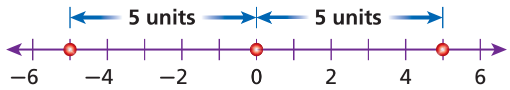
2-6 Solving Absolute-Value Equations

Objective
Solve equations in one variable that contain absolute-value expressions.

Why learn this?
Engineers can solve absolute-value equations to calculate the length of the deck of a bridge. (See Example 3.)

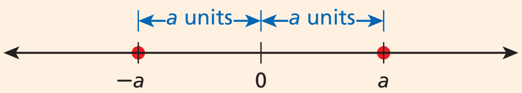
Recall that the absolute value of a number is that number's distance from zero on a number line. For example, $|-5| = 5$ and $|5| = 5$.



For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and −5 have an absolute value of 5.

To write this statement using algebra, you would write $|x| = 5$. This equation asks, "What values of x have an absolute value of 5?" The solutions are 5 and −5. Notice that this equation has two solutions.

Know it! Note — **Absolute-Value Equations**

WORDS	NUMBERS
The equation $|x| = a$ asks, "What values of x have an absolute value of a?" The solutions are a and the opposite of a.	$|x| = 5$ $x = 5$ or $x = -5$
GRAPH	**ALGEBRA**
	$|x| = a$ $x = a$ or $x = -a$ $(a \geq 0)$

To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation. Then you must consider two cases.

EXAMPLE 1 Solving Absolute-Value Equations

Helpful Hint

Be sure to check both solutions when you solve an absolute-value equation.

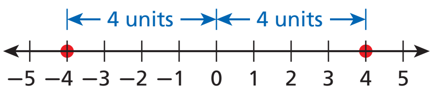
$|x| = 4$	
$|-4|$	4
4	4 ✓

$|x| = 4$	
$|4|$	4
4	4 ✓

Solve each equation.

A $|x| = 4$

$|x| = 4$ *Think: What numbers are 4 units from 0?*



Case 1	**Case 2**
$x = -4$	$x = 4$

Rewrite the equation as two cases.

The solutions are −4 and 4.

Solve each equation.

B $4|x+2| = 24$

$$\frac{4|x+2|}{4} = \frac{24}{4}$$

Since $|x+2|$ is multiplied by 4, divide both sides by 4 to undo the multiplication.

$$|x+2| = 6$$

Think: What numbers are 6 units from 0?

Case 1	Case 2
$x+2 = -6$	$x+2 = 6$
$\underline{-2 \quad -2}$	$\underline{-2 \quad -2}$
$x = -8$	$x = 4$

Rewrite the equation as two cases. Since 2 is added to x, subtract 2 from both sides of the equation.

The solutions are –8 and 4.

CHECK IT OUT! Solve each equation. Check your answer.

1a. $|x| - 3 = 4$ **1b.** $8 = |x - 2.5|$

The table summarizes the steps for solving absolute-value equations.

Know it! Note

Solving an Absolute-Value Equation
1. Use inverse operations to isolate the absolute-value expression.
2. Rewrite the resulting equation as two cases that do not involve absolute values.
3. Solve the equation in each of the two cases.

Not all absolute-value equations have two solutions. If the absolute-value expression equals 0, there is one solution. If an equation states that an absolute value is negative, there are no solutions.

EXAMPLE 2 Special Cases of Absolute-Value Equations

Solve each equation.

A $|x+3| + 4 = 4$

$$|x+3| + 4 = 4$$
$$\underline{\quad -4 \quad -4}$$
$$|x+3| = 0$$

Since 4 is added to $|x+3|$, subtract 4 from both sides to undo the addition.

$$x + 3 = 0$$
$$\underline{-3 \quad -3}$$
$$x = -3$$

There is only one case. Since 3 is added to x, subtract 3 from both sides to undo the addition.

