3-5 Solving Inequalities with Variables on Both Sides

Objective
Solve inequalities that contain variable terms on both sides.

Who uses this?
Business owners can use inequalities to find the most cost-efficient services. (See Example 2.)

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities like you solved equations with variables on both sides.

Use the properties of inequality to "collect" all the variable terms on one side and all the constant terms on the other side.

EXAMPLE 1 Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.

A $x < 3x + 8$

$x < 3x + 8$ — *To collect the variable terms on one side, subtract x from both sides.*

$-x \quad -x$

$0 < 2x + 8$ — *Since 8 is added to 2x, subtract 8 from both sides to undo the addition.*

$-8 \quad -8$

$-8 < 2x$

$\frac{-8}{2} < \frac{2x}{2}$ — *Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.*

$-4 < x$ (or $x > -4$)

B $6x - 1 \leq 3.5x + 4$

$6x - 1 \leq 3.5x + 4$ — *Subtract 6x from both sides.*

$-6x \quad -6x$

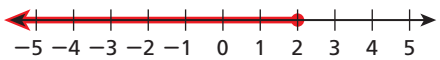
$-1 \leq -2.5x + 4$ — *Since 4 is added to −2.5x, subtract 4 from both sides to undo the addition.*

$-4 \quad -4$

$-5 \leq -2.5x$

$\frac{-5}{-2.5} \geq \frac{-2.5x}{-2.5}$ — *Since x is multiplied by −2.5, divide both sides by−2.5 to undo the multiplication. Reverse the inequality symbol.*

$2 \geq x$



Helpful Hint
Your first step can also be to subtract $3x$ from both sides to get $-2x < 8$. When you divide by a negative number, remember to reverse the inequality symbol.

CHECK IT OUT! **Solve each inequality and graph the solutions.**

1a. $4x \geq 7x + 6$

1b. $5t + 1 < -2t - 6$

EXAMPLE 2 Business Application

The *Daily Info* charges a fee of $650 plus $80 per week to run an ad. The *People's Paper* charges $145 per week. For how many weeks will the total cost at *Daily Info* be less expensive than the cost at *People's Paper*?

Let w be the number of weeks the ad runs in the paper.

Daily Info fee	plus	$80 per week	times	number of weeks	is less expensive than	*People's Paper* charge per week	times	number of weeks.
$650	+	$80	•	w	<	$145	•	w

$$650 + 80w < 145w$$

$$-80w \quad -80w$$

$$650 < 65w$$

Subtract 80w from both sides.

Since w is multiplied by 65, divide both sides by 65 to undo the multiplication.

$$\frac{650}{65} < \frac{65w}{65}$$

$$10 < w$$

The total cost at *Daily Info* is less than the cost at *People's Paper* if the ad runs for more than 10 weeks.

2. A-Plus Advertising charges a fee of $24 plus $0.10 per flyer to print and deliver flyers. Print and More charges $0.25 per flyer. For how many flyers is the cost at A-Plus Advertising less than the cost at Print and More?

You may need to simplify one or both sides of an inequality before solving it. Look for like terms to combine and places to use Distributive Property.

EXAMPLE 3 Simplifying Each Side Before Solving

Solve each inequality and graph the solutions.

A $6(1 - x) < 3x$

$$6(1 - x) < 3x$$

Distribute 6 on the left side of the inequality.

$$6(1) - 6(x) < 3x$$

$$6 - 6x < 3x$$

Add 6x to both sides so that the coefficient of x is positive.

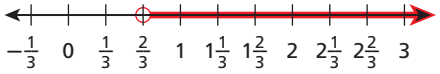
$$+6x \quad +6x$$

$$6 < 9x$$

Since x is multiplied by 9, divide both sides by 9 to undo the multiplication.

$$\frac{6}{9} < \frac{9x}{9}$$

$$\frac{2}{3} < x$$



Helpful Hint

In Example 3B, you can also multiply each term in the inequality by the same power of 10 to clear the decimals.

$10(1.6x) \le 10(-0.2x) + 10(0.9)$

$16x \le -2x + 9$

Solve each inequality and graph the solutions.

B $1.6x \le -0.2x + 0.9$

$1.6x \le -0.2x + 0.9$

Since −0.2x is added to 0.9, subtract −0.2x from both sides. Subtracting −0.2x is the same as adding 0.2x.

