

2-5

Solving for a Variable



Objectives

Solve a formula for a given variable.

Solve an equation in two or more variables for one of the variables.

Vocabulary

formula

literal equation

Who uses this?

Athletes can “rearrange” the distance formula to calculate their average speed.

Many wheelchair athletes compete in marathons, which cover about 26.2 miles. Using the time t it took to complete the race, the distance d , and the formula $d = rt$, racers can find their average speed r .

A **formula** is an equation that states a rule for a relationship among quantities.

In the formula $d = rt$, d is isolated. You can “rearrange” a formula to isolate any variable by using inverse operations. This is called *solving for a variable*.



Solving for a Variable

Step 1 Locate the variable you are asked to solve for in the equation.

Step 2 Identify the operations on this variable and the order in which they are applied.

Step 3 Use inverse operations to undo operations and isolate the variable.

EXAMPLE 1 Sports Application

In 2004, Ernst Van Dyk won the wheelchair race of the Boston Marathon with a time of about 1.3 hours. The race was about 26.2 miles. What was his average speed? Use the formula $d = rt$ and round your answer to the nearest tenth.

The question asks for speed, so first solve the formula $d = rt$ for r .

$$d = rt$$

Locate r in the equation.

$$\frac{d}{t} = \frac{rt}{t}$$

Since r is multiplied by t , divide both sides by t to undo the multiplication.

$$\frac{d}{t} = r, \text{ or } r = \frac{d}{t}$$

Now use this formula and the information given in the problem.

$$r = \frac{d}{t} \approx \frac{26.2}{1.3} \\ \approx 20.2$$

Van Dyk's average speed was about 20.2 miles per hour.

Helpful Hint

A number divided by itself equals 1. For $t \neq 0$, $\frac{t}{t} = 1$.



1. Solve the formula $d = rt$ for t . Find the time in hours that it would take Van Dyk to travel 26.2 miles if his average speed was 18 miles per hour. Round to the nearest hundredth.

EXAMPLE 2 Solving Formulas for a Variable

- A** The formula for a Fahrenheit temperature in terms of degrees Celsius is $F = \frac{9}{5}C + 32$. Solve for C .

$$F = \frac{9}{5}C + 32 \quad \text{Locate } C \text{ in the equation.}$$

$$\begin{array}{r} -32 \\ \hline F - 32 = \frac{9}{5}C \end{array} \quad \begin{array}{l} \text{Since } 32 \text{ is added to } \frac{9}{5}C, \text{ subtract } 32 \text{ from both} \\ \text{sides to undo the addition.} \end{array}$$

$$\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\frac{9}{5}C \quad \begin{array}{l} \text{Since } C \text{ is multiplied by } \frac{9}{5}, \text{ divide both} \\ \text{sides by } \frac{9}{5} \text{ (multiply by } \frac{5}{9}\text{) to undo the} \\ \text{multiplication.} \end{array}$$

$$\frac{5}{9}(F - 32) = C$$

- B** The formula for a person's typing speed is $s = \frac{w - 10e}{m}$, where s is speed in words per minute, w is number of words typed, e is number of errors, and m is number of minutes typing. Solve for w .

$$s = \frac{w - 10e}{m} \quad \text{Locate } w \text{ in the equation.}$$

$$m(s) = m\left(\frac{w - 10e}{m}\right) \quad \begin{array}{l} \text{Since } w - 10e \text{ is divided by } m, \text{ multiply both} \\ \text{sides by } m \text{ to undo the division.} \end{array}$$

$$ms = w - 10e$$

$$\begin{array}{r} +10e \\ \hline ms + 10e = w \end{array} \quad \begin{array}{l} \text{Since } 10e \text{ is subtracted from } w, \text{ add } 10e \text{ to} \\ \text{both sides to undo the subtraction.} \end{array}$$



2. The formula for an object's final velocity f is $f = i - gt$, where i is the object's initial velocity, g is acceleration due to gravity, and t is time. Solve for i .

A formula is a type of *literal equation*. A **literal equation** is an equation with two or more variables. To solve for one of the variables, use inverse operations.

