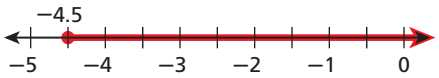
5. $b > -3$ **6.** $2.5 < c$ **7.** $y \le -\sqrt{25}$ **8.** $3 - (4 + 7) \ge h$

Write the inequality shown by each graph.

9.



10.



Write an inequality for the situation and graph the solutions.

11. Madison must run a mile in no more than 9 minutes to qualify for the race.

Solve each inequality and graph the solutions.

12. $d - 5 > -7$ **13.** $f + 4 < -3$ **14.** $4.5 \ge s + 3.2$ **15.** $g + (-2) \le 9$

16. Students need at least 75 hours of volunteer service to meet their graduation requirement. Samir has already completed 48 hours. Write and solve an inequality to determine how many more hours he needs to complete.

Solve each inequality and graph the solutions.

17. $-2c \le 2$ **18.** $3 > \frac{k}{2}$ **19.** $\frac{4}{5}x \le -8$ **20.** $\frac{b}{3} > -7$

21. Marco needs to buy premium gasoline for his car. He has $20 in his wallet. Write and solve an inequality to determine how many gallons of gas Marco can buy.

Gasoline Prices ($)		
Regular	Plus	Premium
2.05	2.12	2.25

Solve each inequality and graph the solutions.

22. $3x - 8 < 4$ **23.** $-2(c - 3) > 4$ **24.** $5 \le \frac{3}{4}n - 2^4$ **25.** $3 - 2a \le -15 + (-9)$

Solve each inequality.

26. $2k - 6 > 3k + 2$ **27.** $2(5 - f) \le f + 12$ **28.** $\frac{3}{2}d \le -\frac{1}{2}d + 6$

Solve each compound inequality and graph the solutions.

29. $-1 \le x - 3 < 3$ **30.** $t + 7 < 3$ OR $t - 1 > 4$ **31.** $4 \le d - 2 < 5$

32. The driving school instructor has asked Lina to stay within 2 miles of the posted speed limits. The current road has a speed limit of 45 mi/h. Write a compound inequality to show Lina's acceptable speeds s.

Solve each inequality.

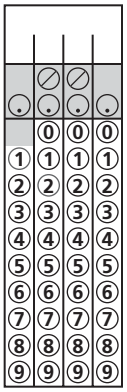
33. $|x - 3| + 7 < 17$ **34.** $6|x| + 4 \ge 16$ **35.** $|x + 12| \le 23$

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 3

FOCUS ON SAT STUDENT-PRODUCED RESPONSES

Ten questions on the SAT require you to enter your answer in a special grid like the one shown. You do not have to write your answer in the boxes at the top of the grid, but doing this may help you avoid errors when filling in the grid. The circles must be filled in correctly for you to receive credit.



HOT TIP!

You cannot enter a zero in the first column of the grid. This is to encourage you to give a more accurate answer when you need to round. For example, $\frac{1}{16}$ written as a decimal is 0.0625. This should be entered in the grid as .063 instead of 0.06.

You may want to time yourself as you take this practice test. It should take you about 9 minutes to complete.

1. Mailing a standard-sized letter in 2005 by first-class mail cost \$0.37 for a letter weighing 1 ounce or less and \$0.23 for each additional ounce. How much did it cost, in dollars, to send a standard-sized letter that weighed 3 ounces?

2. If $p = q - 2$ and $\frac{q}{3} = 9$, what is the value of p?

3. Give the maximum value of x if $12 - 3(x + 1) \geq \frac{1}{2}(3 - 5)$.

4. Give the minimum value of x if $2x + y \leq 7x - 9$ and $y = -3$.

5. For what integer value of x is $2x - 9 < 5$ and $x - 1 > 4$?

6. What is the minimum value of z that satisfies the inequality $z - 7.3 \geq 4.1$?

7. To be eligible for financial aid, Alisa must work at least 15 hours per week in a work-study program. She wants to spend at least 5 more hours studying than working each week. What is the minimum number of hours per day (Monday through Friday) that she must study to meet this goal and be eligible for financial aid?

8. For all real numbers a and b, define the operation # as follows:

$$a \# b = 2a - b$$

Given $a = 3$ and $a \# b = 1$, what is the value of b?

CHAPTER 3

TEST TACKLER

Standardized Test Strategies

Short Response: Understand Short Response Scores

To answer a short-response question completely, you must show how you solved the problem and explain your answer. Short response questions are scored using a 2-point scoring rubric. A sample scoring rubric is provided below.

EXAMPLE 1

Short Response An online company offers free shipping if the cost of the order is at least $35. Your order currently totals $26.50. Write an inequality to show how much more you need to spend to qualify for free shipping. Solve the inequality and explain what your answer means.

