

# 2-9

# Percents

### Objective

Solve problems involving percents.

### Vocabulary

percent

### Who uses this?

Jewelers use percents to determine the purity of precious metals. (See Example 4.)



A **percent** is a ratio that compares a number to 100. For example,  $25\% = \frac{25}{100}$ .

To find the fraction equivalent of a percent, write the percent as a ratio with a denominator of 100. Then simplify.

$$25\% = \frac{25}{100} = \frac{1}{4}$$

To find the decimal equivalent of a percent, divide by 100.

$$25\% = \frac{25}{100} = 0.25$$



Some Common Equivalents											
Percent	10%	20%	25%	$33\frac{1}{3}\%$	40%	50%	60%	$66\frac{2}{3}\%$	75%	80%	100%
Fraction	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	1
Decimal	0.1	0.2	0.25	$0.\bar{3}$	0.4	0.5	0.6	$0.\bar{6}$	0.75	0.8	1.0

The greatest percent shown in the table is 100%, or 1. But percents can be greater than 100%. For example,  $120\% = \frac{120}{100} = 1.2$ . You can also find percents that are less than 1%. For example,  $0.5\% = \frac{0.5}{100} = 0.005$ .

You can use the proportion  $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$  to find unknown values.

## EXAMPLE 1 Finding the Part

**A** Find 50% of 20.

**Method 1** Use a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \text{Use the percent proportion.}$$

$$\frac{x}{20} = \frac{50}{100} \quad \text{Let } x \text{ represent the part.}$$

$$100x = 1000 \quad \text{Find the cross products.}$$

$$x = 10 \quad \text{Since } x \text{ is multiplied by } 100, \text{ divide both sides by } 100 \text{ to undo the multiplication.}$$

50% of 20 is 10.

**Check** 50% is the same as  $\frac{1}{2}$ , and  $\frac{1}{2}$  of 20 is 10. ✓

**B** Find 105% of 72.

**Method 2** Use an equation.

$$x = 105\% \text{ of } 72 \quad \textit{Write an equation. Let } x \textit{ represent the part.}$$

$$x = 1.05(72) \quad \textit{Write the percent as a decimal and multiply.}$$

$$x = 75.6$$

$$105\% \text{ of } 72 \text{ is } 75.6.$$



**1a.** Find 20% of 60.   **1b.** Find 210% of 8.   **1c.** Find 4% of 36.

## EXAMPLE 2 Finding the Percent

### Helpful Hint

Before solving, decide what is a reasonable answer. For Example 2A, 50% of 60 is 30, so the answer will be less than 50% of 60.

**A** What percent of 60 is 15?

**Method 1** Use a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \textit{Use the percent proportion.}$$

$$\frac{15}{60} = \frac{x}{100} \quad \textit{Let } x \textit{ represent the percent.}$$

$$60x = 1500 \quad \textit{Find the cross products.}$$

$$\frac{60x}{60} = \frac{1500}{60} \quad \textit{Since } x \textit{ is multiplied by } 60, \textit{ divide both sides by } 60 \textit{ to undo the multiplication.}$$

$$x = 25$$

$$15 \text{ is } 25\% \text{ of } 60.$$

**B** 440 is what percent of 400?

**Method 2** Use an equation.

$$440 = x \cdot 400 \quad \textit{Write an equation. Let } x \textit{ represent the percent.}$$

$$440 = 400x \quad \textit{Since } x \textit{ is multiplied by } 400, \textit{ divide both sides by } 400 \textit{ to undo the multiplication.}$$

$$\frac{440}{400} = \frac{400x}{400}$$

$$1.1 = x \quad \textit{The answer is a decimal.}$$

$$110\% = x \quad \textit{Write the decimal as a percent. This answer is reasonable; } 440 \textit{ is more than } 100\% \textit{ of } 400.$$

$$440 \text{ is } 110\% \text{ of } 400.$$



**2a.** What percent of 35 is 7?   **2b.** 27 is what percent of 9?

## EXAMPLE 3 Finding the Whole

**A** 40% of what number is 14?

**Method 1** Use a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \textit{Use the percent proportion.}$$

$$\frac{14}{x} = \frac{40}{100} \quad \textit{Let } x \textit{ represent the whole.}$$

