

3-2

Solving Inequalities by Adding or Subtracting

Objectives

Solve one-step inequalities by using addition.

Solve one-step inequalities by using subtraction.

Who uses this?

You can use inequalities to determine how many more photos you can take. (See Example 2.)

Tenea has a cell phone that also takes pictures. After taking some photos, Tenea can use a one-step inequality to determine how many more photos she can take.

Solving one-step inequalities is much like solving one-step equations. To solve an inequality, you need to isolate the variable using the properties of inequality and inverse operations.



Properties of Inequality

Addition and Subtraction

WORDS	NUMBERS	ALGEBRA
Addition You can add the same number to both sides of an inequality, and the statement will still be true.	$3 < 8$ $3 + 2 < 8 + 2$ $5 < 10$	$a < b$ $a + c < b + c$
Subtraction You can subtract the same number from both sides of an inequality, and the statement will still be true.	$9 < 12$ $9 - 5 < 12 - 5$ $4 < 7$	$a < b$ $a - c < b - c$

These properties are also true for inequalities that use the symbols $>$, \geq , and \leq .

EXAMPLE 1 Using Addition and Subtraction to Solve Inequalities

Solve each inequality and graph the solutions.

A $x + 9 < 15$

$$x + 9 < 15$$

$$\begin{array}{r} -9 \quad -9 \\ x \quad < \quad 6 \end{array}$$

Since 9 is added to x , subtract 9 from both sides to undo the addition.



B $d - 3 > -6$

$$d - 3 > -6$$

$$\begin{array}{r} +3 \quad +3 \\ d \quad > \quad -3 \end{array}$$

Since 3 is subtracted from d , add 3 to both sides to undo the subtraction.



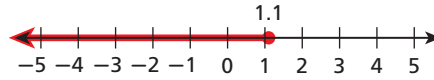
Helpful Hint

Use an inverse operation to “undo” the operation in an inequality. If the inequality contains addition, use subtraction to undo the addition.

Solve each inequality and graph the solutions.

$$\begin{aligned} \mathbf{C} \quad & 0.7 \geq n - 0.4 \\ & 0.7 \geq n - 0.4 \\ & \underline{+ 0.4} \quad \underline{+ 0.4} \\ & 1.1 \geq n \\ & n \leq 1.1 \end{aligned}$$

Since 0.4 is subtracted from n , add 0.4 to both sides to undo the subtraction.



Solve each inequality and graph the solutions.

1a. $s + 1 \leq 10$ 1b. $2\frac{1}{2} > -3 + t$ 1c. $q - 3.5 < 7.5$

Since there can be an infinite number of solutions to an inequality, it is not possible to check all the solutions. You can check the endpoint and the direction of the inequality symbol.

The solutions of $x + 9 < 15$ are given by $x < 6$.

Step 1 Check the endpoint.

Substitute 6 for x in the related equation $x + 9 = 15$. The endpoint should be a solution of the equation.

$$\begin{array}{r|l} x + 9 = 15 & \\ \hline 6 + 9 & 15 \\ 15 & 15 \checkmark \end{array}$$

Step 2 Check the inequality symbol.

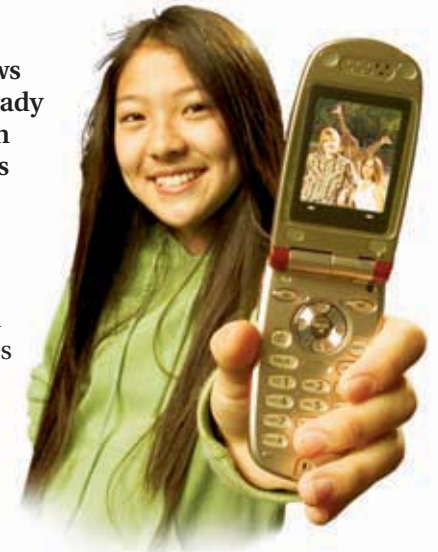
Substitute a number less than 6 for x in the original inequality. The number you choose should be a solution of the inequality.

$$\begin{array}{r|l} x + 9 < 15 & \\ \hline 4 + 9 & < 15 \\ 13 & < 15 \checkmark \end{array}$$

EXAMPLE 2 Problem Solving Application



The memory in Tenea's camera phone allows her to take up to 20 pictures. Tenea has already taken 16 pictures. Write, solve, and graph an inequality to show how many more pictures Tenea could take.



1 Understand the Problem

The **answer** will be an inequality and a graph that show all the possible numbers of pictures that Tenea can take.

List the important information:

- Tenea can take up to, or *at most*, 20 pictures.
- Tenea has taken 16 pictures already.

2 Make a Plan

Write an inequality.

Let p represent the remaining number of pictures Tenea can take.

