2-8 Applications of Proportions

Objectives
Use proportions to solve problems involving geometric figures.

Use proportions and similar figures to measure objects indirectly.

Vocabulary
similar
corresponding sides
corresponding angles
indirect measurement
scale factor

Why learn this?
Proportions can be used to find the heights of tall objects, such as totem poles, that would otherwise be difficult to measure. (See Example 2.)

Similar figures have exactly the same shape but not necessarily the same size.

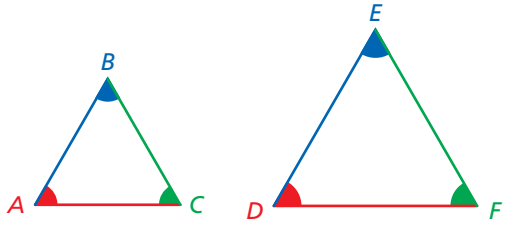
Corresponding sides of two figures are in the same relative position, and **corresponding angles** are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.

Reading Math

- $\overline{AB}$ means segment AB. AB means the length of $\overline{AB}$.
- $\angle A$ means angle A. $m\angle A$ means the measure of angle A.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$m\angle A = m\angle D$
$m\angle B = m\angle E$
$m\angle C = m\angle F$



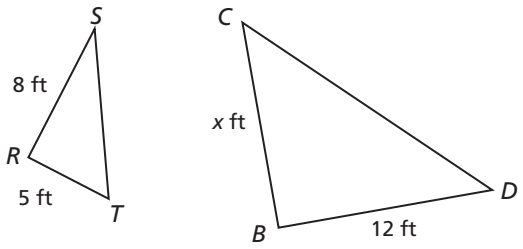
When stating that two figures are similar, use the symbol $\sim$. For the triangles above, you can write $\triangle ABC \sim \triangle DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\triangle ABC \sim \triangle EFD$.

You can use proportions to find missing lengths in similar figures.

EXAMPLE 1 Finding Missing Measures in Similar Figures

Find the value of x in each diagram.

A $\triangle RST \sim \triangle BCD$



R corresponds to B, S corresponds to C, and T corresponds to D.

$$\frac{5}{12} = \frac{8}{x}$$ $\frac{RT}{BD} = \frac{RS}{BC}$

$5x = 96$ *Use cross products.*

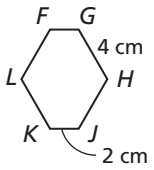
$\frac{5x}{5} = \frac{96}{5}$ *Since x is multiplied by 5, divide both sides by 5 to undo the multiplication.*

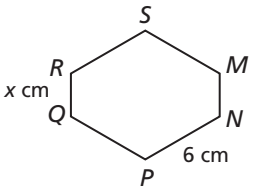
$x = 19.2$

The length of $\overline{BC}$ is 19.2 ft.

Find the value of x in each diagram.

B $FGHJKL \sim MNPQRS$





$\frac{6}{4} = \frac{x}{2}$ — $\frac{NP}{GH} = \frac{RQ}{KJ}$

$4x = 12$ — *Use cross products.*

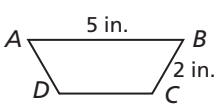
$\frac{4x}{4} = \frac{12}{4}$ — *Since x is multiplied by 4, divide both sides by 4 to undo the multiplication.*

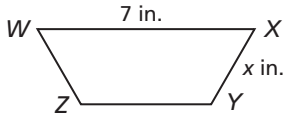
$x = 3$

The length of $\overline{QR}$ is 3 cm.

CHECK IT OUT!

1. Find the value of x in the diagram if $ABCD \sim WXYZ$.

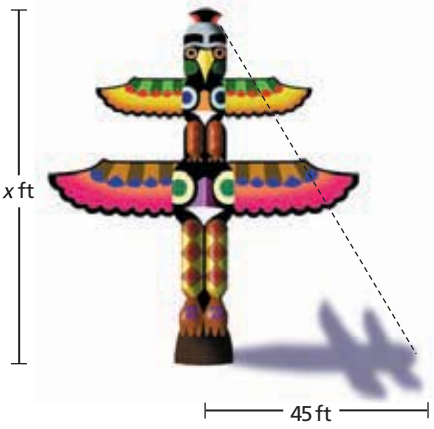




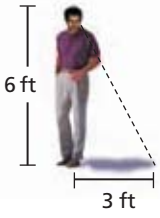
You can solve a proportion involving similar triangles to find a length that is not easily measured. This method of measurement is called **indirect measurement**. If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

EXAMPLE 2 *Measurement Application*

A totem pole casts a shadow 45 feet long at the same time that a 6-foot-tall man casts a shadow that is 3 feet long. Write and solve a proportion to find the height of the totem pole.



Both the man and the totem pole form right angles with the ground, and their shadows are cast at the same angle. You can form two similar right triangles.