$$40x = 1400 \quad \textit{Find the cross products.}$$

$$\frac{40x}{40} = \frac{1400}{40} \quad \textit{Since } x \textit{ is multiplied by } 40, \textit{ divide both sides by } 40 \textit{ to undo the multiplication.}$$

$$x = 35$$

$$40\% \text{ of } 35 \text{ is } 14.$$

**B** 40 is 0.8% of what number?

**Method 2** Use an equation.

$$40 = 0.8\% \text{ of } x$$

$$40 = 0.008 \cdot x$$

$$\frac{40}{0.008} = \frac{0.008x}{0.008}$$

$$5000 = x$$

40 is 0.8% of 5000.

*Write an equation. Let  $x$  represent the whole.*

*Write the percent as a decimal.*

*Since  $x$  is multiplied by 0.008, divide both sides by 0.008 to undo the multiplication.*



**3a.** 120% of what number is 90?

**3b.** 48 is 15% of what number?

#### EXAMPLE 4 Career Application

Jewelers use the karat system to determine the amount of pure gold in jewelry. Pure gold is 24 karat, meaning the item is 100% gold. A 14-karat gold ring contains 14 parts gold and 10 parts other metal. What percent of the ring is gold? Round your answer to the nearest percent.



$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

*Use the percent proportion.*

$$\frac{14}{24} = \frac{x}{100}$$

*Let  $x$  represent the percent.*

$$24x = 1400$$

*Find the cross products.*

$$\frac{24x}{24} = \frac{1400}{24}$$

*Since  $x$  is multiplied by 24, divide both sides by 24 to undo the multiplication.*

$$x = 58.\bar{3}$$

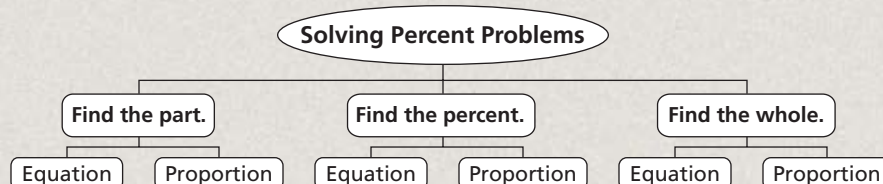
A 14-karat gold ring is approximately 58% gold.



**4.** Use the information above to find the number of karats in a bracelet that is 42% gold. Round your answer to the nearest whole number.

### THINK AND DISCUSS

- Describe the numerical value of the percent when the part is greater than the whole. Give an example.
- 64% of a number is 32. Is the number greater than or less than 32? Explain.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write and solve an example using the given method.



### GUIDED PRACTICE

1. **Vocabulary** In your own words, write a definition of *percent*.

SEE EXAMPLE 1

p. 133

2. Find 75% of 40.

3. Find  $12\frac{1}{2}\%$  of 168.

4. Find 115% of 57.

5. Find 70% of 8.

SEE EXAMPLE 2

p. 134

6. What percent of 40 is 25?

7. What percent of 225 is 180?

8. 57 is what percent of 30?

9. 1 is what percent of 8?

SEE EXAMPLE 3

p. 134

10. 28 is 32% of what number?

11. 4% of what number is 7?

12. 16 is 10% of what number?

13. 105% of what number is 37.8?

SEE EXAMPLE 4

p. 135

14. **Nutrition** A certain granola bar has 2 grams of fiber. This is 8% of the recommended daily value. How many grams of fiber are recommended daily?

### PRACTICE AND PROBLEM SOLVING

Find each value. Round to the nearest tenth if necessary.

15. 60% of 80

16. 35% of 90

17.  $\frac{1}{2}\%$  of 500

18. 210% of 30

19. What percent of 52 is 13?

20. What percent of 9 is 27?

21. 11 is what percent of 22?

22. 5 is what percent of 67?

23. 36 is 90% of what number?

24. 8.2 is 2% of what number?