Number taken	plus	number remaining	is at most	20 pictures.
16	+	p	\leq	20

3 Solve

$$16 + p \leq 20 \quad \text{Since 16 is added to } p, \text{ subtract 16 from both sides to undo the addition.}$$

$$\begin{array}{r} 16 + p \leq 20 \\ -16 \quad -16 \\ \hline p \leq 4 \end{array}$$

It is not reasonable for Tenea to take a negative or fractional number of pictures, so graph the nonnegative integers less than or equal to 4.

Tenea could take 0, 1, 2, 3, or 4 more pictures.



4 Look Back

Check Check the endpoint, 4.

$$16 + p = 20$$

$$\begin{array}{r} 16 + p = 20 \\ 16 + 4 \quad | \quad 20 \\ \hline 20 \quad | \quad 20 \checkmark \end{array}$$

Check a number less than 4.

$$16 + p \leq 20$$

$$\begin{array}{r} 16 + p \leq 20 \\ 16 + 2 \leq 20 \\ \hline 18 \leq 20 \checkmark \end{array}$$

Adding 0, 1, 2, 3, or 4 more pictures will not exceed 20.



2. The Recommended Dietary Allowance (RDA) of iron for a female in Sarah's age group (14–18 years) is 15 mg per day. Sarah has consumed 11 mg of iron today. Write and solve an inequality to show how many more milligrams of iron Sarah can consume without exceeding the RDA.

EXAMPLE 3 Sports Application

Josh can bench press 220 pounds. He wants to bench press at least 250 pounds. Write and solve an inequality to determine how many more pounds Josh must lift to reach his goal. Check your answer.

Let p represent the number of additional pounds Josh must lift.

220 pounds plus additional pounds is at least 250 pounds.

$$220 \quad + \quad p \quad \geq \quad 250$$

$220 + p \geq 250$ Since 220 is added to p , subtract 220 from both sides to undo the addition.

$$\begin{array}{r} 220 + p \geq 250 \\ -220 \quad -220 \\ \hline p \geq 30 \end{array}$$

Check Check the endpoint, 30.

$$220 + p = 250$$

$$\begin{array}{r} 220 + p = 250 \\ 220 + 30 \quad | \quad 250 \\ \hline 250 \quad | \quad 250 \checkmark \end{array}$$

Check a number greater than 30.

$$220 + p \geq 250$$

$$\begin{array}{r} 220 + p \geq 250 \\ 220 + 40 \quad | \quad \geq \quad 250 \\ \hline 260 \quad | \quad \geq \quad 250 \checkmark \end{array}$$

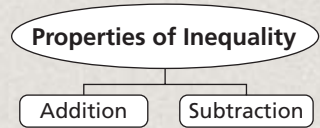
Josh must lift at least 30 additional pounds to reach his goal.



3. **What if...?** Josh has reached his goal of 250 pounds and now wants to try to break the school record of 282 pounds. Write and solve an inequality to determine how many more pounds Josh needs to break the school record. Check your answer.

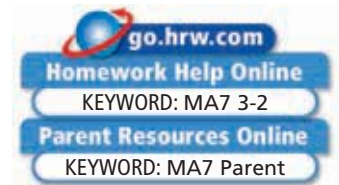
THINK AND DISCUSS

1. Show how to check your solution to Example 1B.
2. Explain how the Addition and Subtraction Properties of Inequality are like the Addition and Subtraction Properties of Equality.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an inequality that you must use the specified property to solve. Then solve and graph the inequality.



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Exercises



GUIDED PRACTICE

SEE EXAMPLE 1

p. 176

Solve each inequality and graph the solutions.

1. $12 < p + 6$

2. $w + 3 \geq 4$

3. $-5 + x \leq -20$

4. $z - 2 > -11$

SEE EXAMPLE 2

p. 177

5. **Health** For adults, the maximum safe water temperature in a spa is 104°F . The water temperature in Bill's spa is 102°F . The temperature is increased by $t^\circ\text{F}$. Write, solve, and graph an inequality to show the values of t for which the water temperature is still safe.

SEE EXAMPLE 3

p. 178

6. **Consumer Economics** A local restaurant will deliver food to your house if the purchase amount of your order is at least \$25.00. The total for part of your order is \$17.95. Write and solve an inequality to determine how much more you must spend for the restaurant to deliver your order.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
7–10	1
11	2
12	3

Solve each inequality and graph the solutions.

7. $a - 3 \geq 2$

8. $2.5 > q - 0.8$

9. $-45 + x < -30$

10. $r + \frac{1}{4} \leq \frac{3}{4}$

11. **Engineering** The maximum load for a certain elevator is 2000 pounds. The total weight of the passengers on the elevator is 1400 pounds. A delivery man who weighs 243 pounds enters the elevator with a crate of weight w . Write, solve, and graph an inequality to show the values of w that will not exceed the weight limit of the elevator.

12. **Transportation** The gas tank in Mindy's car holds at most 15 gallons. She has already filled the tank with 7 gallons of gas. She will continue to fill the tank with g gallons more. Write and solve an inequality that shows all values of g that Mindy can add to the car's tank.