EXAMPLE 3 Solving Literal Equations for a Variable

- A** Solve $m - n = 5$ for m .

$$m - n = 5 \quad \text{Locate } m \text{ in the equation.}$$

$$\begin{array}{r} +n \\ \hline m = 5 + n \end{array} \quad \begin{array}{l} \text{Since } n \text{ is subtracted from } m, \text{ add } n \text{ to both sides to} \\ \text{undo the subtraction.} \end{array}$$

- B** Solve $\frac{m}{k} = x$ for k .

$$\frac{m}{k} = x \quad \text{Locate } k \text{ in the equation.}$$

$$k\left(\frac{m}{k}\right) = kx \quad \begin{array}{l} \text{Since } k \text{ appears in the denominator, multiply both} \\ \text{sides by } k. \end{array}$$

$$m = kx$$

$$\begin{array}{r} \frac{m}{x} = \frac{kx}{x} \\ \hline \frac{m}{x} = k \end{array} \quad \begin{array}{l} \text{Since } k \text{ is multiplied by } x, \text{ divide both sides by } x \text{ to} \\ \text{undo the multiplication.} \end{array}$$



- 3a. Solve $5 - b = 2t$ for t .

3b. Solve $D = \frac{m}{V}$ for V .

Remember!

Dividing by a fraction is the same as multiplying by the reciprocal.

THINK AND DISCUSS

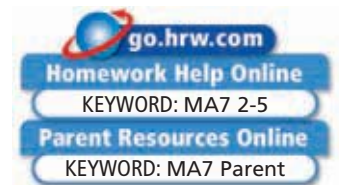
- Describe a situation in which a formula could be used more easily if it were “rearranged.” Include the formula in your description.
- Explain how to solve $P = 2\ell + 2w$ for w .
- GET ORGANIZED** Copy and complete the graphic organizer. Write a formula that is used in each subject. Then solve the formula for each of its variables.

Common Formulas	
Subject	Formula
Geometry	
Physical science	
Earth science	



2-5

Exercises



GUIDED PRACTICE

SEE EXAMPLE 1
p. 107

- Vocabulary** Explain why a *formula* is a type of *literal equation*.
- Construction** The formula $a = 46c$ gives the floor area a in square meters that can be wired using c circuits.
 - Solve $a = 46c$ for c .
 - If a room is 322 square meters, how many circuits are required to wire this room?

SEE EXAMPLE 2
p. 108

- The formula for the volume of a rectangular prism with length ℓ , width w , and height h is $V = \ell wh$. Solve this formula for w .

SEE EXAMPLE 3
p. 108

- Solve $st + 3t = 6$ for s .
- Solve $m - 4n = 8$ for m .
- Solve $\frac{f+4}{g} = 6$ for f .
- Solve $b + c = \frac{10}{a}$ for a .

PRACTICE AND PROBLEM SOLVING

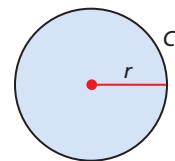
Independent Practice

For Exercises	See Example
8	1
9	2
10–13	3

Extra Practice

Skills Practice p. S6
Application Practice p. S29

- Geometry** The formula $C = 2\pi r$ relates the circumference C of a circle to its radius r . (Recall that π is the constant ratio of circumference to diameter.)
 - Solve $C = 2\pi r$ for r .
 - If a circle's circumference is 15 inches, what is its radius? Leave the symbol π in your answer.
- Finance** The formula $A = P + I$ shows that the total amount of money A received from an investment equals the principal P (the original amount of money invested) plus the interest I . Solve this formula for I .



C is the distance around the circle.

r is the distance from the center of the circle to any point on the circle.

- Solve $-2 = 4r + s$ for s .
- Solve $xy - 5 = k$ for x .
- Solve $\frac{m}{n} = p - 6$ for n .
- Solve $\frac{x-2}{y} = z$ for y .

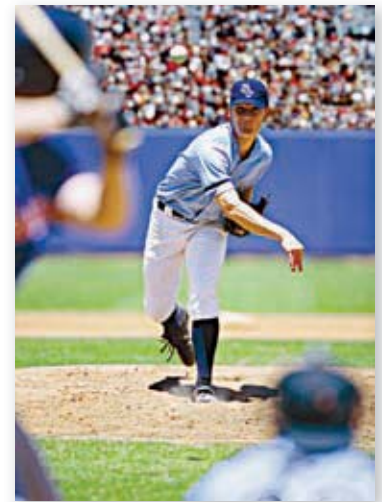
Solve for the indicated variable.


14. $S = 180n - 360$ for n 15. $\frac{x}{5} - g = a$ for x 16. $A = \frac{1}{2}bh$ for b
 17. $y = mx + b$ for x 18. $a = 3n + 1$ for n 19. $PV = nRT$ for T
 20. $T + M = R$ for T 21. $M = T - R$ for T 22. $PV = nRT$ for R
 23. $2a + 2b = c$ for b 24. $5p + 9c = p$ for c 25. $ax + r = 7$ for r
 26. $3x + 7y = 2$ for y 27. $4y + 3x = 5$ for x 28. $y = 3x + 3b$ for b

29. **Estimation** The table shows the flying time and distance traveled for five flights on a certain airplane.
- Use the data in the table to write a rule that *estimates* the relationship between flying time t and distance traveled d .
 - Use your rule from part **a** to estimate the time that it takes the airplane to fly 1300 miles.
 - Solve your rule for d .
 - Use your rule from part **c** to estimate the distance the airplane can fly in 8 hours.

