3-6 Solving Compound Inequalities

Objectives
Solve compound inequalities in one variable.

Graph solution sets of compound inequalities in one variable.

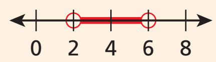
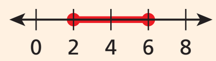
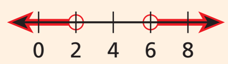
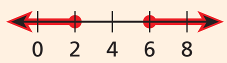
Vocabulary
compound inequality
intersection
union

Who uses this?
A lifeguard can use compound inequalities to describe the safe pH levels in a swimming pool. (See Example 1.)

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a **compound inequality**.

Know it! Note

Compound Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers greater than 2 AND less than 6	$x > 2$ AND $x < 6$ $2 < x < 6$	
All real numbers greater than or equal to 2 AND less than or equal to 6	$x \geq 2$ AND $x \leq 6$ $2 \leq x \leq 6$	
All real numbers less than 2 OR greater than 6	$x < 2$ OR $x > 6$	
All real numbers less than or equal to 2 OR greater than or equal to 6	$x \leq 2$ OR $x \geq 6$	

EXAMPLE 1

Chemistry Application

A water analyst recommends that the pH level of swimming pool water be between 7.2 and 7.6 inclusive. Write a compound inequality to show the pH levels that are within the recommended range. Graph the solutions.

Helpful Hint

The phrase "between 7.2 and 7.6 *inclusive*" means that the numbers 7.2 and 7.6 are included in the solutions. Use a solid circle for endpoints that are solutions.

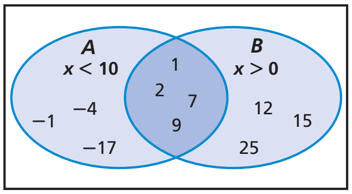
Let p be the pH level of swimming pool water.

7.2	is less than or equal to	pH level	is less than or equal to	7.6
7.2	$\leq$	p	$\leq$	7.6

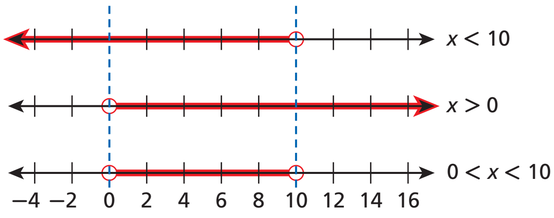
$7.2 \leq p \leq 7.6$

CHECK IT OUT! **1.** The free chlorine level in a pool should be between 1.0 and 3.0 parts per million inclusive. Write a compound inequality to show the levels that are within this range. Graph the solutions.

In this diagram, oval A represents some integer solutions of $x < 10$, and oval B represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of *both* $x < 10$ *and* $x > 0$.



You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the **intersection** and shows the numbers that are solutions of both inequalities.



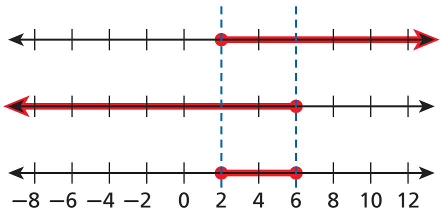
EXAMPLE 2 Solving Compound Inequalities Involving AND

Solve each compound inequality and graph the solutions.

A $4 \le x + 2 \le 8$

$$\begin{array}{lcl} 4 \le x + 2 & \text{AND} & x + 2 \le 8 \\ \underline{-2 \quad\; -2} & & \underline{-2 \quad\; -2} \\ 2 \le x & \text{AND} & x \le 6 \end{array}$$

Write the compound inequality using AND.
Solve each simple inequality.



Graph $2 \le x$.

Graph $x \le 6$.

Graph the intersection by finding where the two graphs overlap.

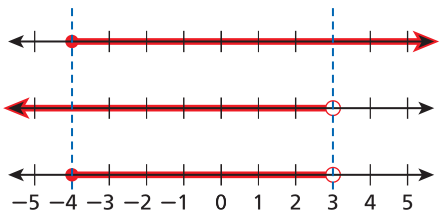
Remember!
The statement $-5 \le 2x + 3 \le 9$ consists of two inequalities connected by AND. Example 2B shows a "shorthand" method for solving this type of inequality.

