2-3 Solving Two-Step and Multi-Step Equations

Objective
Solve equations in one variable that contain more than one operation.

Why learn this?
Equations containing more than one operation can model real-world situations, such as the cost of a music club membership.

Alex belongs to a music club. In this club, students can buy a student discount card for $19.95. This card allows them to buy CDs for $3.95 each. After one year, Alex has spent $63.40.

To find the number of CDs c that Alex bought, you can solve an equation.

Cost of discount card

Cost per CD → $3.95c + 19.95 = 63.40$ ← Total cost

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

$$3.95c + 19.95 = 63.40$$

Operations in the Equation		To Solve
❶ First c is **multiplied** by 3.95.	Work Backward →	❶ **Subtract** 19.95 from both sides of the equation.
❷ Then 19.95 is **added**.		❷ Then **divide** both sides by 3.95.

EXAMPLE 1 Solving Two-Step Equations

Solve $10 = 6 - 2x$. Check your answer.

$10 = 6 - 2x$ — *First x is multiplied by –2. Then 6 is added.*

$-6 \quad -6$ — *Work backward: Subtract 6 from both sides.*

$4 = -2x$

$\frac{4}{-2} = \frac{-2x}{-2}$ — *Since x is multiplied by –2, divide both sides by –2 to undo the multiplication.*

$-2 = 1x$

$-2 = x$

Check

$10 = 6 - 2x$	
10	$6 - 2(-2)$
10	$6 - (-4)$
10	10 ✓

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $-4 + 7x = 3$ **1b.** $1.5 = 1.2y - 5.7$ **1c.** $\frac{n}{7} + 2 = 2$

EXAMPLE 2 Solving Two-Step Equations That Contain Fractions

Solve $\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$.

Method 1 Use fraction operations.

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$ — *Since $\frac{1}{5}$ is subtracted from $\frac{q}{15}$, add $\frac{1}{5}$ to both sides to undo the subtraction.*

$+\frac{1}{5} \quad +\frac{1}{5}$

$\frac{q}{15} = \frac{4}{5}$ — *Since q is divided by 15, multiply both sides by 15 to undo the division.*

$15\left(\frac{q}{15}\right) = 15\left(\frac{4}{5}\right)$

$q = \frac{15 \cdot 4}{5}$ — *Simplify.*

$q = \frac{60}{5}$

$q = 12$

Method 2 Multiply by the least common denominator (LCD) to clear the fractions.

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$

$15\left(\frac{q}{15} - \frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$ — *Multiply both sides by 15, the LCD of the fractions.*

$15\left(\frac{q}{15}\right) - 15\left(\frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$ — *Distribute 15 on the left side.*

$q - 3 = 9$ — *Simplify.*

$+3 \quad +3$

$q = 12$ — *Since 3 is subtracted from q, add 3 to both sides to undo the subtraction.*

Solve each equation. Check your answer.

2a. $\frac{2x}{5} - \frac{1}{2} = 5$ **2b.** $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$ **2c.** $\frac{1}{5}n - \frac{1}{3} = \frac{8}{3}$

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

EXAMPLE 3 Simplifying Before Solving Equations

Solve each equation.

A $6x + 3 - 8x = 13$

$6x + 3 - 8x = 13$

$6x - 8x + 3 = 13$ — *Use the Commutative Property of Addition.*

$-2x + 3 = 13$ — *Combine like terms.*

$-3 \quad -3$ — *Since 3 is added to −2x, subtract 3 from both sides to undo the addition.*

$-2x = 10$

$\frac{-2x}{-2} = \frac{10}{-2}$ — *Since x is multiplied by −2, divide both sides by −2 to undo the multiplication.*

$x = -5$

Helpful Hint

You can think of an opposite sign as a coefficient of −1. $-(x+2) = -1(x+2)$ and $-x = -1x$.

Solve each equation.

