

3-3

Solving Inequalities by Multiplying or Dividing

Objectives

Solve one-step inequalities by using multiplication.

Solve one-step inequalities by using division.

Who uses this?

You can solve an inequality to determine how much you can buy with a certain amount of money. (See Example 3.)

Remember, solving inequalities is similar to solving equations. To solve an inequality that contains multiplication or division, undo the operation by dividing or multiplying both sides of the inequality by the same number.

The rules below show the properties of inequality for multiplying or dividing by a positive number. The rules for multiplying or dividing by a negative number appear later in this lesson.



Jonny Hawkins/CartoonResource.com

“This is all I have, so I’ll take 3 pencils, 3 notebooks, a binder, and 0.9 calculators.”



Properties of Inequality

Multiplication and Division by Positive Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication You can multiply both sides of an inequality by the same <i>positive</i> number, and the statement will still be true.	$7 < 12$ $7(3) < 12(3)$ $21 < 36$	If $a < b$ and $c > 0$, then $ac < bc$.
Division You can divide both sides of an inequality by the same <i>positive</i> number, and the statement will still be true.	$15 < 35$ $\frac{15}{5} < \frac{35}{5}$ $3 < 7$	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for inequalities that use the symbols $>$, \geq , and \leq .

EXAMPLE 1 Multiplying or Dividing by a Positive Number

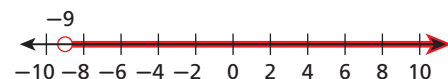
Solve each inequality and graph the solutions.

A $3x > -27$

$3x > -27$

$\frac{3x}{3} > \frac{-27}{3}$

$x > -9$



Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.

Solve each inequality and graph the solutions.

B $\frac{2}{3}r < 6$

$$\frac{2}{3}r < 6$$

Since r is multiplied by $\frac{2}{3}$, multiply both sides by the reciprocal of $\frac{2}{3}$.

$$\frac{3}{2}\left(\frac{2}{3}r\right) < \frac{3}{2}(6)$$

$$r < 9$$



Solve each inequality and graph the solutions.

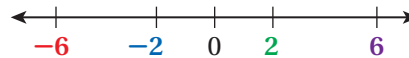
1a. $4k > 24$

1b. $-50 \geq 5q$

1c. $\frac{3}{4}g > 27$

What happens when you multiply or divide both sides of an inequality by a negative number?

Look at the number line below.



$$2 < 6$$

$$-2 \square -6$$

$$-2 > -6$$

Multiply both sides by -1 .

Use the number line to determine the direction of the inequality.

$$6 > -2$$

$$-6 \square 2$$

$$-6 < 2$$

Multiply both sides by -1 .

Use the number line to determine the direction of the inequality.

Notice that when you multiply (or divide) both sides of an inequality by a negative number, you must reverse the inequality symbol. This means there is another set of properties of inequality for multiplying or dividing by a negative number.



Properties of Inequality

Multiplication and Division by Negative Numbers

WORDS	NUMBERS	ALGEBRA
<p>Multiplication</p> <p>If you multiply both sides of an inequality by the same <i>negative</i> number, you must reverse the inequality symbol for the statement to still be true.</p>	$8 > 4$ $8(-2) < 4(-2)$ $-16 < -8$	<p>If $a > b$ and $c < 0$, then $ac < bc$.</p>
<p>Division</p> <p>If you divide both sides of an inequality by the same <i>negative</i> number, you must reverse the inequality symbol for the statement to still be true.</p>	$12 > 4$ $\frac{12}{-4} < \frac{4}{-4}$ $-3 < -1$	<p>If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.</p>

These properties are also true for inequalities that use the symbols $<$, \geq , and \leq .

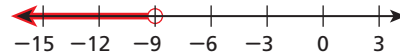
EXAMPLE 2 Multiplying or Dividing by a Negative Number

Caution!

Do not change the direction of the inequality symbol just because you see a negative sign. For example, you do not change the symbol when solving $4x < -24$.

A $-8x > 72$
 $\frac{-8x}{-8} < \frac{72}{-8}$
 $x < -9$

Since x is multiplied by -8 , divide both sides by -8 .
Change $>$ to $<$.



B $-3 \leq \frac{x}{-5}$
 $-5(-3) \geq -5\left(\frac{x}{-5}\right)$
 $15 \geq x$ (or $x \leq 15$)

Since x is divided by -5 , multiply both sides by -5 .
Change \leq to \geq .



Solve each inequality and graph the solutions.