Flying Times		
Flight	Time (h)	Distance (mi)
A	2	1018
B	3	1485
C	4	2103
D	5	2516
E	6	2886

30. **Sports** To find a baseball pitcher's earned run average (ERA), you can use the formula $Ei = 9r$, where E represents ERA, i represents number of innings pitched, and r represents number of earned runs allowed. Solve the equation for E . What is a pitcher's ERA if he allows 5 earned runs in 18 innings pitched?
31. **Meteorology** For altitudes up to 36,000 feet, the relationship between temperature and altitude can be described by the formula $t = -0.0035a + g$, where t is the temperature in degrees Fahrenheit, a is the altitude in feet, and g is the ground temperature in degrees Fahrenheit. Solve this formula for a .



-  32. **Write About It** In your own words, explain how to solve a literal equation for one of the variables.
33. **Critical Thinking** How is solving $a - ab = c$ for a different from the problems in this lesson? How might you solve this equation for a ?

**MULTI-STEP
TEST PREP**



34. This problem will prepare you for the Multi-Step Test Prep on page 118.
- Suppose firefighters can extinguish a wildfire at a rate of 60 acres per day. Use this information to complete the table.
 - Use the last row in the table to write an equation for acres A extinguished in terms of the number of days d .
 - Graph the points in the table with *Days* on the horizontal axis and *Acres* on the vertical axis. Describe the graph.

Days	Acres
1	60
2	■
3	180
4	■
5	■
d	■

35. Which equation is the result of solving $9 + 3x = 2y$ for x ?

- (A) $\frac{9 + 3y}{2} = x$ (B) $\frac{2}{3}y - 9 = x$ (C) $x = \frac{2}{3}y - 3$ (D) $x = 2y - 3$

36. Which of the following is a correct method for solving $2a - 5b = 10$ for b ?

- (F) Add $5b$ to both sides, then divide both sides by 2.
 (G) Subtract $5b$ from both sides, then divide both sides by 2.
 (H) Divide both sides by 5, then add $2a$ to both sides.
 (J) Subtract $2a$ from both sides, then divide both sides by -5 .

37. The formula for the volume of a rectangular prism is $V = \ell wh$. Anna wants to make a cardboard box with a length of 7 inches, a width of 5 inches, and a volume of 210 cubic inches. Which variable does Anna need to solve for in order to build her box?

- (A) V (B) ℓ (C) w (D) h

CHALLENGE AND EXTEND

Solve for the indicated variable.

38. $3.3x + r = 23.1$ for x 39. $\frac{2}{5}a - \frac{3}{4}b = c$ for a 40. $\frac{3}{5}x + 1.4y = \frac{2}{5}$ for y

41. $t = \frac{d}{500} + \frac{1}{2}$ for d 42. $s = \frac{1}{2}gt^2$ for g 43. $v^2 = u^2 + 2as$ for s

44. Solve $y = mx + 6$ for m . What can you say about y if $m = 0$?

45. **Entertainment** The formula $S = \frac{h \cdot w \cdot f \cdot t}{35,000}$ gives the approximate size in kilobytes (Kb) of a compressed video. The variables h and w represent the height and width of the frame measured in pixels, f is the number of frames per second (fps) the video plays, and t is the time the video plays in seconds. Estimate the time a movie trailer will play if it has a frame height of 320 pixels, has a frame width of 144 pixels, plays at 15 fps, and has a size of 2370 Kb.



SPIRAL REVIEW

46. Jill spent $\frac{1}{4}$ of the money she made baby-sitting. She made \$40 baby-sitting. How much did she spend? (*Previous course*)
 47. In one class, $\frac{3}{5}$ of the students are boys. There are 30 students in the class. How many are girls? (*Previous course*)

Evaluate each expression for the given value of x . (*Lesson 1-6*)

48. $3 + 2 \cdot x + 4$ for $x = 3$ 49. $24 \div 4 - x$ for $x = 12$ 50. $43 - 62 + x$ for $x = 15$

Solve each equation. Check your answer. (*Lesson 2-1*)

51. $18 = -2 + w$ 52. $2 = -3 + c$ 53. $-8 + k = 4$ 54. $-15 + a = -27$