B $5 = |x+2| + 8$

$$5 = |x+2| + 8$$
$$\underline{-8 \qquad -8}$$
$$-3 = |x+2| \text{ ✗}$$

Since 8 is added to $|x+2|$, subtract 8 from both sides to undo the addition.

Absolute value cannot be negative.

This equation has no solution.

Remember!

Absolute value must be nonnegative because it represents a distance.

CHECK IT OUT! Solve each equation.

2a. $2 - |2x - 5| = 7$ **2b.** $-6 + |x - 4| = -6$

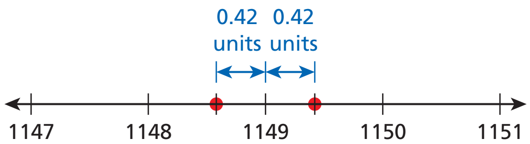
EXAMPLE 3 *Engineering Application*

Sydney Harbour Bridge in Australia is 1149 meters long. Because of changes in temperature, the bridge can expand or contract by as much as 420 millimeters. Write and solve an absolute-value equation to find the minimum and maximum lengths of the bridge.

First convert millimeters to meters.

420 mm = 0.4 2 0 m *Move the decimal point three places to the left.*

The length of the bridge can vary by 0.42 m, so find two numbers that are 0.42 units away from 1149 on a number line.



You can find these numbers by using the absolute-value equation $|x - 1149| = 0.42$. Solve the equation by rewriting it as two cases.

Case 1	**Case 2**
$x - 1149 = -0.42$	$x - 1149 = 0.42$
$+1149 \quad +1149$	$+1149 \quad +1149$
$x = 1148.58$	$x = 1149.42$

Since 1149 is subtracted from x, add 1149 to both sides of each equation.

The minimum length of the bridge is 1148.58 m, and the maximum length is 1149.42 m.

CHECK IT OUT!

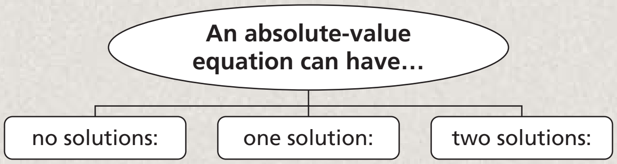
3. Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

THINK AND DISCUSS

1. Explain the steps you would use to solve the equation $\frac{1}{5}|x - 3| = 2$.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an absolute-value equation that has the indicated number of solutions, and then solve.



2-6 Exercises

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Homework Help Online
KEYWORD: MA11 2-6
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Solve each equation.

SEE EXAMPLE 1
p. 112

1. $|x| = 6$ **2.** $9 = |x + 5|$ **3.** $|3x| + 2 = 8$

4. $2|x| = 18$ **5.** $\left|x + \frac{1}{2}\right| = 1$ **6.** $|x - 3| - 6 = 2$

SEE EXAMPLE 2
p. 113

7. $-8 = |x|$ **8.** $|x| = 0$ **9.** $|x + 4| = -7$

10. $7 = |3x + 9| + 7$ **11.** $|2.8 - x| + 1.5 = 1.5$ **12.** $5|x + 7| + 14 = 8$

SEE EXAMPLE 3
p. 114

13. Communication Barry's walkie-talkie has a range of 2 mi. Barry is traveling on a straight highway and is at mile marker 207. Write and solve an absolute-value equation to find the minimum and maximum mile marker from 207 that Barry's walkie-talkie will reach.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–22	1
23–28	2
29	3

Extra Practice
Skills Practice p. S6
Application Practice p. S29

Solve each equation.

14. $|x| = \frac{1}{5}$ **15.** $|2x - 4| = 22$ **16.** $18 = 3|x - 1|$

17. $-2|x| = -4$ **18.** $3|x| - 12 = 18$ **19.** $|x - 42.04| = 23.24$

20. $\left|\frac{2}{3}x - \frac{2}{3}\right| = \frac{2}{3}$ **21.** $|3x + 1| = 13$ **22.** $|-2x + 3| = 5.8$

23. $|4x| + 9 = 9$ **24.** $8 = 7 - |x|$ **25.** $|x| + 6 = 12 - 6$

26. $|x - 3| + 14 = 5$ **27.** $0 = \left|\frac{2}{3} - x\right|$ **28.** $3 + |x - 1| = 3$

29. Space Shuttle The diameter of a valve for the space shuttle must be within 0.001 mm of 5 mm. Write and solve an absolute-value equation to find the boundary values for the acceptable diameters of the valve.

