Solving Two-Step and Multi-Step Inequalities

Objective
Solve inequalities that contain more than one operation.

Who uses this?
Contestants at a county fair can solve an inequality to find how many pounds a prize-winning pumpkin must weigh. (See Example 3.)

At the county fair, contestants can enter contests that judge animals, recipes, crops, art projects, and more. Sometimes an average score or average weight is used to determine the winner of the blue ribbon. A contestant can use a multi-step inequality to determine what score or weight is needed in order to win.

Inequalities that contain more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.

EXAMPLE 1 **Solving Multi-Step Inequalities**

Solve each inequality and graph the solutions.

A $160 + 4f \leq 500$
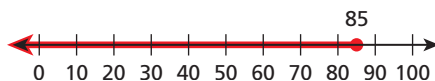
$$160 + 4f \leq 500$$
$$\underline{-160 \qquad -160}$$
$$4f \leq 340$$
$$\frac{4f}{4} \leq \frac{340}{4}$$
$$f \leq 85$$

Since 160 is added to 4f, subtract 160 from both sides to undo the addition.

Since f is multiplied by 4, divide both sides by 4 to undo the multiplication.

0 10 20 30 40 50 60 70 80 **85** 90 100
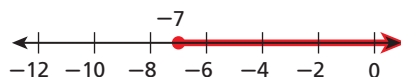
B $7 - 2t \leq 21$

$$7 - 2t \leq 21$$
$$\underline{-7 \qquad -7}$$
$$-2t \leq 14$$
$$\frac{-2t}{-2} \geq \frac{14}{-2}$$
$$t \geq -7$$

Since 7 is added to −2t, subtract 7 from both sides to undo the addition.

Since t is multiplied by −2, divide both sides by −2 to undo the multiplication. Change ≤ to ≥.

−12 −10 −8 **−7** −6 −4 −2 0

CHECK IT OUT!

Solve each inequality and graph the solutions.

1a. $-12 \geq 3x + 6$ **1b.** $\dfrac{x+5}{-2} > 3$ **1c.** $\dfrac{1-2n}{3} \geq 7$

To solve more complicated inequalities, you may first need to simplify the expressions on one or both sides by using the order of operations, combining like terms, or using the Distributive Property.

EXAMPLE 2 | Simplifying Before Solving Inequalities

Solve each inequality and graph the solutions.

A $-4 + (-8) < -5c - 2$

$$-12 < -5c - 2$$

Combine like terms. Since 2 is subtracted from $-5c$, add 2 to both sides to undo the subtraction.
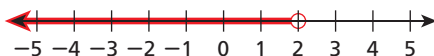
$$\underline{+2 \qquad\quad +2}$$
$$-10 < -5c$$

$$\frac{-10}{-5} > \frac{-5c}{-5}$$

Since c is multiplied by -5, divide both sides by -5 to undo the multiplication. Change $<$ to $>$.

$$2 > c \ (\text{or } c < 2)$$

B $-3(3 - x) < 4^2$

$$-3(3 - x) < 4^2$$

$$-3(3) - (-3)x < 4^2$$ *Distribute -3 on the left side.*

$$-9 + 3x < 4^2$$

$$-9 + 3x < 16$$ *Simplify the right side.*

$$-9 + 3x < 16$$ *Since -9 is added to $3x$, add 9 to both sides to undo the addition.*
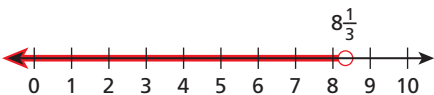
$$\underline{+9 \qquad\quad +9}$$
$$3x < 25$$

$$\frac{3x}{3} < \frac{25}{3}$$ *Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.*

$$x < 8\frac{1}{3}$$

C $\frac{4}{5}x + \frac{1}{2} > \frac{3}{5}$

$$10\left(\frac{4}{5}x + \frac{1}{2}\right) > 10\left(\frac{3}{5}\right)$$ *Multiply both sides by 10, the LCD of the fractions.*

$$10\left(\frac{4}{5}x\right) + 10\left(\frac{1}{2}\right) > 10\left(\frac{3}{5}\right)$$ *Distribute 10 on the left side.*

$$8x + 5 > 6$$ *Since 5 is added to $8x$, subtract 5 from both sides to undo the addition.*

$$\underline{-5 \quad -5}$$
$$8x > 1$$

$$\frac{8x}{8} > \frac{1}{8}$$ *Since x is multiplied by 8, divide both sides by 8 to undo the multiplication.*

$$x > \frac{1}{8}$$
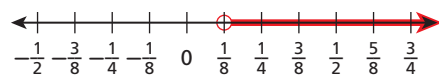
CHECK IT OUT! **Solve each inequality and graph the solutions.**