2-point response:

Let c be the amount I must add to my order.
c plus the amount I already ordered must be at least $35.
$c + 26.50 \geq 35$
$c + 26.50 \geq 35$
$c + 26.50 - 26.50 \geq 35 - 26.50$
$c \geq 8.50$
Check:
$8.50 + 26.50 \geq 35$ ✓
To get free shipping on the order, I must spend at least $8.50 more since $8.50 + $26.50 is at least $35.

The student wrote and solved an inequality correctly. The student showed all work and explained the meaning of the solution to the inequality.

1-point response:

$c + 26.50 > 35$
$c > 8.50$
$8.50

The student gave a correct answer, but the inequality symbol shown in the student's work is incorrect. No explanation was given.

0-point response:

$9.25

The student gave an answer that satisfies the problem but did not show any work or give an explanation.

Scoring Rubric:

2 points: The student writes and correctly solves an inequality, showing all work. Student defines the variable, answers the question in a complete sentence, and provides an explanation.

1 point: The student writes and correctly solves an inequality but does not show all work, does not define the variable, or does not provide an explanation.

1 point: The student writes and solves an inequality but gives an incorrect answer. The student shows all work and provides an explanation for the answer.

0 points: The student gives no response or provides a solution without showing any work or explanation.

Read short-response test items carefully. If you are allowed to write in the test booklet, underline or circle the parts of the question that tell you what your answer must include. Be sure to explain how you get your answer in complete sentences.

Read each sample and answer the questions that follow by using the scoring rubric below.

Scoring Rubric:

2 points: The student demonstrates a thorough understanding of the concept, correctly answers the question, and provides a complete explanation.

1 point: The student correctly answers the question but does not show all work or does not provide an explanation.

1 point: The student makes minor errors resulting in an incorrect solution but shows and explains understanding of the concept.

0 points: The student gives a response but shows no work or explanation, or the student gives no response.

Sample A

Short Response Write a real-world situation that can be modeled by the inequality $25s - 75 \geq 250$. Solve for s and explain how the value of s relates to your situation.

Student's Answer

A painter rents a booth at the county fair for $75. The artist sells his paintings for $25 each. If he makes at least $250 in profit, he can buy a new easel.

The artist has to sell at least 13 paintings.

1. What score should the student's answer receive? Explain your reasoning.
2. What additional information, if any, should the student's answer include in order to receive full credit?

Sample B

Short Response How do the solutions of $3s - 10 < 15 - 2s$ and $-34 + 9s \leq 4s - 9$ differ? How are the solutions alike? Include a graph in your explanation.

Student's Answer

Solve both inequalities.

$3s - 10 < 15 - 2s$

$+2s \quad +2s$

$5s - 10 < 15$

$+10 \quad +10$

$5s < 25$

$\frac{5s}{5} < \frac{25}{5}$

$s < 5$

$-34 + 9s \leq 4s - 9$

$-4s \quad -4s$

$-34 + 5s \leq -9$

$+34 \quad +34$

$5s \leq 25$

$\frac{5s}{5} \leq \frac{25}{5}$

$s \leq 5$

Blue graph: $s < 5$ Red graph: $s \leq 5$

3. What score should the student's answer receive? Explain your reasoning.
4. What additional information, if any, should the student's answer include in order to receive full credit?

Sample C

Short Response Explain the difference between the solution of the equation $x - 6 = 2x + 9$ and the solutions of the inequality $x - 6 < 2x + 9$.

Student's Answer

The equation has a solution of $x = -15$, and the inequality has a solution of $x > -15$. The equation is true only when x equals –15. The inequality is true for all values greater than –15.

5. What score should the student's answer receive? Explain your reasoning.
6. What additional information, if any, should the student's answer include in order to receive full credit?

CHAPTER 3

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–3

Multiple Choice

1. Which algebraic expression means "5 less than y"?

Ⓐ $5 - y$
Ⓑ $y - 5$
Ⓒ $5 < y$
Ⓓ $5 \div y$

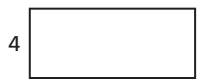
2. Which expression is equivalent to $5 + 2(x - 5)$?

Ⓕ $2x$
Ⓖ $2x + 5$
Ⓗ $2x - 5$
Ⓙ $7x - 35$

3. If $t + 8 = 2$, find the value of $2t$.

Ⓐ -12
Ⓑ -6
Ⓒ 12
Ⓓ 20

4. The length of the rectangle is $2(x + 1)$ meters and the perimeter is 60 meters. What is the length of the rectangle?

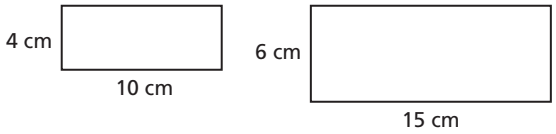


Ⓕ 12 meters
Ⓖ 26 meters
Ⓗ 28 meters
Ⓙ 56 meters

5. Samantha opened a bank account in June and deposited some money. She deposited twice that amount in August. At the end of August, Samantha had less than $600 in her account. If she made no other withdrawals or deposits, which inequality could be used to determine the maximum amount Samantha could have deposited in June?