30. The two numbers that are 5 units from 3 on the number line are represented by the equation $|n - 3| = 5$. What are these two numbers? Graph the solutions.

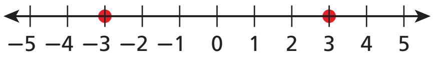
31. Write and solve an absolute-value equation that represents two numbers x that are 2 units from 7 on a number line. Graph the solutions.

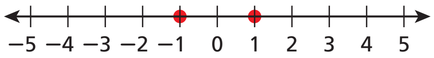
32. Manufacturing A quality control inspector at a bolt factory examines random bolts that come off the assembly line. Any bolt whose diameter differs by more than 0.04 mm from 6.5 mm is sent back. Write and solve an absolute-value equation to find the maximum and minimum diameters of an acceptable bolt.

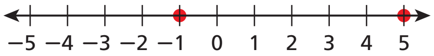
33. Construction A brick company guarantees to fill a contractor's order to within 5% accuracy. A contractor orders 1500 bricks. Write and solve an absolute-value equation to find the maximum and minimum number of bricks guaranteed.

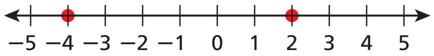
34. Multi-Step A machine prints posters and then trims them to the correct size. The equation $|\ell - 65.1| = 0.2$ gives the maximum and minimum acceptable lengths for the posters in inches. Does a poster with a length of 64.8 inches fall within the acceptable range? Why or why not?

Write an absolute-value equation whose solutions are graphed on the number line.

35. 

36. 

37. 

38. 

Tell whether each statement is sometimes, always, or never true. Explain.

39. An absolute-value equation has two solutions.

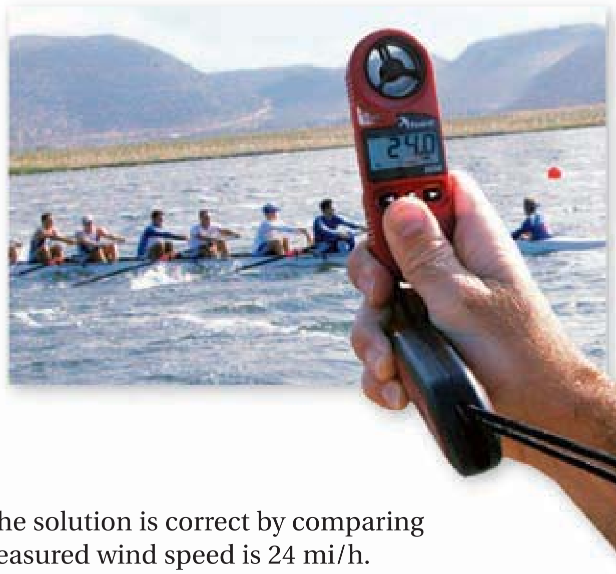
40. The value of $|x + 4|$ is equal to the value of $|x| + 4$.

41. The absolute value of a number is nonnegative.

42. **Temperature** A thermostat is set so that the temperature in a laboratory freezer stays within 2.5 °F of 2 °F. Write and solve an absolute-value equation to find the maximum and minimum temperatures in the freezer.

43. **Recreation** To ensure safety, boaters must be aware of wind conditions while they are on the water. A particular anemometer gives a measurement of wind speed within a certain amount of the true wind speed, as shown in the table.