Extra Practice

Skills Practice p. S8
Application Practice p. S30

Write an inequality to represent each statement. Solve the inequality and graph the solutions.

13. Ten less than a number x is greater than 32.
14. A number n increased by 6 is less than or equal to 4.
15. A number r decreased by 13 is at most 15.

Solve each inequality and graph the solutions.

16. $x + 4 \leq 2$ 17. $-12 + q > 39$ 18. $x + \frac{3}{5} < 7$ 19. $4.8 \geq p + 4$
 20. $-12 \leq x - 12$ 21. $4 < 206 + c$ 22. $y - \frac{1}{3} > \frac{2}{3}$ 23. $x + 1.4 \geq 1.4$



Health



Special-effects contact lenses are sometimes part of costumes for movies. All contact lenses should be worn under an eye doctor's supervision.

24. Use the inequality $s + 12 \geq 20$ to fill in the missing numbers.

- a. $s \geq \square$ b. $s + \square \geq 30$ c. $s - 8 \geq \square$

25. **Health** A particular type of contact lens can be worn up to 30 days in a row. Alex has been wearing these contact lenses for 21 days. Write, solve, and graph an inequality to show how many more days Alex could wear his contact lenses.

Solve each inequality and match the solutions to the correct graph.

26. $1 \leq x - 2$ A. 27. $8 > x - (-5)$ B. 28. $x + 6 > 9$ C. 29. $-4 \geq x - 7$ D.

30. **Estimation** Is $x < 10$ a reasonable estimate for the solutions to the inequality $11.879 + x < 21.709$? Explain your answer.

31. **Sports** At the Seattle Mariners baseball team's home games, there are 45,611 seats in the four areas listed in the table. Suppose all the suite level and club level seats during a game are filled. Write and solve an inequality to determine how many people p could be sitting in the other types of seats.

Mariners Home Game Seating	
Type of Seat	Number of Seats
Main bowl	24,399
Upper bowl	16,022
Club level	4,254
Suite level	936



32. **Critical Thinking** Recall that in Chapter 2 a balance scale was used to model solving equations. Describe how a balance scale could model solving inequalities.

33. **Critical Thinking** Explain why $x + 4 \geq 6$ and $x - 4 \geq -2$ have the same solutions.



34. **Write About It** How do the solutions of $x + 2 \geq 3$ differ from the solutions of $x + 2 > 3$? How do the graphs of the solutions differ?

MULTI-STEP TEST PREP



35. This problem will prepare you for the Multi-Step Test Prep on page 188.
- Daryl finds that the distance from Columbus, Ohio, to Washington, D.C., is 411 miles. What is the round-trip distance?
 - Daryl can afford to drive a total of 1000 miles. Write an inequality to show the number of miles m he can drive while in Washington, D.C.
 - Solve the inequality and graph the solutions on a number line. Show that your answer is reasonable.

36. Which is a reasonable solution of $4.7367 + p < 20.1784$?
 (A) 15 (B) 16 (C) 24 (D) 25
37. Which statement can be modeled by $x + 3 \leq 12$?
 (F) Sam has 3 bottles of water. Together, Sam and Dave have at most 12 bottles of water.
 (G) Jennie sold 3 cookbooks. To earn a prize, Jennie must sell at least 12 cookbooks.
 (H) Peter has 3 baseball hats. Peter and his brothers have fewer than 12 baseball hats.
 (J) Kathy swam 3 laps in the pool this week. She must swim more than 12 laps.
38. Which graph represents the solutions of $p + 3 < 17$?
 (A) (C)
 (B) (D)
39. Which inequality does NOT have the same solutions as $n + 12 \leq 26$?
 (F) $n \leq 14$ (G) $n + 6 \leq 20$ (H) $10 \geq n - 4$ (J) $n - 12 \leq 14$

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

40. $6\frac{9}{10} \geq 4\frac{4}{5} + x$ 41. $r - 1\frac{2}{5} \leq 3\frac{7}{10}$ 42. $6\frac{2}{3} + m > 7\frac{1}{6}$

Determine whether each statement is sometimes, always, or never true. Explain.

43. $a + b > a - b$
 44. If $a > c$, then $a + b > c + b$.
 45. If $a > b$ and $c > d$, then $a + c > b + d$.
 46. If $x + b > c$ and $x > 0$ have the same solutions, what is the relationship between b and c ?

SPIRAL REVIEW

Solve each equation for the indicated variable. (Lesson 2-5)

47. $2x + 3y = 9$ for y 48. $P = 4s$ for s 49. $2 + ab = c$ for a
 50. $p + e = f$ for e 51. $2s - k = 11$ for k 52. $5m + n = 0$ for m

Find the value of x in each diagram of similar figures. (Lesson 2-8)

53. 54.

Write the inequality shown by each graph. (Lesson 3-1)

55. 56.