2a. $10 \geq -x$

2b. $4.25 > -0.25h$

EXAMPLE 3 Consumer Application

Ryan has a \$16 gift card for a health store where a smoothie costs \$2.50 with tax. What are the possible numbers of smoothies that Ryan can buy?

Let s represent the number of smoothies Ryan can buy.

\$2.50 times number of smoothies is at most \$16.00.

2.50 • s ≤ 16.00

$2.50s \leq 16.00$

$\frac{2.50s}{2.50} \leq \frac{16.00}{2.50}$

Since s is multiplied by 2.50, divide both sides by 2.50.
The symbol does not change.

$s \leq 6.4$

Ryan can buy only a whole number of smoothies.

Ryan can buy 0, 1, 2, 3, 4, 5, or 6 smoothies.



3. A pitcher holds 128 ounces of juice. What are the possible numbers of 10-ounce servings that one pitcher can fill?

THINK AND DISCUSS

- Compare the Multiplication and Division Properties of Inequality and the Multiplication and Division Properties of Equality.
- GET ORGANIZED** Copy and complete the graphic organizer. In each cell, write and solve an inequality.



Solving Inequalities by Using Multiplication and Division		
	By a Positive Number	By a Negative Number
Divide		
Multiply		

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1

p. 182

1. $3b > 27$ 2. $-40 \geq 8b$ 3. $\frac{d}{3} > 6$ 4. $24d \leq 6$
 5. $1.1m \leq 1.21$ 6. $\frac{2}{3}k > 6$ 7. $9s > -18$ 8. $\frac{4}{5} \geq \frac{r}{2}$

SEE EXAMPLE 2

p. 184

9. $-2x < -10$ 10. $\frac{b}{-2} \geq 8$ 11. $-3.5n < 1.4$ 12. $4 > -8g$
 13. $\frac{d}{-6} < \frac{1}{2}$ 14. $-10h \geq -6$ 15. $12 > \frac{t}{-6}$ 16. $-\frac{1}{2}m \geq -7$

SEE EXAMPLE 3

p. 184

17. **Travel** Tom saved \$550 to go on a school trip. The cost for a hotel room, including tax, is \$80 per night. What are the possible numbers of nights Tom can stay at the hotel?

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

Independent Practice

For Exercises See Example

18–29 1

30–41 2

42 3

Extra Practice

Skills Practice p. 58

Application Practice p. 530



18. $10 < 2t$ 19. $\frac{1}{3}j \leq 4$ 20. $-80 < 8c$ 21. $21 > 3d$
 22. $\frac{w}{4} \geq -2$ 23. $\frac{h}{4} \leq \frac{2}{7}$ 24. $6y < 4.2$ 25. $12c \leq -144$
 26. $\frac{4}{5}x \geq \frac{2}{5}$ 27. $6b \geq \frac{3}{5}$ 28. $-25 > 10p$ 29. $\frac{b}{8} \leq -2$
 30. $-9a > 81$ 31. $\frac{1}{2} < \frac{r}{-3}$ 32. $-6p > 0.6$ 33. $\frac{y}{-4} > -\frac{1}{2}$
 34. $-\frac{1}{6}f < 5$ 35. $-2.25t < -9$ 36. $24 \leq -10w$ 37. $-11z > 121$
 38. $\frac{3}{5} < \frac{f}{-5}$ 39. $-k \geq 7$ 40. $-2.2b < -7.7$ 41. $16 \geq -\frac{4}{3}p$

42. **Camping** The rope Roz brought with her camping gear is 54 inches long. Roz needs to cut shorter pieces of rope that are each 18 inches long. What are the possible number of pieces Roz can cut?

Solve each inequality and graph the solutions.

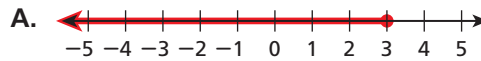
43. $-8x < 24$ 44. $3t \leq 24$ 45. $\frac{1}{4}x < 5$ 46. $\frac{4}{5}p \geq -24$
 47. $54 \leq -9p$ 48. $3t > -\frac{1}{2}$ 49. $-\frac{3}{4}b > -\frac{3}{2}$ 50. $216 > 3.6r$

Write an inequality for each statement. Solve the inequality and graph the solutions.