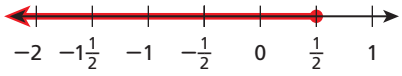
$+0.2x \quad +0.2x$

$1.8x \le 0.9$

Since x is multiplied by 1.8, divide both sides by 1.8 to undo the multiplication.

$\frac{1.8x}{1.8} \le \frac{0.9}{1.8}$

$x \le \frac{1}{2}$



CHECK IT OUT! **Solve each inequality and graph the solutions. Check your answer.**

3a. $5(2 - r) \ge 3(r - 2)$ **3b.** $0.5x - 0.3 + 1.9x < 0.3x + 6$

Some inequalities are true no matter what value is substituted for the variable. For these inequalities, all real numbers are solutions.

Some inequalities are false no matter what value is substituted for the variable. These inequalities have no solutions.

If both sides of an inequality are fully simplified and the same variable term appears on both sides, then the inequality has all real numbers as solutions or it has no solutions. Look at the other terms in the inequality to decide which is the case.

EXAMPLE 4 All Real Numbers as Solutions or No Solutions

Solve each inequality.

A $x + 5 \ge x + 3$

$x + 5 \ge x + 3$

The same variable term (x) appears on both sides. Look at the other terms.

For any number x, adding 5 will always result in a greater number than adding 3.

All values of x make the inequality true.
All real numbers are solutions.

B $2(x + 3) < 5 + 2x$

$2x + 6 < 5 + 2x$ *Distribute 2 on the left side.*

The same variable term ($2x$) appears on both sides. Look at the other terms.

For any number $2x$, adding 6 will never result in a lesser number than adding 5.

No values of x make the inequality true.
There are no solutions.

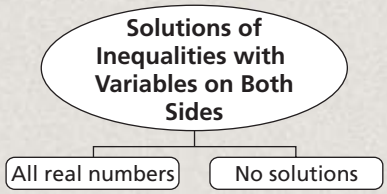
CHECK IT OUT! **Solve each inequality.**

4a. $4(y - 1) \ge 4y + 2$ **4b.** $x - 2 < x + 1$

THINK AND DISCUSS

1. Explain how you would collect the variable terms to solve the inequality $5c - 4 > 8c + 2$.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of an inequality of the indicated type.



3-5 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 196

Solve each inequality and graph the solutions.

1. $2x > 4x - 6$

2. $7y + 1 \leq y - 5$

3. $27x + 33 > 58x - 29$

4. $-3r < 10 - r$

5. $5c - 4 > 8c + 2$

6. $4.5x - 3.8 \geq 1.5x - 2.3$

SEE EXAMPLE 2 p. 197

7. School The school band will sell pizzas to raise money for new uniforms. The supplier charges \$100 plus \$4 per pizza. If the band members sell the pizzas for \$7 each, how many pizzas will they have to sell to make a profit?

SEE EXAMPLE 3 p. 197

Solve each inequality and graph the solutions.

8. $5(4 + x) \leq 3(2 + x)$

9. $-4(3 - p) > 5(p + 1)$

10. $2(6 - x) < 4x$

11. $4x > 3(7 - x)$

12. $\frac{1}{2}f + \frac{3}{4} \geq \frac{1}{4}f$

13. $-36.72 + 5.65t < 0.25t$

SEE EXAMPLE 4 p. 198

Solve each inequality.

14. $2(x - 2) \leq -2(1 - x)$

15. $4(y + 1) < 4y + 2$

16. $4v + 1 < 4v - 7$

17. $b - 4 \geq b - 6$

18. $3(x - 5) > 3x$

19. $2k + 7 \geq 2(k + 14)$

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

20. $3x \leq 5x + 8$

21. $9y + 3 > 4y - 7$

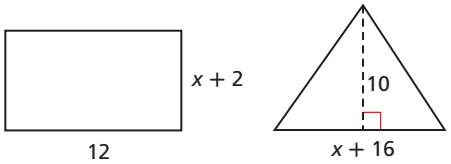
22. $1.5x - 1.2 < 3.1x - 2.8$

23. $7 + 4b \geq 3b$

24. $7 - 5t < 4t - 2$

25. $2.8m - 5.2 > 0.8m + 4.8$

26. Geometry For what values of x is the area of the rectangle greater than the area of the triangle?



Independent Practice

For Exercises	See Example
20–25	1
26	2
27–32	3
33–38	4

Extra Practice

Skills Practice p. S9

Application Practice p. S30

Solve each inequality and graph the solutions.