B $9 = 6 - (x + 2)$

$9 = 6 + (-1)(x + 2)$ *Write subtraction as addition of the opposite.*

$9 = 6 + (-1)(x) + (-1)(2)$ *Distribute −1 on the right side.*

$9 = 6 - x - 2$ *Simplify.*

$9 = 6 - 2 - x$ *Use the Commutative Property of Addition.*

$9 = 4 - x$ *Combine like terms.*

$-4 \quad -4$ *Since 4 is added to −x, subtract 4 from both sides to undo the addition.*

$5 = -x$

$\frac{5}{-1} = \frac{-x}{-1}$ *Since x is multiplied by −1, divide both sides by −1 to undo the multiplication.*

$-5 = x$

CHECK IT OUT! **Solve each equation. Check your answer.**

3a. $2a + 3 - 8a = 8$

3b. $-2(3 - d) = 4$

3c. $4(x - 2) + 2x = 40$

EXAMPLE 4 *Problem-Solving Application*

Alex belongs to a music club. In this club, students can buy a student discount card for $19.95. This card allows them to buy CDs for $3.95 each. After one year, Alex has spent $63.40. Write and solve an equation to find how many CDs Alex bought during the year.

PROBLEM SOLVING

1 Understand the Problem

The **answer** will be the number of CDs that Alex bought during the year.

List the important information:

- Alex paid $19.95 for a student discount card.
- Alex pays $3.95 for each CD purchased.
- After one year, Alex has spent $63.40.

2 Make a Plan

Let c represent the number of CDs that Alex purchased. That means Alex has spent $3.95c. However, Alex must also add the amount spent on the card. Write an equation to represent this situation.

total cost	=	cost of compact discs	+	cost of discount card
63.40	=	3.95c	+	19.95

3 Solve

$63.40 = 3.95c + 19.95$

$-19.95 \quad -19.95$

$43.45 = 3.95c$

Since 19.95 is added to 3.95c, subtract 19.95 from both sides to undo the addition.

$\frac{43.45}{3.95} = \frac{3.95c}{3.95}$

$11 = c$

Since c is multiplied by 3.95, divide both sides by 3.95 to undo the multiplication.

Alex bought 11 CDs during the year.

4 Look Back

Check that the answer is reasonable. The cost per CD is about \$4, so if Alex bought 11 CDs, this amount is about $11(4) = \$44$.

Add the cost of the discount card, which is about \$20: $44 + 20 = 64$. So the total cost was about \$64, which is close to the amount given in the problem, \$63.40.

CHECK IT OUT!

4. Sara paid \$15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was \$735.95. How much was the monthly fee?

EXAMPLE 5 Solving Equations to Find an Indicated Value

If $3a + 12 = 30$, find the value of $a + 4$.

Step 1 Find the value of a.

$3a + 12 = 30$

$-12 \quad -12$

$3a = 18$

Since 12 is added to 3a, subtract 12 from both sides to undo the addition.

$\frac{3a}{3} = \frac{18}{3}$

$a = 6$

Since a is multiplied by 3, divide both sides by 3 to undo the multiplication.

Step 2 Find the value of $a + 4$.

$a + 4$

$6 + 4$

10

To find the value of a + 4, substitute 6 for a.

Simplify.

CHECK IT OUT!

5. If $2x + 4 = -24$, find the value of $3x$.

THINK AND DISCUSS

1. Explain the steps you would follow to solve $2x + 1 = 7$. How is this procedure different from the one you would follow to solve $2x - 1 = 7$?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and solve a multi-step equation. Use addition, subtraction, multiplication, and division at least one time each.

Solving Multi-Step Equations	

2-3 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 2-3
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Solve each equation. Check your answer.

SEE EXAMPLE 1 p. 92

1. $4a + 3 = 11$
2. $8 = 3r - 1$
3. $42 = -2d + 6$
4. $x + 0.3 = 3.3$
5. $15y + 31 = 61$
6. $9 - c = -13$

SEE EXAMPLE 2 p. 93

7. $\frac{x}{6} + 4 = 15$
8. $\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}$
9. $\frac{2}{7}j - \frac{1}{7} = \frac{3}{14}$
10. $15 = \frac{a}{3} - 2$
11. $4 - \frac{m}{2} = 10$
12. $\frac{x}{8} - \frac{1}{2} = 6$

SEE EXAMPLE 3 p. 93

13. $28 = 8x + 12 - 7x$
14. $2y - 7 + 5y = 0$
15. $2.4 = 3(m + 4)$
16. $3(x - 4) = 48$
17. $4t + 7 - t = 19$
18. $5(1 - 2w) + 8w = 15$

SEE EXAMPLE 4 p. 94

19. **Transportation** Paul bought a student discount card for the bus. The card cost $7 and allows him to buy daily bus passes for $1.50. After one month, Paul spent $29.50. How many daily bus passes did Paul buy?