25.  $4\frac{1}{2}\%$  of what number is 23?

26. 16% of what number is 94?

27. **Nutrition** A certain can of iced tea contains 4% of the recommended daily allowance of sodium. The recommended daily allowance is 2500 milligrams. How many milligrams of sodium are in the can of iced tea?

Write each decimal or fraction as a percent.

28.  $\frac{5}{4}$

29. 0.02

30. 0.27

31.  $\frac{2}{25}$

32.  $\frac{7}{7}$

33. 0.64

34.  $\frac{31}{100}$

35. 0.85

36. 0.003

37.  $\frac{17}{20}$

Write each percent as a decimal and as a fraction.

38. 23%

39. 52%

40. 12.5%

41. 90%

42. 87.2%

43. 112%

44. 29%

45. 6%

46. 1.5%

47.  $\frac{3}{5}\%$

48. **Estimation** To estimate 26% of 400, think:

$$26\% \text{ is close to } 25\% \text{ and } 25\% = \frac{1}{4}$$

$$\frac{1}{4} \text{ of } 400 = 100.$$

Therefore, 26% of 400 is about 100.

Use a similar method to estimate 48% of 610 and 73% of 820. Then check your estimates by finding each percent.

#### Independent Practice

For Exercises	See Example
15–18	1
19–22	2
23–26	3
27	4

#### Extra Practice

Skills Practice p. S7  
 Application Practice p. S29

49. **Critical Thinking** Which is greater, 0.5 or  $\frac{1}{2}\%$ ? Explain.

Write each list in order from least to greatest.

50.  $\frac{1}{20}$ , 5.3%, 5.1, 0.005,  $\frac{1}{2}$

51. 1.1, 11%,  $\frac{1}{10}$ , 0.001, 1%

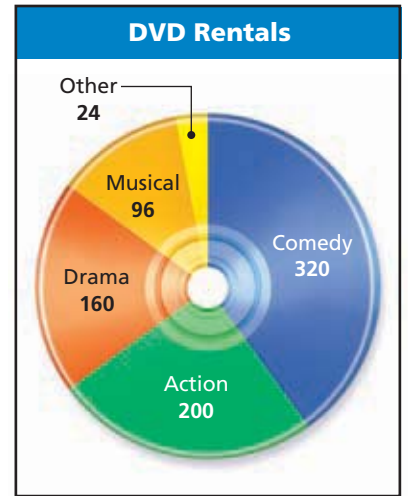
52.  $\frac{3}{8}$ , 29%,  $\frac{2}{5}$ , 0.25, 38%

53. 0.49, 82%, 0.94,  $\frac{4}{5}$ ,  $\frac{5}{9}$

54. **Biology** On average, sloths spend 16.5 hours per day sleeping. What percent of the day do sloths spend sleeping? Round your answer to the nearest percent.

55. **Entertainment** The numbers of various types of movies rented over a period of time are indicated in the graph.

- What percent of the movies rented were comedies?
- What type of movie made up 25% of the rentals?
- What percent of the movies rented were in the "other" category?
- What if...?** If 25 of the comedy rentals had instead been action rentals, what percent of the movies rented would have been comedies? Round your answer to the nearest tenth.



56. **Multi-Step** According to the 2000 U.S. Census, 138,053,563 Americans are male, and 143,368,343 Americans are female. About what percent of the population is male? female? Round your answers to the nearest percent.

57. Complete each statement in the table below. Describe any patterns you see in the completed table.

1% of 400 is 4.	100% of <input type="text"/> is 12.	<input type="text"/> % of 80 is 20.
2% of <input type="text"/> is 4.	50% of <input type="text"/> is 12.	<input type="text"/> % of 40 is 20.
4% of <input type="text"/> is 4.	25% of <input type="text"/> is 12.	<input type="text"/> % of 20 is 20.
8% of <input type="text"/> is 4.	12.5% of <input type="text"/> is 12.	<input type="text"/> % of 10 is 20.



58. **Write About It** Explain the advantages of using the proportion method to solve percent problems. Then, explain the advantages of using the equation method to solve percent problems.