Measured Wind Speed (mi/h)	True Wind Speed (mi/h)
20	15–25
22	17–27
24	19–29
26	21–31
28	23–33
30	25–35



a. Use the table to write an absolute-value equation for the minimum and maximum possible true wind speeds *t* for the measured wind speed shown on the anemometer.

b. Solve your equation from part **a.** Check that the solution is correct by comparing it to the values given in the table when the measured wind speed is 24 mi/h.

c. Will your equation work for all of the values in the table? Explain.

d. Explain what your equation says about the instrument's measurements.

MULTI-STEP TEST PREP

44. This problem will help prepare you for the Multi-Step Test Prep on page 118.

The water pumps on a wildland fire apparatus can pump at various rates. The center of the acceptable range of pumping rates is 55 gallons per minute.

a. Write an absolute-value expression that gives the distance on the number line of a pump's rate *r* from 55.

b. The smallest and largest pumps have rates that differ by 45 gallons per minute from the rate at the center of the range. Write an absolute-value equation for the rates of these pumps.

c. Find the least and greatest rates that are acceptable for a wildland fire apparatus.

45. **Write About It** Do you agree with the following statement: "To solve an absolute-value equation, you need to solve two equations." Why or why not?

46. **Critical Thinking** Is there a value of a for which the equation $|x - a| = 1$ has exactly one solution? Explain.

TEST PREP

47. Which situation could be modeled by the equation $|x - 65| = 3$?
   Ⓐ Two numbers on the number line are 65 units away from 3.
   Ⓑ The length of a carpet is 3 inches less than 65 inches.
   Ⓒ The maximum and minimum weights of wrestlers on the team are within 3 kg of 65 kg.
   Ⓓ The members of an exercise club for seniors are all between 63 and 67 years old.

48. For which of the following is $n = -3$ a solution?
   Ⓕ $|n - 1| = 2$ Ⓖ $|n + 2| = -1$ Ⓗ $|n - 2| = 1$ Ⓙ $|n + 1| = 2$

49. The minimum and maximum sound levels at a rock concert are 90 decibels and 95 decibels. Which equation models this situation?
   Ⓐ $|x - 90| = 5$ Ⓑ $|x - 92.5| = 2.5$ Ⓒ $|x - 92.5| = 5$ Ⓓ $|x - 95| = 2.5$

CHALLENGE AND EXTEND

50. The perimeter of a rectangle is 100 inches. The length of the rectangle is $|2x - 4|$ inches, and the width is x inches. What are the possible values of x? Explain.

51. Fill in the missing reasons to justify each step in solving the equation $3|2x + 1| = 21$.

Statements	Reasons
1. $3|2x + 1| = 21$	1. Given
2. $|2x + 1| = 7$	2. ____?____
3. $2x + 1 = -7$ or $2x + 1 = 7$	3. Definition of absolute value
4. $2x = -8$ or $2x = 6$	4. ____?____
5. $x = -4$ or $x = 3$	5. ____?____

52. Solve $|x| = |x + 1|$. (*Hint:* Consider two cases: $x \geq 0$ and $x < 0$.)

SPIRAL REVIEW

53. Sabrina is building a bookcase for her room. She has a piece of wood that is $4\frac{3}{8}$ ft long and estimates that she wants to use about $\frac{2}{5}$ of it. How much wood does she want to use? *(Lesson 1-3)*

Solve each equation. Check your answer. *(Lesson 2-1)*

54. $5 = p - 4.5$ 55. $-2 = y + 6\frac{1}{2}$ 56. $-12 + q = 3$ 57. $y - 4.3 = -5.7$

Solve for the indicated variable. *(Lesson 2-5)*

58. $m + 5n = 7$ for m 59. $S = T + R$ for T 60. $2y + 3x = 1$ for y

61. $\frac{3 + w}{z} = x$ for w 62. $c + d = \frac{5}{e}$ for e 63. $6M - N = S$ for N