SEE EXAMPLE 5 p. 95

20. If $3x - 13 = 8$, find the value of $x - 4$.
21. If $3(x + 1) = 7$, find the value of $3x$.
22. If $-3(y - 1) = 9$, find the value of $\frac{1}{2}y$.
23. If $4 - 7x = 39$, find the value of $x + 1$.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
24–29	1
30–35	2
36–41	3
42	4
43–46	5

Extra Practice

Skills Practice p. S6
Application Practice p. S29

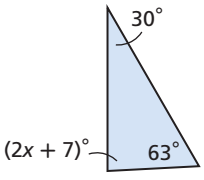
Solve each equation. Check your answer.

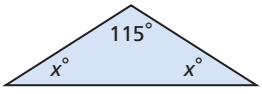
24. $5 = 2g + 1$
25. $6h - 7 = 17$
26. $0.6v + 2.1 = 4.5$
27. $3x + 3 = 18$
28. $0.6g + 11 = 5$
29. $32 = 5 - 3t$
30. $2d + \frac{1}{5} = \frac{3}{5}$
31. $1 = 2x + \frac{1}{2}$
32. $\frac{z}{2} + 1 = \frac{3}{2}$
33. $\frac{2}{3} = \frac{4j}{6}$
34. $\frac{3}{4} = \frac{3}{8}x - \frac{3}{2}$
35. $\frac{1}{5} - \frac{x}{5} = -\frac{2}{5}$
36. $6 = -2(7 - c)$
37. $5(h - 4) = 8$
38. $-3x - 8 + 4x = 17$
39. $4x + 6x = 30$
40. $2(x + 3) = 10$
41. $17 = 3(p - 5) + 8$

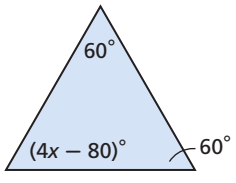
42. **Consumer Economics** Jennifer is saving money to buy a bike. The bike costs $245. She has $125 saved, and each week she adds $15 to her savings. How long will it take her to save enough money to buy the bike?

43. If $2x + 13 = 17$, find the value of $3x + 1$.
44. If $-(x - 1) = 5$, find the value of $-4x$.
45. If $5(y + 10) = 40$, find the value of $\frac{1}{4}y$.
46. If $9 - 6x = 45$, find the value of $x - 4$.

Geometry Write and solve an equation to find the value of x for each triangle. (*Hint:* The sum of the angle measures in any triangle is 180°.)

47.


48.


49.


Write an equation to represent each relationship. Solve each equation.

50. Seven less than twice a number equals 19.

51. Eight decreased by 3 times a number equals 2.

52. The sum of two times a number and 5 is 11.

53. **History** In 1963, Dr. Martin Luther King Jr. began his famous "I have a dream" speech with the words "Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation." The proclamation was signed by President Abraham Lincoln in 1863.

a. Using the dates given, write and solve an equation that can be used to find the number of years in a score.

b. How many score would represent 60?

LINK

History

Martin Luther King Jr. entered college at age 15. During his life he earned 3 degrees and was awarded 20 honorary degrees.

Source: lib.lsu.edu

Solve each equation. Check your answer.

54. $3t + 44 = 50$

55. $3(x - 2) = 18$

56. $15 = \frac{c}{3} - 2$

57. $2x + 6.5 = 15.5$

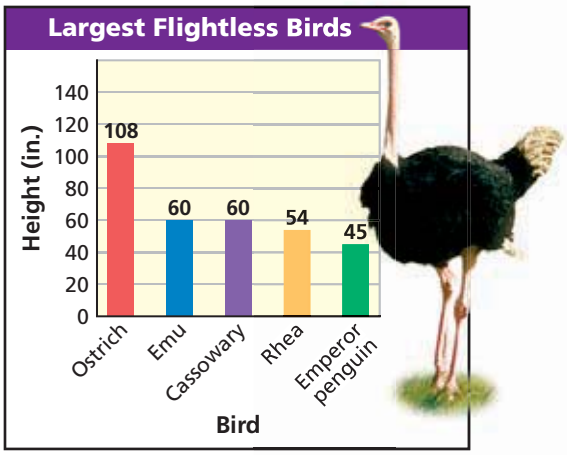
58. $3.9w - 17.9 = -2.3$

59. $17 = x - 3(x + 1)$

60. $5x + 9 = 39$

61. $15 + 5.5m = 70$

Biology **Use the graph for Exercises 62 and 63.**



Source: The Top Ten of Everything

62. The height of an ostrich is 20 inches more than 4 times the height of a kiwi. Write and solve an equation to find the height of a kiwi. Show that your answer is reasonable.