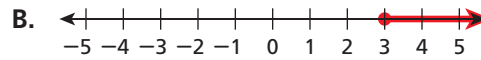
51. The product of a number and 7 is not less than 21.
 52. The quotient of h and -6 is at least 5.
 53. The product of $-\frac{4}{5}$ and b is at most -16 .
 54. Ten is no more than the quotient of t and 4.
 55. **Write About It** Explain how you know whether to reverse the inequality symbol when solving an inequality.
 56. **Geometry** The area of a rectangle is at most 21 square inches. The width of the rectangle is 3.5 inches. What are the possible measurements for the length of the rectangle?

Solve each inequality and match the solution to the correct graph.

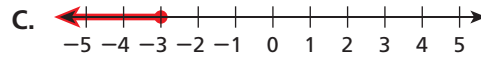
57. $-0.5t \geq 1.5$



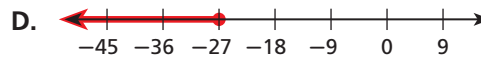
58. $\frac{1}{9}t \leq -3$



59. $-13.5 \leq -4.5t$



60. $\frac{t}{-6} \leq -\frac{1}{2}$



Animals



Orangutans weigh about 3.5 pounds at birth. As adults, female orangutans can weigh as much as 110 pounds, and male orangutans can weigh up to 300 pounds.

61. **Animals** A wildlife shelter is home to birds, mammals, and reptiles. If cat chow is sold in 20 lb bags, what is the least number of bags of cat chow needed for one year at this shelter?

Food Consumed at a Wildlife Shelter per Week	
Type of Food	Amount of Food (lb)
Grapes	4
Mixed seed	10
Peanuts	5
Cat chow	10
Kitten chow	5

62. **Education** In order to earn an A in a college math class, a student must score no less than 90% of all possible points. One semester, students with at least 567 points earned an A in the class. Write an inequality to show the numbers of points possible.
63. **Critical Thinking** Explain why you cannot solve an inequality by multiplying both sides by zero.
64. **///ERROR ANALYSIS///** Two students have different answers for a homework problem. Which answer is incorrect? Explain the error.

A.
$$\begin{aligned} 9m &\geq -27 \\ \frac{9m}{9} &\geq \frac{-27}{9} \\ m &\leq -3 \end{aligned}$$

B.
$$\begin{aligned} 9m &\geq -27 \\ \frac{9m}{9} &\geq \frac{-27}{9} \\ m &\geq -3 \end{aligned}$$

65. Jan has a budget of \$800 for catering. The catering company charges \$12.50 per guest. Write and solve an inequality to show the numbers of guests Jan can invite.

MULTI-STEP TEST PREP



66. This problem will prepare you for the Multi-Step Test Prep on page 188.
- The Swimming Club can spend a total of \$250 for hotel rooms for its spring trip. One hotel costs \$75 per night. Write an inequality to find the number of rooms the club can reserve at this hotel. Let n be the number of rooms.
 - Solve the inequality you wrote in part a. Graph the solutions on a number line. Make sure your answer is reasonable.
 - Another hotel offers a rate of \$65 per night. Does this allow the club to reserve more rooms? Explain your reasoning.

67. Which inequality does NOT have the same solutions as $-\frac{2}{3}y > 4$?

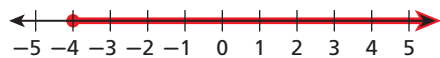
(A) $12 < -2y$

(C) $-\frac{3}{4}y > \frac{9}{2}$

(B) $\frac{y}{2} < -12$

(D) $-3y > 18$

68. The solutions of which inequality are NOT represented by the following graph?



(F) $\frac{x}{2} \geq -2$

(H) $3x \geq -12$

(G) $-5x \geq 20$

(J) $-7x \leq 28$

69. Which inequality can be used to find the number of 39-cent stamps you can purchase for \$4.00?

(A) $0.39s \geq 4.00$

(C) $\frac{s}{0.39} \leq 4.00$

(B) $0.39s \leq 4.00$

(D) $\frac{4.00}{0.39} \leq s$

70. **Short Response** Write three different inequalities that have the same solutions as $x > 4$. Show your work and explain each step.

CHALLENGE AND EXTEND

Solve each inequality.

71. $2\frac{1}{3} \leq -\frac{5}{6}g$

72. $\frac{2x}{3} < 8.25$

73. $2\frac{5}{8}m > \frac{7}{10}$

74. $3\frac{3}{5}f \geq 14\frac{2}{5}$

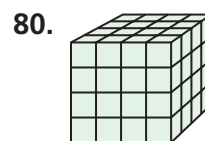
75. **Estimation** What is the greatest possible integer solution of the inequality $3.806x < 19.902$?