$\frac{6}{x} = \frac{3}{45}$

$3x = 270$

$\frac{3x}{3} = \frac{270}{3}$

$x = 90$

$\frac{\text{man's height}}{\text{pole's height}} = \frac{\text{man's shadow}}{\text{pole's shadow}}$

Use cross products. Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.

The totem pole is 90 feet tall.

Helpful Hint

A height of 90 ft seems reasonable for a totem pole. If you got 900 or 9000 ft, that would not be reasonable, and you should check your work.

CHECK IT OUT!

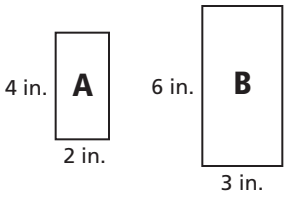
2a. A forest ranger who is 150 cm tall casts a shadow 45 cm long. At the same time, a nearby tree casts a shadow 195 cm long. Write and solve a proportion to find the height of the tree.

2b. A woman who is 5.5 feet tall casts a shadow 3.5 feet long. At the same time, a building casts a shadow 28 feet long. Write and solve a proportion to find the height of the building.

If every dimension of a figure is multiplied by the same number, the result is a similar figure. The multiplier is called a **scale factor**.

EXAMPLE 3 Changing Dimensions

A **Every dimension of a 2-by-4-inch rectangle is multiplied by 1.5 to form a similar rectangle. How is the ratio of the perimeters related to the ratio of corresponding sides? How is the ratio of the areas related to the ratio of corresponding sides?**



	Rectangle A	Rectangle B
$P = 2\ell + 2w$	$2(2) + 2(4) = 12$	$2(6) + 2(3) = 18$
$A = \ell w$	$4(2) = 8$	$6(3) = 18$

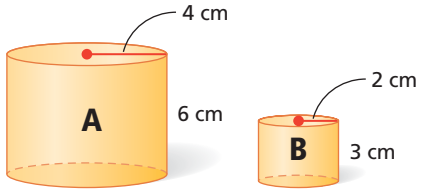
Sides: $\frac{4}{6} = \frac{2}{3}$ Perimeters: $\frac{12}{18} = \frac{2}{3}$ Areas: $\frac{8}{18} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$

The ratio of the perimeters is equal to the ratio of corresponding sides. The ratio of the areas is the square of the ratio of corresponding sides.

Helpful Hint

A scale factor between 0 and 1 reduces a figure. A scale factor greater than 1 enlarges it.

B **Every dimension of a cylinder with radius 4 cm and height 6 cm is multiplied by $\frac{1}{2}$ to form a similar cylinder. How is the ratio of the volumes related to the ratio of corresponding dimensions?**



	Cylinder A	Cylinder B
$V = \pi r^2 h$	$\pi(4)^2(6) = 96\pi$	$\pi(2)^2(3) = 12\pi$

Radii: $\frac{4}{2} = \frac{2}{1} = 2$ Heights: $\frac{6}{3} = \frac{2}{1} = 2$ Volumes: $\frac{96\pi}{12\pi} = \frac{8}{1} = 8 = 2^3$

The ratio of the volumes is the cube of the ratio of corresponding dimensions.

CHECK IT OUT!

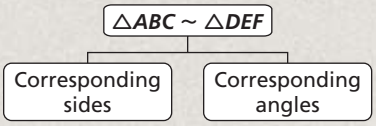
3. A rectangle has width 12 inches and length 3 inches. Every dimension of the rectangle is multiplied by $\frac{1}{3}$ to form a similar rectangle. How is the ratio of the perimeters related to the ratio of the corresponding sides?

THINK AND DISCUSS

1. Name some pairs of real-world items that appear to be similar figures.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In the top box, sketch and label two similar triangles. Then list the corresponding sides and angles in the bottom boxes.



2-8 Exercises

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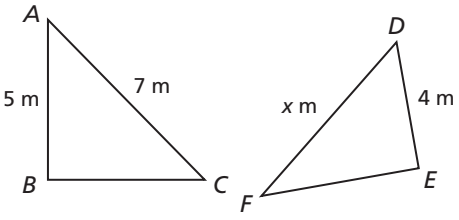
GUIDED PRACTICE

1. **Vocabulary** What does it mean for two figures to be *similar*?

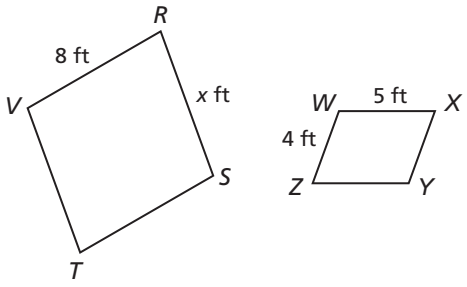
SEE EXAMPLE 1 p. 127

Find the value of x in each diagram.