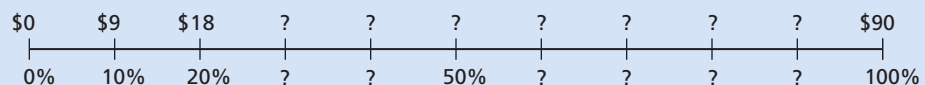
**MULTI-STEP  
TEST PREP**



59. This problem will prepare you for the Multi-Step Test Prep on page 152.

Kathryn found a new dress at the mall. The price tag reads \$90. The sign above the rack of dresses says that all items on the rack are 40% off.

- Set up a proportion to find 40% of \$90. This is the amount of the discount.
- How much will Kathryn pay for the dress? Show that your answer is reasonable.
- Copy and complete the model below. Explain how this model can help you answer the above questions in another way.





60. Which proportion can be used to find 14% of 60?
- (A)  $\frac{x}{100} = \frac{60}{14}$       (B)  $\frac{14}{100} = \frac{60}{x}$       (C)  $\frac{x}{100} = \frac{14}{60}$       (D)  $\frac{14}{100} = \frac{x}{60}$
61. Raul surveyed 35 students about their preferred lunch. Fourteen preferred chicken. Half of those students preferred chicken with barbecue sauce. What percent should Raul report as preferring chicken with barbecue sauce?
- (F) 20%      (G) 40%      (H) 50%      (J) 80%
62. After an election in a small town, the newspaper reported that 42% of the registered voters actually voted. If 12,000 people voted, which equation can be used to find the number of registered voters?
- (A)  $v = 42 \cdot 12,000$       (B)  $v = 0.42 \cdot 12,000$       (C)  $42v = 12,000$       (D)  $0.42v = 12,000$
63. Which list is in order from least to greatest?
- (F)  $\frac{1}{2}$ , 20%, 33%, 0.625,  $\frac{1}{8}$ , 1      (H)  $\frac{1}{8}$ ,  $\frac{1}{2}$ , 0.625, 1, 20%, 30%
- (G)  $\frac{1}{8}$ , 20%, 33%,  $\frac{1}{2}$ , 0.625, 1      (J) 0.625,  $\frac{1}{8}$ ,  $\frac{1}{2}$ , 1, 20%, 30%
64. Moises saves 8% of his weekly paycheck in his savings account. He deposited \$18.80 from his last paycheck into his savings account. Which is the best estimate of the total amount of Moises's last paycheck?
- (A) \$26      (B) \$100      (C) \$160      (D) \$200

## CHALLENGE AND EXTEND

Find each value. Round to the nearest tenth if necessary.

65. What percent of 16 is 2.75?      66. 22 is 73.5% of what number?
67. 121.3% of 73 is what number?      68. What percent of 8000 is 6525?
69. Find 10% of 8 and 8% of 10. What do you notice? Try this with several other pairs of numbers. Do you think this relationship will be true for all pairs of numbers? Why or why not?
70. **Chemistry** A chemist has 20 milliliters of a solution that is 40% acid. She wants to increase the acid content of the solution to make it a 50%-acid solution. How many milliliters of pure acid should she add to the solution? (*Hint*: Begin by finding the number of milliliters of acid in the original solution.)

## SPIRAL REVIEW

Simplify each expression. (*Lesson 1-7*)

71.  $32 + 47 + 28 + 13$       72.  $4 \cdot 23 \cdot 25$
73.  $8 \cdot 4 \cdot 5$       74.  $44 + 27 + 56$
75. A picture has a width of 4 in. and a length of 6 in. It is enlarged on a copier, and the new length is 9 in. What is the new width? (*Lesson 2-8*)
76. A picture has a width of 4 in. and a length of 6 in. It is reduced on a copier, and the new length is 4.8 in. What is the new width? (*Lesson 2-8*)
77. A rectangle has an area of  $9 \text{ ft}^2$ . Every dimension is multiplied by a scale factor, and the new rectangle has an area of  $81 \text{ ft}^2$ . What was the scale factor? (*Lesson 2-8*)