76. **Critical Thinking** The Transitive Property of Equality states that if $a = b$ and $b = c$, then $a = c$. Is there a Transitive Property of Inequality using the symbol $<$? Give an example to support your answer.

77. **Critical Thinking** The Symmetric Property of Equality states that if $a = b$, then $b = a$. Is there a Symmetric Property of Inequality? Give an example to support your answer.

SPIRAL REVIEW

Write the power represented by each geometric model. (Lesson 1-4)



Find each unit rate. (Lesson 2-7)

81. Twelve gallons of gas cost \$22.68. Find the unit rate in dollars per gallon.

82. A tree grows four feet in six years. Find the unit rate in feet per year.

83. A student types 105 words in 3 minutes. Find the unit rate in words per minute.

Solve each inequality and graph the solutions. (Lesson 3-2)

84. $x + 5 \geq 3$

85. $t - \frac{1}{4} < \frac{3}{4}$

86. $4 > x - 1$

87. $6 > b - 8$



Simple Inequalities

Remember the Alamo! The Spanish Club is planning a trip for next summer. They plan to travel from Fort Worth, Texas, to San Antonio, Texas. They can spend only \$550 for the entire trip.

1. The treasurer of the club budgets \$60 for gasoline. The current gas price is \$1.95/gallon. The school van gets an average of 20 miles per gallon of gasoline. Determine how many miles they can drive on this budget. Round your answer to the nearest mile.
2. The distance from Fort Worth to San Antonio is 266 miles. Write an inequality that can be used to solve for the number of miles m that they can drive while in San Antonio. Solve your inequality and graph the solutions.
3. The treasurer budgeted \$200 for hotel rooms for one night. The club chose a hotel that charges \$58 per night. Write an inequality that can be used to solve for the number of rooms they can reserve n . What is the maximum number of rooms that they can reserve in the hotel?
4. Use the maximum number of rooms you found in part 3. How much will the club spend on hotel rooms?
5. The club members plan to spend \$80 on food. They also want to see attractions in San Antonio, such as SeaWorld and the Alamo.

Write an inequality that can be solved to find the amount of money available for seeing attractions. What is the maximum amount the club can spend seeing attractions?
6. Write a summary of the budget for the Spanish Club trip. Include the amount they plan to spend on gasoline, hotel rooms, food, and attractions.



Gara Karen Aimee
Janet Nancy Jaime
Loni ☺ Ron Diana

Quiz for Lessons 3-1 Through 3-3

3-1 Graphing and Writing Inequalities

Describe the solutions of each inequality in words.

1. $-2 < r$

2. $t - 1 \leq 7$

3. $2s \geq 6$

4. $4 > 5 - x$

Graph each inequality.

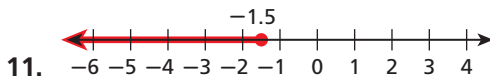
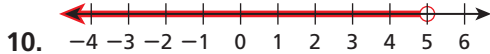
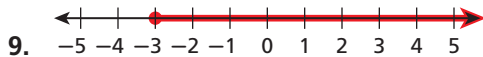
5. $x > -2$

6. $m \leq 1\frac{1}{2}$

7. $g < \sqrt{8+1}$

8. $h \geq 2^3$

Write the inequality shown by each graph.



Write an inequality for each situation and graph the solutions.

12. You must purchase at least 5 tickets to receive a discount.

13. Children under 13 are not admitted to certain movies without an adult.

14. A cell phone plan allows up to 250 free minutes per month.

3-2 Solving One-Step Inequalities by Adding or Subtracting

Solve each inequality and graph the solutions.

15. $k + 5 \leq 7$

16. $4 > p - 3$

17. $r - 8 \geq -12$

18. $-3 + p < -6$

19. Allie must sell at least 50 gift baskets for the band fund-raiser. She already sold 36 baskets. Write and solve an inequality to determine how many more baskets Allie must sell for the fund-raiser.

20. Dante has at most \$12 to spend on entertainment each week. So far this week, he spent \$7.50. Write and solve an inequality to determine how much money Dante can spend on entertainment the rest of the week.

3-3 Solving One-Step Inequalities by Multiplying or Dividing

Solve each inequality and graph the solutions.

21. $-4x < 8$

22. $\frac{d}{3} \geq -3$

23. $\frac{3}{4}t \leq 12$

24. $8 > -16c$

25. A spool of ribbon is 80 inches long. Riley needs to cut strips of ribbon that are 14 inches long. What are the possible numbers of strips that Riley can cut?