27. $4(2 - x) \leq 5(x - 2)$

28. $-3(n + 4) < 6(1 - n)$

29. $9(w + 2) \leq 12w$

30. $4.5 + 1.3t > 3.8t - 3$

31. $\frac{1}{2}r + \frac{2}{3} \geq \frac{1}{3}r$

32. $2(4 - n) < 3n - 7$

Solve each inequality.

33. $3(2 - x) < -3(x - 1)$

34. $7 - y > 5 - y$

35. $3(10 + z) \leq 3z + 36$

36. $-5(k - 1) \geq 5(2 - k)$

37. $4(x - 1) \leq 4x$

38. $3(v - 9) \geq 15 + 3v$

Solve each inequality and graph the solutions.

39. $3t - 12 > 5t + 2$

40. $-5(y + 3) - 6 < y + 3$

41. $3x + 9 - 5x < x$

42. $18 + 9p > 12p - 31$

43. $2(x - 5) < -3x$

44. $-\frac{2}{5}x \leq \frac{4}{5} - \frac{3}{5}x$

45. $-2(x - 7) - 4 - x < 8x + 32$

46. $-3(2r - 4) \geq 2(5 - 3r)$

47. $-7x - 10 + 5x \geq 3(x + 4) + 8$

48. $-\frac{1}{3}(n + 8) + \frac{1}{3}n \leq 1 - n$

LINK

Recreation

The American Kitefliers Association has over 4000 members in 35 countries. Kitefliers participate in festivals, competitions, and kite-making workshops.

49. Recreation A red kite is 100 feet off the ground and is rising at 8 feet per second. A blue kite is 180 feet off the ground and is rising at 5 feet per second. How long will it take for the red kite to be higher than the blue kite? Round your answer to the nearest second.

50. Education The table shows the enrollment in Howard High School and Phillips High School for three school years.

School Enrollment

	Year 1	Year 2	Year 3
Howard High School	1192	1188	1184
Phillips High School	921	941	961

a. How much did the enrollment change each year at Howard?

b. Use the enrollment in year 1 and your answer from part **a** to write an expression for the enrollment at Howard in any year x.

c. How much did the enrollment change each year at Phillips?

d. Use the enrollment in year 1 and your answer from part **c** to write an expression for the enrollment at Phillips in any year x.

e. Assume that the pattern in the table continues. Use your expressions from parts **b** and **d** to write an inequality that can be solved to find the year in which the enrollment at Phillips High School will be greater than the enrollment at Howard High School. Solve your inequality and graph the solutions.

MULTI-STEP TEST PREP

51. This problem will prepare you for the Multi-Step Test Prep on page 218.

a. The school orchestra is creating a CD of their last concert. The total cost is \$400 + 4.50 per CD. Write an expression for the cost of creating the CDs based on the number of CDs n.

b. The orchestra plans to sell the CDs for \$12. Write an expression for the amount the orchestra earns from the sale of n CDs.

c. In order for the orchestra to make a profit, the amount they make selling the CDs must be greater than the cost of creating the CDs. Write an inequality that can be solved to find the number of CDs the orchestra must sell in order to make a profit. Solve your inequality.

Write an inequality to represent each relationship. Solve your inequality.

52. Four more than twice a number is greater than two-thirds of the number.

53. Ten less than five times a number is less than six times the number decreased by eight.

54. The sum of a number and twenty is less than four times the number decreased by one.

55. Three-fourths of a number is greater than or equal to five less than the number.

56. **Entertainment** Use the table to determine how many movies you would have to rent for Video View to be less expensive than Movie Place.

	Membership Fee ($)	Cost per Rental ($)
Movie Place	None	2.99
Video View	19.99	1.99

57. **Geometry** In an acute triangle, all angles measure less than 90°. Also, the sum of the measures of any two angles is greater than the measure of the third angle. Can the measures of an acute triangle be x, $x - 1$, and $2x$? Explain.

58. **Write About It** Compare the steps you would follow to solve an inequality to the steps you would follow to solve an equation.

59. **Critical Thinking** How can you tell just by looking at the inequality $x > x + 1$ that it has no solutions?

60. **ERROR ANALYSIS** Two students solved the inequality $5x < 3 - 4x$. Which is incorrect? Explain the error.

A

$5x < 3 - 4x$

$+4x \quad +4x$

$9x < 3$

$x < \frac{1}{3}$

B

$5x < 3 - 4x$

$-4x \quad -4x$

$x < 3$

TEST PREP

61. If $a - b > a + b$, which statement is true?

Ⓐ The value of a is positive.
Ⓑ The value of b is positive.
Ⓒ The value of a is negative.
Ⓓ The value of b is negative.