2a. $2m + 5 > 5^2$ **2b.** $3 + 2(x + 4) > 3$ **2c.** $\frac{5}{8} < \frac{3}{8}x - \frac{1}{4}$

EXAMPLE 3 **Gardening Application**

To win the blue ribbon for the Heaviest Pumpkin Crop at the county fair, the average weight of John's two pumpkins must be greater than 819 lb. One of his pumpkins weighs 887 lb. What is the least number of pounds the second pumpkin could weigh in order for John to win the blue ribbon?

Let p represent the weight of the second pumpkin. The average weight of the pumpkins is the sum of each weight divided by 2.

(887	plus	p)	divided by	2	must be greater than	819.
(887	+	p)	÷	2	>	819

$\dfrac{887 + p}{2} > 819$ *Since 887 + p is divided by 2, multiply both sides by 2 to undo the division.*

$2\left(\dfrac{887 + p}{2}\right) > 2(819)$

$887 + p > 1638$ *Since 887 is added to p, subtract 887 from both sides to undo the addition.*

$\dfrac{-\,887 \qquad\quad -\,887}{p > \qquad 751}$

The second pumpkin must weigh more than 751 pounds.

Check   Check the endpoint, 751.   Check a number greater than 751.

$\dfrac{887 + p}{2} = 819$ $\dfrac{887 + p}{2} > 819$

$\dfrac{887 + 751}{2} \,\Big|\, 819$ $\dfrac{887 + 755}{2} \,>\,\Big|\, 819$

$\dfrac{1638}{2} \,\Big|\, 819$ $\dfrac{1642}{2} \,>\,\Big|\, 819$

$819 \,\Big|\, 819\ ✓$ $821 \,>\,\Big|\, 819\ ✓$

CHECK IT OUT!

3. The average of Jim's two test scores must be at least 90 to make an A in the class. Jim got a 95 on his first test. What scores can Jim get on his second test to make an A in the class?

THINK AND DISCUSS

1. The inequality $v \geq 25$ states that 25 is the ___?___. (*value of v, minimum value of v*, or *maximum value of v*)

2. Describe two sets of steps for solving the inequality $\frac{x+5}{3} > 7$.

3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

Solving Multi-Step Equations and Inequalities

How are they alike?   How are they different?

3-4 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA7 3-4
Parent Resources Online
KEYWORD: MA7 Parent
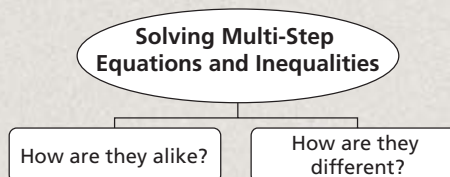
GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE **1**
p. 190

1. $2m + 1 > 13$ **2.** $2d + 21 \leq 11$ **3.** $6 \leq -2x + 2$ **4.** $4c - 7 > 5$

5. $\dfrac{4 + x}{3} > -4$ **6.** $1 < 0.2x - 0.7$ **7.** $\dfrac{3 - 2x}{3} \leq 7$ **8.** $2x + 5 \geq 2$

SEE EXAMPLE **2**
p. 191

9. $4(x + 2) > 6$ **10.** $\dfrac{1}{4}x + \dfrac{2}{3} < \dfrac{3}{4}$ **11.** $4 - x + 6^2 \geq 21$

12. $4 - x > 3(4 - 2)$ **13.** $0.2(x - 10) > -1.8$ **14.** $3(j + 41) \leq 35$

SEE EXAMPLE **3**
p. 192

15. **Business** A sales representative is given a choice of two paycheck plans. One choice includes a monthly base pay of $300 plus 10% commission on his sales. The second choice is a monthly salary of $1200. For what amount of sales would the representative make more money with the first plan?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–27	1
28–36	2
37	3

Extra Practice
Skills Practice p. S9
Application Practice p. S30

Solve each inequality and graph the solutions.