Ⓐ $2x < 600$
Ⓑ $2x > 600$
Ⓒ $3x < 600$
Ⓓ $3x > 600$

6. Which proportion could be used to determine the ratio of the areas of these similar rectangles?

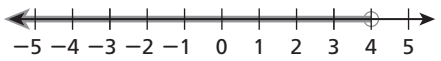


Ⓕ $\frac{2}{3}$
Ⓖ $\frac{2}{5}$
Ⓗ $\frac{4}{9}$
Ⓙ $\frac{4}{25}$

7. For which inequality is -2 a solution?

Ⓐ $2x < -4$
Ⓑ $-2x < 4$
Ⓒ $-2x > -4$
Ⓓ $-2x < -4$

8. Which graph shows the solutions of $-2(1 - x) < 3(x - 2)$?

Ⓕ


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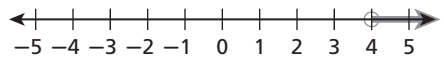

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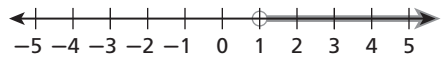
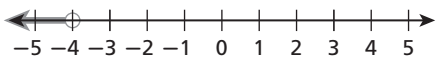


Ⓙ


9. Which compound inequality has no solution?

Ⓐ $x > 1$ OR $x < -2$
Ⓑ $x < 1$ AND $x > -2$
Ⓒ $x < 1$ OR $x < -2$
Ⓓ $x > 1$ AND $x < -2$

To check your answer, use a different method to solve the problem from the one you originally used. If you made a mistake the first time, you are unlikely to make the same mistake when you solve the problem a different way.

10. Which inequality has the same solutions as $p < -2$?

Ⓕ $p + 1 < -2$

Ⓖ $p + 4 < 2$

Ⓗ $2p + 1 < -4$

Ⓙ $3p < -12$

11. What is the greatest integer solution of $5 - 3m > 11$?

Ⓐ 0

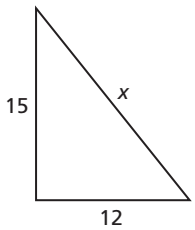
Ⓑ -1

Ⓒ -2

Ⓓ -3

Gridded Response

12. The sum of the measures of any two sides of a triangle must be greater than the measure of the third side. What is the greatest possible integer value for x?



13. After 2 years, the simple interest paid on an investment of $2500 was $175. What percent was the interest rate?

14. Amy's bowling score in her third game was 10 points less than her score in the first game and 5 points more than her score in the second game. The total points for all three games was no more than 275. What is the greatest number of points Amy could have scored in her first game?

15. Trevor needs a 93 on his second quiz to have a quiz average of 90. What score did Trevor receive on his first quiz?

16. The radius of a circle can be determined by the formula $r = \sqrt{\frac{A}{\pi}}$. What is the length in meters of the radius of a circle that has an area of 314 square meters? (Use 3.14 for π.)

Short Response

17. Write 2 different inequalities that have the same solution as $n > 3$ such that

a. the first inequality uses the symbol $>$ and requires addition or subtraction to solve.

b. the second inequality uses the symbol $<$ and requires multiplication or division to solve.

18. Alison has twice as many video games as Kyle. Maurice has 5 more video games than Alison. The total number of video games is less than 40.

a. Write an inequality to represent this situation.

b. Solve the inequality to determine the greatest number of video games Maurice could have. Justify each step in your solution.

19. Donna's Deli delivers lunches for $7 per person plus a $35 delivery fee. Larry's Lunches delivers lunches for $11 per person.

a. Write an expression to represent the cost of x lunches from Donna's Deli. Write an expression to represent the cost of ordering x lunches from Larry's Lunches.

b. Write an inequality to determine the number of lunches for which the cost of Larry's Lunches is less than the cost of Donna's Deli.

c. Solve the inequality and explain what the answer means. Which restaurant charges less for an order of 10 lunches?

Extended Response

20. Aleya has two employment opportunities. Company A offered her a yearly salary of $31,000. Company B offered her a similar position with a yearly salary of $27,000 plus 2.5% commission on her total sales for the year.

a. Let x represent Aleya's total sales for the year at company B. Write an expression to represent the total income after one year at company B.

b. Use your expression from part **a** to write an inequality that could be solved to determine the amount of sales for which the yearly income at company A would be greater than that at company B.

c. Solve the inequality from part **b** and explain the meaning of the solution in relation to Aleya's decision to work for company A or company B.

d. How much more than the salary at company A would Aleya make after one year at company B if her total sales for the year were $200,000?