2. $\triangle ABC \sim \triangle DEF$



3. $RSTV \sim WXYZ$



SEE EXAMPLE 2 p. 128

4. Roger is 5 feet tall and casts a shadow 3.5 feet long. At the same time, the flagpole outside his school casts a shadow 14 feet long. Write and solve a proportion to find the height of the flagpole.

SEE EXAMPLE 3 p. 129

5. A rectangle has length 12 feet and width 8 feet. Every dimension of the rectangle is multiplied by $\frac{3}{4}$ to form a similar rectangle. How is the ratio of the areas related to the ratio of corresponding sides?

PRACTICE AND PROBLEM SOLVING

Independent Practice

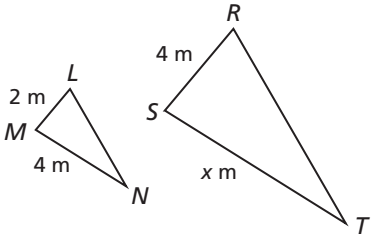
For Exercises	See Example
6–7	1
8	2
9	3

Extra Practice

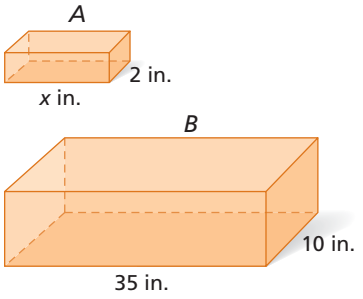
Skills Practice p. S7
Application Practice p. S29

Find the value of x in each diagram.

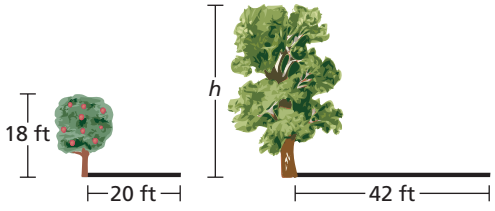
6. $\triangle LMN \sim \triangle RST$



7. prism $A \sim$ prism B



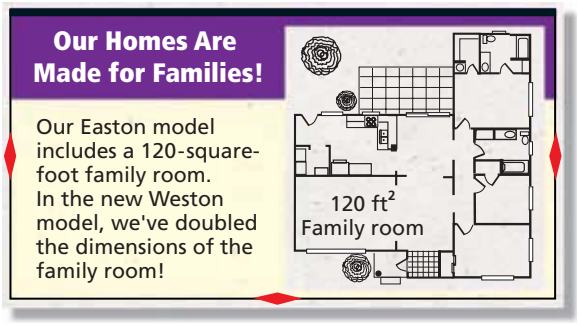
8. Write and solve a proportion to find the height of the taller tree in the diagram at right.



9. A triangle has side lengths of 5 inches, 12 inches, and 15 inches. Every dimension is multiplied by $\frac{1}{5}$ to form a new triangle. How is the ratio of the perimeters related to the ratio of corresponding sides?

10. **Hobbies** For a baby shower gift, Heather crocheted a baby blanket whose length was $2\frac{1}{2}$ feet and whose width was 2 feet. She plans to crochet a proportionally larger similar blanket for the baby's mother. If she wants the length of the mother's blanket to be $6\frac{1}{4}$ feet, what should the width be? Show that your answer is reasonable.

11. Real Estate Refer to the home builder's advertisement. The family rooms in both models are rectangular. How much carpeting is needed to carpet the family room in the Weston model?

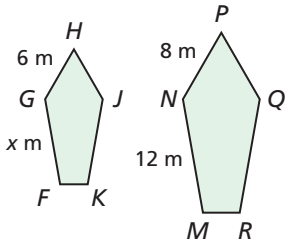


12. A rectangle has an area of 16 ft^2. Every dimension is multiplied by a scale factor, and the new rectangle has an area of 64 ft^2. What was the scale factor?

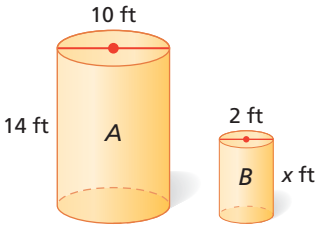
13. A cone has a volume of 98π cm^3. Every dimension is multiplied by a scale factor, and the new cone has a volume of 6272π cm^3. What was the scale factor?

Find the value of x in each diagram.

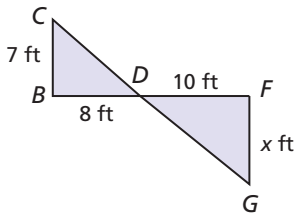
14. $FGHJK \sim MNPQR$



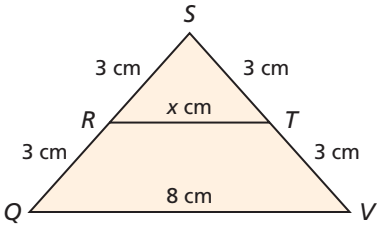
15. cylinder $A \sim$ cylinder B



16. $\triangle BCD \sim \triangle FGD$



17. $\triangle RST \sim \triangle QSV$



18. A tower casts a 450 ft shadow at the same time that a 4 ft child casts a 6 ft shadow. Write and solve a proportion to find the height of the tower.