16. $4r - 9 > 7$ **17.** $3 \leq 5 - 2x$ **18.** $\dfrac{w + 3}{2} > 6$ **19.** $11w + 99 < 77$

20. $9 \geq \dfrac{1}{2}v + 3$ **21.** $-4x - 8 > 16$ **22.** $8 - \dfrac{2}{3}z \leq 2$ **23.** $f + 2\dfrac{1}{2} < -2$

24. $\dfrac{3n - 8}{5} \geq 2$ **25.** $-5 > -5 - 3w$ **26.** $10 > \dfrac{5 - 3p}{2}$ **27.** $2v + 1 > 2\dfrac{1}{3}$

28. $4(x + 3) > -24$ **29.** $4 > x - 3(x + 2)$ **30.** $-18 \geq 33 - 3h$

31. $-2 > 7x - 2(x - 4)$ **32.** $9 - (9)^2 > 10x - x$ **33.** $2a - (-3)^2 \geq 13$

34. $6 - \dfrac{x}{3} + 1 > \dfrac{2}{3}$ **35.** $12(x - 3) + 2x > 6$ **36.** $15 \geq 19 + 2(q - 18)$

37. **Communications** One cell phone company offers a plan that costs $29.99 and includes unlimited night and weekend minutes. Another company offers a plan that costs $19.99 and charges $0.35 per minute during nights and weekends. For what numbers of night and weekend minutes does the second company's plan cost more than the first company's plan?
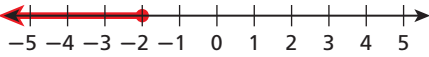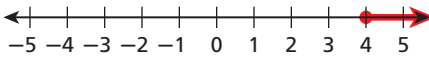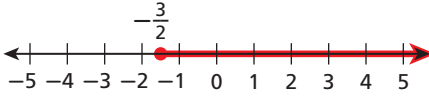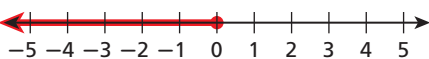
Solve each inequality and graph the solutions.

38. $-12 > -4x - 8$ **39.** $5x + 4 \leq 14$ **40.** $\dfrac{2}{3}x - 5 > 7$

41. $x - 3x > 2 - 10$ **42.** $5 - x - 2 > 3$ **43.** $3 < 2x - 5(x + 3)$

44. $\dfrac{1}{6} - \dfrac{2}{3}m \geq \dfrac{1}{4}$ **45.** $4 - (r - 2) > 3 - 5$ **46.** $0.3 - 0.5n + 1 \geq 0.4$

47. $6^2 > 4(x + 2)$ **48.** $-4 - 2n + 4n > 7 - 2^2$ **49.** $\dfrac{1}{4}(p - 10) \geq 6 - 4$

50. Use the inequality $-4t - 8 \leq 12$ to fill in the missing numbers.

　　a. $t \geq \blacksquare$ **b.** $t + 4 \geq \blacksquare$ **c.** $t - \blacksquare \geq 0$

　　d. $t + 10 \geq \blacksquare$ **e.** $3t \geq \blacksquare$ **f.** $\dfrac{t}{\blacksquare} \geq -5$

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. One-half of a number, increased by 9, is less than 33.

52. Six is less than or equal to the sum of 4 and $-2x$.

53. The product of 4 and the sum of a number and 12 is at most 16.

54. The sum of half a number and two-thirds of the number is less than 14.

**Solve each inequality and match the solution to the correct graph.**

55. $4x - 9 \geq 7$

A.

-5 -4 -3 -2 -1 0 1 2 3 4 5

56. $-6 \geq 3(x - 2)$

B.

-5 -4 -3 -2 -1 0 1 2 3 4 5

57. $-2x - 6 \geq -4 + 2$

C. $-\frac{3}{2}$

-5 -4 -3 -2 -1 0 1 2 3 4 5

58. $\frac{1}{2} - \frac{1}{3}x \leq \left(\frac{2}{3} + \frac{1}{3}\right)^2$

D.