B $-5 \le 2x + 3 < 9$

$$\begin{array}{l} -5 \le 2x + 3 < 9 \\ \underline{-3 \qquad\;\; -3 \quad -3} \\ -8 \le 2x < 6 \\ \frac{-8}{2} \le \frac{2x}{2} < \frac{6}{2} \\ -4 \le x < 3 \end{array}$$

Since 3 is added to 2x, subtract 3 from each part of the inequality.

Since x is multiplied by 2, divide each part of the inequality by 2.



Graph $-4 \le x$.

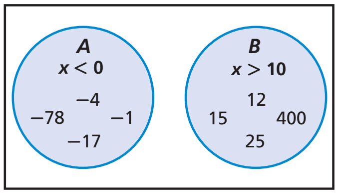
Graph $x < 3$.

Graph the intersection by finding where the two graphs overlap.

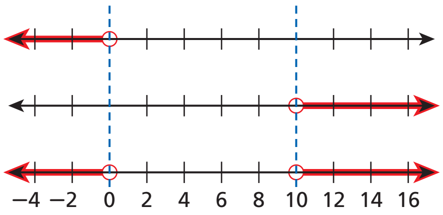
CHECK IT OUT! **Solve each compound inequality and graph the solutions.**

2a. $-9 < x - 10 < -5$ **2b.** $-4 \le 3n + 5 < 11$

In this diagram, circle A represents some integer solutions of $x < 0$, and circle B represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of *either* $x < 0$ *or* $x > 10$.



You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combined regions are called the **union** and show the numbers that are solutions of either inequality.



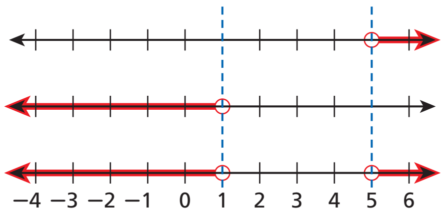
EXAMPLE 3 Solving Compound Inequalities Involving OR

Solve each compound inequality and graph the solutions.

A $\mathbf{-4 + a > 1 \text{ OR } -4 + a < -3}$

$$-4 + a > 1 \text{ OR } -4 + a < -3$$
$$\underline{+4} \quad \underline{+4} \qquad \underline{+4} \quad \underline{+4}$$
$$a > 5 \text{ OR } a < 1$$

Solve each simple inequality.



Graph a > 5.

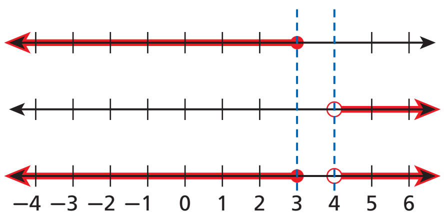
Graph a < 1.

Graph the union by combining the regions.

B $\mathbf{2x \le 6 \text{ OR } 3x > 12}$

$$2x \le 6 \text{ OR } 3x > 12$$
$$\frac{2x}{2} \le \frac{6}{2} \qquad \frac{3x}{3} > \frac{12}{3}$$
$$x \le 3 \text{ OR } x > 4$$

Solve each simple inequality.



Graph x ≤ 3.

Graph x > 4.

Graph the union by combining the regions.

CHECK IT OUT! **Solve each compound inequality and graph the solutions.**

3a. $2 + r < 12 \text{ OR } r + 5 > 19$

3b. $7x \ge 21 \text{ OR } 2x < -2$

Every solution of a compound inequality involving AND must be a solution of both parts of the compound inequality. If no numbers are solutions of *both* simple inequalities, then the compound inequality has no solutions.

The solutions of a compound inequality involving OR are not always two separate sets of numbers. There may be numbers that are solutions of both parts of the compound inequality.

EXAMPLE 4 Writing a Compound Inequality from a Graph

Write the compound inequality shown by each graph.

A

The shaded portion of the graph is not between two values, so the compound inequality involves OR.

On the left, the graph shows an arrow pointing left, so use either $<$ or $\leq$.
The solid circle at −1 means −1 is a solution, so use $\leq$.

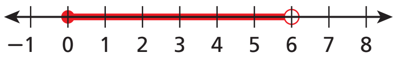
$x \leq -1$

On the right, the graph shows an arrow pointing right, so use either $>$ or $\geq$.
The solid circle at 7 means 7 is a solution, so use $\geq$.