19. Write About It At Pizza Palace, a pizza with a diameter of 8 inches costs \$6.00. The restaurant manager says that a 16-inch pizza should be priced at \$12.00 because it is twice as large. Do you agree? Explain why or why not.

20. This problem will prepare you for the Multi-Step Test Prep on page 152.

Another common application of proportion is *percents*. A percent is a ratio of a number to 100. For example, $80\% = \frac{80}{100}$.

a. Write 12%, 18%, 25%, 67%, and 98% as ratios.

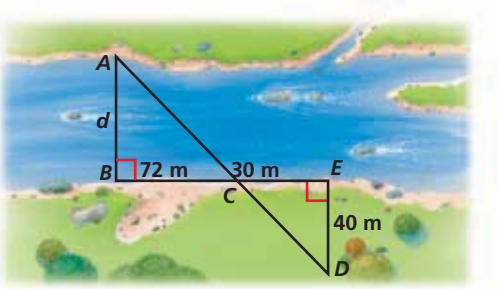
b. Percents can also be written as decimals. Write each of your ratios from part **a** as a decimal.

c. What do you notice about a percent and its decimal equivalent?

You will learn more about percents and their connections to proportions in upcoming lessons.

21. A lighthouse casts a shadow that is 36 meters long. At the same time, a person who is 1.5 meters tall casts a shadow that is 4.5 meters long. Write and solve a proportion to find the height of the lighthouse.

22. In the diagram, $\triangle ABC \sim \triangle DEC$. What is the distance across the river from A to B?



23. Critical Thinking If every dimension of a two-dimensional figure is multiplied by k, by what quantity is the area multiplied?

TEST PREP

24. A beach ball holds 800 cubic inches of air. Another beach ball has a radius that is half that of the larger ball. How much air does the smaller ball hold?

Ⓐ 400 cubic inches
Ⓑ 200 cubic inches
Ⓒ 100 cubic inches
Ⓓ 80 cubic inches

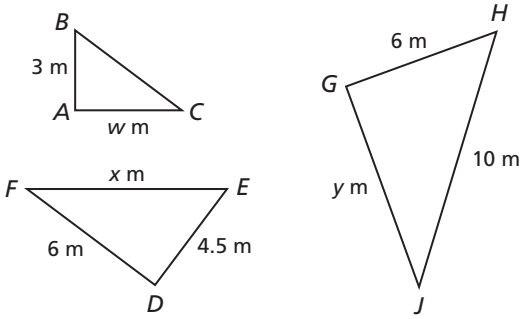
25. For two similar triangles, $\frac{SG}{MW} = \frac{GT}{WR} = \frac{TS}{RM}$. Which statement below is NOT correct?

Ⓕ $\triangle SGT \sim \triangle MWR$
Ⓖ $\triangle GST \sim \triangle MRW$
Ⓗ $\triangle TGS \sim \triangle RWM$
Ⓙ $\triangle GTS \sim \triangle WRM$

26. Gridded Response A rectangle has length 5 centimeters and width 3 centimeters. A similar rectangle has length 7.25 centimeters. What is the width in centimeters of this rectangle?

CHALLENGE AND EXTEND

27. Find the values of w, x, and y given that $\triangle ABC \sim \triangle DEF \sim \triangle GHJ$.



28. $\triangle RST \sim \triangle VWX$ and $\frac{RT}{VX} = b$.
What is $\frac{\text{area of } \triangle RST}{\text{area of } \triangle VWX}$?

29. Multi-Step Rectangles A and B are similar. The area of A is 30.195 cm^2. The length of B is 6.1 cm. Each dimension of B is $\frac{2}{3}$ the corresponding dimension of A. What is the perimeter of B?

SPIRAL REVIEW

Add or subtract. *(Lesson 1-2)*

30. $-9 - 2$ **31.** $-7 + (-5)$ **32.** $12 - (-18)$ **33.** $19 - 65$

Generate ordered pairs for each function for $x = -2, -1, 0, 1, 2$. *(Lesson 1-8)*

34. $y = 2x$ **35.** $y = x^2$ **36.** $y = 6 - x$ **37.** $y = 3x - 1$

Solve each proportion. *(Lesson 2-7)*

38. $\frac{x}{8} = \frac{1}{4}$ **39.** $\frac{6}{x} = \frac{3}{16}$ **40.** $\frac{5}{12} = \frac{-4}{f}$ **41.** $\frac{3}{10} = \frac{x+1}{15}$