-5 -4 -3 -2 -1 0 1 2 3 4 5

59. Entertainment A digital video recorder (DVR) records television shows on an internal hard drive. To use a DVR, you need a subscription with a DVR service company. Two companies advertise their charges for a DVR machine and subscription service.

EASY ELECTRONICS
$225 for DVR machine
$400 for lifetime subscription

CABLE SOLUTIONS
$275 for DVR machine
$15 per month for service

For what numbers of months will a consumer pay less for the machine and subscription at Easy Electronics than at Cable Solutions?

60. Geometry The area of the triangle shown is less than 55 square inches.
   a. Write an inequality that can be used to find x.
   b. Solve the inequality you wrote in part **a.**
   c. What is the maximum height of the triangle?

$(2x + 3)$ in.
5 in.
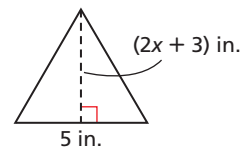
MULTI-STEP TEST PREP

   a. A band wants to create a CD of their last concert. They received a donation of $500 to cover the cost. The total cost is $350 plus $3 per CD. Complete the table to find a relationship between the number of CDs and the total cost.

   b. Write an equation for the cost c of the CDs based on the number of CDs n.

Number	Process	Cost
1	350 + 3	353
2		
3		
10		
n		

   c. Write an inequality that can be used to determine how many CDs can be made with the $500 donation. Solve the inequality and determine how many CDs the band can have made from the $500 donation.

62. Critical Thinking What is the least whole number that is a solution of $4r - 4.9 > 14.95$?

63. Write About It Describe two sets of steps to solve $2(x + 3) > 10$.

64. What are the solutions of $3y > 2x + 4$ when $y = 6$?

Ⓐ $7 > x$　　　Ⓑ $x > 7$　　　Ⓒ $x > 11$　　　Ⓓ $11 > x$

65. Cecilia has $30 to spend at a carnival. Admission costs $5.00, lunch will cost $6.00, and each ride ticket costs $1.25. Which inequality represents the number of ride tickets x that Cecilia can buy?

Ⓕ $30 - (5 - 6) + 1.25x \leq 30$　　　Ⓗ $30 - (5 + 6) \leq 1.25x$

Ⓖ $5 + 6 + 1.25x \leq 30$　　　Ⓙ $30 + 1.25x \leq 5 + 6$

66. Which statement is modeled by $2p + 5 < 11$?

Ⓐ The sum of 5 and 2 times p is at least 11.

Ⓑ Five added to the product of 2 and p is less than 11.

Ⓒ Two times p plus 5 is at most 11.

Ⓓ The product of 2 and p added to 5 is 11.

67. Gridded Response A basketball team scored 8 points more in its second game than in its first. In its third game, the team scored 42 points. The total number of points scored in the three games was more than 150. What is the least number of points the team might have scored in its *second* game?

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

68. $3(x + 2) - 6x + 6 \leq 0$　　　**69.** $-18 > -(2x + 9) - 4 + x$　　　**70.** $\dfrac{2 + x}{2} - (x - 1) > 1$

Write an inequality for each statement. Graph the solutions.

71. x is a positive number.　　　**72.** x is a negative number.

73. x is a nonnegative number.　　　**74.** x is not a positive number.

75. x times negative 3 is positive.　　　**76.** The opposite of x is greater than 2.

SPIRAL REVIEW

Find each root. *(Lesson 1-5)*

77. $\sqrt{49}$　　　**78.** $-\sqrt{144}$　　　**79.** $\sqrt{\dfrac{4}{9}}$

80. $\sqrt{196}$　　　**81.** $\sqrt[3]{-729}$　　　**82.** $\sqrt{10{,}000}$

83. Video rental store A charges a membership fee of $25 and $2 for each movie rental. Video rental store B charges a membership fee of $10 and $2.50 for each movie. Find the number of movie rentals for which both stores' charges are the same. *(Lesson 2-4)*

Solve each inequality and graph the solutions. *(Lesson 3-3)*

84. $2x < -8$　　　**85.** $\dfrac{a}{-2} \leq -3$　　　**86.** $\dfrac{1}{4} < \dfrac{t}{12}$