62. If $-a < b$, which statement is always true?

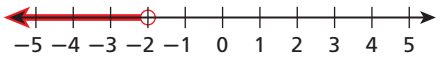
Ⓕ $a < b$ Ⓖ $a > b$ Ⓗ $a < -b$ Ⓙ $a > -b$

63. Which is a solution of the inequality $7(2 - x) > 4(x - 2)$?

Ⓐ -2 Ⓑ 2 Ⓒ 4 Ⓓ 7

64. Which is the graph of $-5x < -2x - 6$?

Ⓕ


Ⓗ


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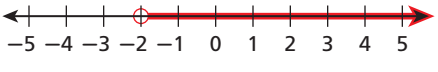

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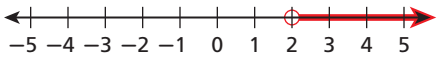

65. **Short Response** Write a real-world situation that could be modeled by the inequality $7x + 4 > 4x + 13$. Explain how the inequality relates to your situation.

CHALLENGE AND EXTEND

Solve each inequality.

66. $2\frac{1}{2} + 2x \geq 5\frac{1}{2} + 2\frac{1}{2}x$

67. $1.6x - 20.7 > 6.3x - (-2.2x)$

68. $1.3x - 7.5x < 8.5x - 29.4$

69. $-4w + \frac{-8 - 37}{9} \leq \frac{75 - 3}{9} + 3w$

70. Replace the square and circle with numbers so that the inequality has all real numbers as solutions. $\square - 2x < \bigcirc - 2x$

71. Replace the square and circle with numbers so that the inequality has no solutions. $\square - 2x < \bigcirc - 2x$

72. **Critical Thinking** Explain whether there are any numbers that can replace the square and circle so that the inequality has all real numbers as solutions. $\square + 2x < \bigcirc + x$

SPIRAL REVIEW

73. The ratio of the width of a rectangle to the length is 2:5. The length is 65 inches. Find the width. *(Lesson 2-7)*

74. Find the simple interest paid after 6 months on a loan of $5000 borrowed at a rate of 9%. *(Lesson 2-10)*

Define a variable and write an inequality for each situation. Graph the solutions. *(Lesson 3-1)*

75. Participants must be at least 14 years old.

76. The maximum speed on a certain highway is 60 miles per hour.

Career Path

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Katie Flannigan
Culinary Arts program

Q: **What math classes did you take in high school?**
A: Algebra 1, Geometry, and Algebra 2

Q: **What math classes have you taken since high school?**
A: I have taken a basic accounting class and a business math class.

Q: **How do you use math?**
A: I use math to estimate how much food I need to buy. I also use math when adjusting recipe amounts to feed large groups of people.

Q: **What are your future plans?**
A: I plan to start my own catering business. The math classes I took will help me manage the financial aspects of my business.

3-6 Algebra LAB

Truth Tables and Compound Statements

A compound statement is formed by combining two or more simple statements. A compound statement is either true or false depending on whether its simple statements are true or false.

Use with Lesson 3-6

Activity 1

• Let *P* be "Cindy is at least 17 years old." • Let *Q* be "Cindy has a driver's license."

If...	then *P* is	and *Q* is	so *P* AND *Q* is
Cindy is 18 years old. Cindy has a driver's license.	True	True	True
Cindy is 17 years old. Cindy does not have a driver's license.	True	False	False
Cindy is 16 years old. Cindy has a driver's license.	False	True	False
Cindy is 15 years old. Cindy does not have a driver's license.	False	False	False

P* AND *Q is true when _____?_____.

Try This

For each pair of simple statements, tell whether *P* AND *Q* is true or false.

1. *P:* Many birds can fly; *Q:* A zebra is an animal.

Activity 2

• Let *P* be "Paul plays tennis." • Let *Q* be "Paul has brown eyes."

If...	then *P* is	and *Q* is	so *P* OR *Q* is
Paul plays tennis. Paul has brown eyes.	True	True	True
Paul plays tennis. Paul has green eyes.	True	False	True
Paul does not play tennis. Paul has brown eyes.	False	True	True
Paul does not play tennis. Paul has green eyes.	False	False	False

P* OR *Q is true when _____?_____.

Try This

For each pair of simple statements, tell whether *P* OR *Q* is true or false.

2. *P:* The number 12 is even; *Q:* The number 12 is a composite number.