MULTI-STEP TEST PREP

Equations and Formulas

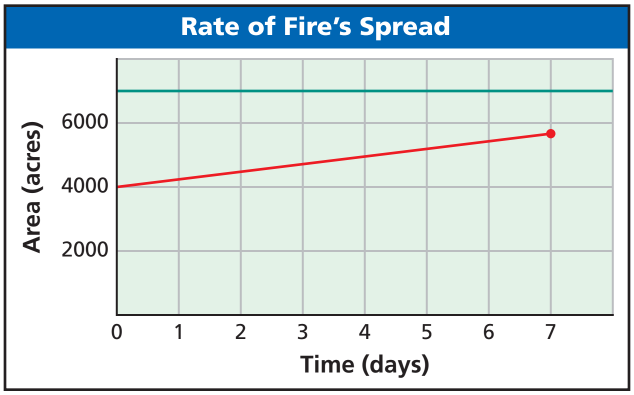
All Fired Up A large forest fire in the western United States burns for 14 days, spreading to cover approximately 3850 acres. Firefighters do their best to contain the fire, but hot temperatures and high winds may prompt them to request additional support.

1. The fire spreads at an average rate of how many acres per day?

2. Officials estimate that the fire will spread to cover 9075 acres before it is contained. At this rate, how many more days will it take for the fire to cover an area of 9075 acres? Answer this question using at least two different methods.

3. Additional help arrives, and the firefighters contain the fire in 7 more days. In total, how many acres does the fire cover before it is contained?

4. If the fire had spread to cover an area of 7000 acres, it would have reached Bowman Valley. Explain how the graph shows that firefighters stopped the spread of the fire before it reached Bowman Valley.



5. The total cost of fighting the fire for 21 days was approximately $1,440,000. What was the approximate cost per acre of fighting the fire?

READY TO GO ON?

Quiz for Lessons 2-1 Through 2-6

2-1 Solving Equations by Adding or Subtracting

Solve each equation.

1. $x - 32 = -18$ **2.** $1.1 = m - 0.9$ **3.** $j + 4 = -17$ **4.** $\frac{9}{8} = g + \frac{1}{2}$

5. When she first purchased it, Soledad's computer had 400 GB of hard drive space. After six months, there were only 313 GB available. Write and solve an equation to find the amount of hard drive space that Soledad used in the first six months.

2-2 Solving Equations by Multiplying or Dividing

Solve each equation.

6. $\frac{h}{3} = -12$ **7.** $-2.8 = \frac{w}{-3}$ **8.** $42 = 3c$ **9.** $-0.1b = 3.7$

10. A fund-raiser raised \$2400, which was $\frac{3}{5}$ of the goal. Write and solve an equation to find the amount of the goal.

2-3 Solving Two-Step and Multi-Step Equations

Solve each equation.

11. $2r + 20 = 200$ **12.** $\frac{3}{5}k + 5 = 7$ **13.** $5n + 6 - 3n = -12$ **14.** $4(x - 7) = 2$

15. A taxicab company charges \$2.10 plus \$0.80 per mile. Carmen paid a fare of \$11.70. Write and solve an equation to find the number of miles she traveled.

2-4 Solving Equations with Variables on Both Sides

Solve each equation.

16. $4x - 3 = 2x + 5$ **17.** $3(2x - 5) = 2(3x - 2)$

18. $2(2t - 3) = 6(t + 2)$ **19.** $7(x + 5) = -7(x + 5)$

20. On the first day of the year, Diego had \$700 in his savings account and started spending \$35 a week. His brother Juan had \$450 and started saving \$15 a week. After how many weeks will the brothers have the same amount? What will that amount be?

2-5 Solving for a Variable

21. Solve $2x + 3y = 12$ for x. **22.** Solve $\frac{x}{r} = v$ for x.

23. Solve $5j + s = t - 2$ for t. **24.** Solve $h + p = 3(k - 8)$ for k.

2-6 Solving Absolute-Value Equations

Solve each equation.

25. $|r| = 7$ **26.** $|h + 4| = 11$

27. $|2x + 4| = 0$ **28.** $16 = 7|p + 3| + 30$