$x \geq 7$

The compound inequality is $x \leq -1$ OR $x \geq 7$.

B



The shaded portion of the graph is between the values 0 and 6, so the compound inequality involves AND.

The shaded values are to the right of 0, so use $>$ or $\geq$.
The solid circle at 0 means 0 is a solution, so use $\geq$.

$x \geq 0$

The shaded values are to the left of 6, so use $<$ or $\leq$.
The empty circle at 6 means 6 is not a solution, so use $<$.

$x < 6$

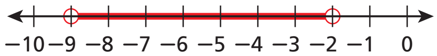
The compound inequality is $x \geq 0$ AND $x < 6$.

Writing Math

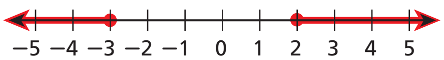
The compound inequality in Example 4B can also be written with the variable between the two endpoints.
$0 \leq x < 6$

CHECK IT OUT! **Write the compound inequality shown by the graph.**

4a.



4b.

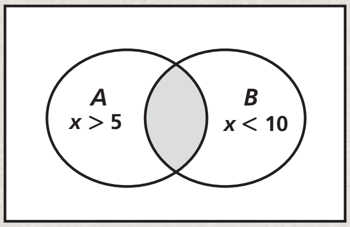


THINK AND DISCUSS

1. Describe how to write the compound inequality $y > 4$ AND $y \leq 12$ without using the joining word AND.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizers. Write three solutions in each of the three sections of the diagram. Then write each of your nine solutions in the appropriate column or columns of the table.



$x > 5$ AND $x < 10$	$x > 5$ OR $x < 10$

3-6 Exercises

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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** The graph of a(n) __?__ shows all values that are solutions to both simple inequalities that make a compound inequality. (*union* or *intersection*)

SEE EXAMPLE 1 p. 204

2. **Biology** An iguana needs to live in a warm environment. The temperature in a pet iguana's cage should be between 70 °F and 95 °F inclusive. Write a compound inequality to show the temperatures that are within the recommended range. Graph the solutions.

Solve each compound inequality and graph the solutions.

SEE EXAMPLE 2 p. 205

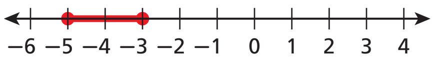
3. $-3 < x + 2 < 7$
4. $5 \le 4x + 1 \le 13$
5. $2 < x + 2 < 5$
6. $11 < 2x + 3 < 21$

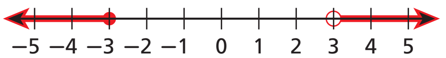
SEE EXAMPLE 3 p. 206

7. $x + 2 < -6$ OR $x + 2 > 6$
8. $r - 1 < 0$ OR $r - 1 > 4$
9. $n + 2 < 3$ OR $n + 3 > 7$
10. $x - 1 < -1$ OR $x - 5 > -1$

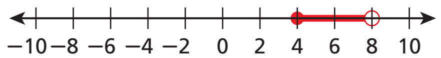
SEE EXAMPLE 4 p. 207

Write the compound inequality shown by each graph.

11. 

12. 

13. 

14. 

PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
15	1
16–19	2
20–23	3
24–27	4

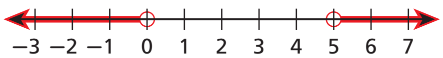
Extra Practice
Skills Practice p. S9
Application Practice p. S30

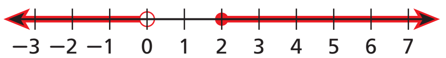
15. **Meteorology** One layer of Earth's atmosphere is called the stratosphere. At one point above Earth's surface the stratosphere extends from an altitude of 16 km to an altitude of 50 km. Write a compound inequality to show the altitudes that are within the range of the stratosphere. Graph the solutions.

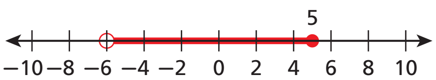
Solve each compound inequality and graph the solutions.

16. $-1 < x + 1 < 1$
17. $1 \le 2n - 5 \le 7$
18. $-2 < x - 2 < 2$
19. $5 < 3x - 1 < 17$
20. $x - 4 < -7$ OR $x + 3 > 4$
21. $2x + 1 < 1$ OR $x + 5 > 8$
22. $x + 1 < 2$ OR $x + 5 > 8$
23. $x + 3 < 0$ OR $x - 2 > 0$

Write the compound inequality shown by each graph.

24. 

25. 

26. 

27. 

28. **Music** A typical acoustic guitar has a range of three octaves. When the guitar is tuned to "concert pitch," the range of frequencies for those three octaves is between 82.4 Hz and 659.2 Hz inclusive. Write a compound inequality to show the frequencies that are within the range of a typical acoustic guitar. Graph the solutions.

MULTI-STEP TEST PREP

29. This problem will prepare you for the Multi-Step Test Prep on page 218.

Jenna's band is going to record a CD at a recording studio. They will pay \$225 to use the studio for one day and \$80 per hour for sound technicians. Jenna has \$200 and can reasonably expect to raise up to an additional \$350 by taking pre-orders for the CDs.

a. Explain how the inequality $200 \leq 225 + 80n \leq 550$ can be used to find the number of hours Jenna and her band can afford to use the studio and sound technicians.

b. Solve the inequality. Are there any numbers in the solution set that are not reasonable in this situation?

c. Suppose Jenna raises \$350 in pre-orders. How much more money would she need to raise if she wanted to use the studio and sound technicians for 6 hours?

Write and graph a compound inequality for the numbers described.

30. all real numbers between -6 and 6

31. all real numbers less than or equal to 2 and greater than or equal to 1

32. all real numbers greater than 0 and less than 15

33. all real numbers between -10 and 10 inclusive

34. Transportation The cruise-control function on Georgina's car should keep the speed of the car within 3 mi/h of the set speed. Write a compound inequality to show the acceptable speeds s if the set speed is 55 mi/h. Graph the solutions.

35. Chemistry Water is not a liquid if its temperature is above 100 °C or below 0 °C. Write a compound inequality for the temperatures t when water is not a liquid.

LINK

Chemistry

The element gallium is in a solid state at room temperature but becomes a liquid at about 30 °C. Gallium stays in a liquid state until it reaches a temperature of about 2204 °C.

Solve each compound inequality and graph the solutions.

36. $5 \leq 4b - 3 \leq 9$

37. $-3 < x - 1 < 4$

38. $r + 2 < -2$ OR $r - 2 > 2$

39. $2a - 5 < -5$ OR $3a - 2 > 1$

40. $x - 4 \geq 5$ AND $x - 4 \leq 5$

41. $n - 4 < -2$ OR $n + 1 > 6$

42. Sports The ball used in a soccer game may not weigh more than 16 ounces or less than 14 ounces at the start of the match. After $1\frac{1}{2}$ ounces of air was added to a ball, the ball was approved for use in a game. Write and solve a compound inequality to show how much the ball might have weighed before the air was added.

43. Meteorology Tornado damage is rated using the Fujita scale shown in the table. A tornado has a wind speed of 200 miles per hour. Write and solve a compound inequality to show how many miles per hour the wind speed would need to increase for the tornado to be rated "devastating" but not "incredible."

Fujita Tornado Scale

Category	Type	Wind Speed (mi/h)
F0	Weak	40 to 72
F1	Moderate	73 to 112
F2	Significant	113 to 157
F3	Severe	158 to 206
F4	Devastating	207 to 260
F5	Incredible	261 to 318

44. Give a real-world situation that can be described by a compound inequality. Write the inequality that describes your situation.

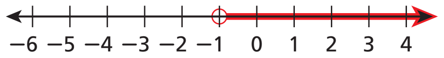
45. Write About It How are the graphs of the compound inequality $x < 3$ AND $x < 7$ and the compound inequality $x < 3$ OR $x < 7$ different? How are the graphs alike? Explain.

46. **Critical Thinking** If there is no solution to a compound inequality, does the compound inequality involve OR or AND? Explain.

TEST PREP

47. Which of the following describes the solutions of $-x + 1 > 2$ OR $x - 1 > 2$?
   - (A) all real numbers greater than 1 or less than 3
   - (B) all real numbers greater than 3 or less than 1
   - (C) all real numbers greater than −1 or less than 3
   - (D) all real numbers greater than 3 or less than −1

48. Which of the following is a graph of the solutions of $x - 3 < 2$ AND $x + 3 > 2$?

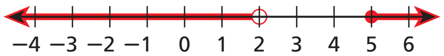
(F) 

(H) 

(G) 

(J) 

49. Which compound inequality is shown by the graph?



   - (A) $x \le 2$ OR $x > 5$
   - (C) $x \le 2$ OR $x \ge 5$
   - (B) $x < 2$ OR $x \ge 5$
   - (D) $x \ge 2$ OR $x > 5$

50. Which of the following is a solution of $x + 1 \ge 3$ AND $x + 1 \le 3$?
   (F) 0 (G) 1 (H) 2 (J) 3

CHALLENGE AND EXTEND

Solve and graph each compound inequality.

51. $2c - 10 < 5 - 3c < 7c$

52. $5p - 10 < p + 6 < 3p$

53. $2s \le 18 - s$ OR $5s \ge s + 36$

54. $9 - x \ge 5x$ OR $20 - 3x \le 17$

55. Write a compound inequality that represents all values of x that are NOT solutions to $x < -1$ OR $x > 3$.

56. For the compound inequality $x + 2 \ge a$ AND $x - 7 \le b$, find values of a and b for which the only solution is $x = 1$.

SPIRAL REVIEW

Simplify each expression. Justify each step. *(Lesson 1-7)*

57. $4(x - 3) + 7$

58. $5x - 4y - x + 3y$

59. $6a - 3(a - 1)$

Generate ordered pairs for each function for $x = -2, -1, 0, 1,$ and 2. Graph the ordered pairs and describe the pattern. *(Lesson 1-8)*

60. $y = -2x + 2$

61. $y = x^2 - 1$

62. $y = x^2 + (-2)$

Solve each inequality and graph the solutions. *(Lesson 3-4)*

63. $3m - 5 < 1$

64. $2(x + 4) > 6$

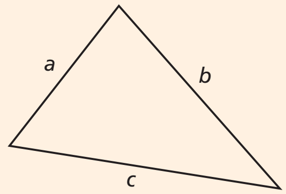
65. $11 \le 7 - 2x$

Connecting Algebra to **Geometry**

Triangle Inequality

For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

The sides of this triangle are labeled a, b, and c. You can use the Triangle Inequality to write three statements about the triangle.



$a + b > c \qquad a + c > b \qquad b + c > a$

Unless all three of the inequalities are true, the lengths a, b, and c cannot form a triangle.

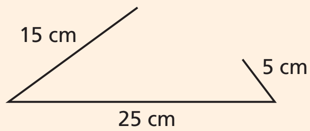
Example 1

Can three side lengths of 25 cm, 15 cm, and 5 cm form a triangle?

a. $25 + 15 > 5$
$40 > 5$ *True*

b. $25 + 5 > 15$
$30 > 15$ *True*

c. $15 + 5 > 25$
$20 > 25$ *False*



One of the inequalities is false, so the three lengths will not make a triangle. The situation is shown in the figure to the right.

Example 2

Two sides of a triangle measure 8 ft and 10 ft. What is the range of lengths of the third side?

Start by writing three statements about the triangle. Use x for the unknown side length.

a. $8 + 10 > x$
$18 > x$
The third side must be shorter than 18 ft.

b. $8 + x > 10$
$8 + x - 8 > 10 - 8$
$x > 2$
The third side must be longer than 2 ft.

c. $x + 10 > 8$
$x + 10 - 10 > 8 - 10$
$x > -2$
This provides no new useful information.

From part **a,** the third side must be shorter than 18 ft. And from part **b,** it must be longer than 2 ft. An inequality showing this is $2 < x < 18$.

Try This

Decide whether the three lengths given can form a triangle. If not, explain.

1. 14 ft, 30 ft, 10 ft

2. 11 cm, 8 cm, 17 cm

3. $6\frac{1}{2}$ yd, 3 yd, $2\frac{3}{4}$ yd

Write a compound inequality for the range of lengths of the third side of each triangle.

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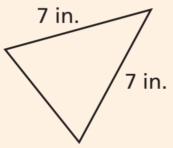
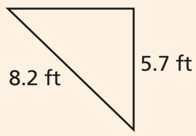


5.


6.