63. Five times the height of a kakapo minus 70 equals the height of an emu. Write and solve an equation to find the height of a kakapo. Show that your answer is reasonable.

64. The sum of two consecutive whole numbers is 57. What are the two numbers? (*Hint:* Let n represent the first number. Then $n + 1$ is the next consecutive whole number.)

65. Stan's, Mark's, and Wayne's ages are consecutive whole numbers. Stan is the youngest, and Wayne is the oldest. The sum of their ages is 111. Find their ages.

66. The sum of two consecutive even whole numbers is 206. What are the two numbers? (*Hint:* Let n represent the first number. What expression can you use to represent the second number?)

MULTI-STEP TEST PREP

67. This problem will prepare you for the Multi-Step Test Prep on page 118.

a. The cost of fighting a certain forest fire is \$225 per acre. Complete the table.

b. Write an equation for the relationship between the cost c of fighting the fire and the number of acres n.

Cost of Fighting Fire	
Acres	Cost (\$)
100	22,500
200	
500	
1000	
1500	
n	

68. Critical Thinking The equation $2(m - 8) + 3 = 17$ has more than one solution method. Give at least two different "first steps" to solve this equation.

69. Write About It Write a series of steps that you can use to solve any multi-step equation.

TEST PREP

70. Lin sold 4 more shirts than Greg. Fran sold 3 times as many shirts as Lin. In total, the three sold 51 shirts. Which represents the number of shirts Greg sold?

Ⓐ $3g = 51$ Ⓑ $3 + g = 51$ Ⓒ $8 + 5g = 51$ Ⓓ $16 + 5g = 51$

71. If $\frac{4m - 3}{7} = 3$, what is the value of $7m - 5$?

Ⓕ 6 Ⓖ 10.5 Ⓗ 37 Ⓙ 68.5

72. The equation $c = 48 + 0.06m$ represents the cost c of renting a car and driving m miles. Which statement best describes this cost?

Ⓐ The cost is a flat rate of \$0.06 per mile.

Ⓑ The cost is \$0.48 for the first mile and \$0.06 for each additional mile.

Ⓒ The cost is a \$48 fee plus \$0.06 per mile.

Ⓓ The cost is a \$6 fee plus \$0.48 per mile.

73. Gridded Response A telemarketer earns \$150 a week plus \$2 for each call that results in a sale. Last week she earned a total of \$204. How many of her calls resulted in sales?

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

74. $\frac{9}{2}x + 18 + 3x = \frac{11}{2}$

75. $\frac{15}{4}x - 15 = \frac{33}{4}$

76. $(x + 6) - (2x + 7) - 3x = -9$

77. $(4x + 2) - (12x + 8) + 2(5x - 3) = 6 + 11$

78. Find a value for b so that the solution of $4x + 3b = -1$ is $x = 2$.

79. Find a value for b so that the solution of $2x - 3b = 0$ is $x = -9$.

80. Business The formula $p = nc - e$ gives the profit p when a number of items n are each sold at a cost c and expenses e are subtracted.

a. If $p = 2500$, $n = 2000$, and $e = 800$, what is the value of c?

b. If $p = 2500$, $n = 1000$, and $e = 800$, what is the value of c?

c. What if...? If n is divided in half while p and e remain the same, what is the effect on c?

SPIRAL REVIEW

Write all classifications that apply to each real number. *(Lesson 1-5)*

81. $\sqrt{3}$ **82.** -58 **83.** $2\frac{1}{3}$ **84.** 0.17

Write each product using the Distributive Property. Then simplify. *(Lesson 1-7)*

85. $8(61)$ **86.** $9(28)$ **87.** $11(28)$ **88.** $13(21)$

Solve each equation. Check your answer. *(Lesson 2-1)*

89. $17 = k + 4$ **90.** $x - 18 = 3$ **91.** $a + 6 = -12$ **92.** $-7 = q - 7$

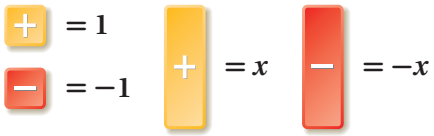
2-4 Algebra LAB

Model Equations with Variables on Both Sides

Algebra tile models can help you understand how to solve equations with variables on both sides.

Use with Lesson 2-4

KEY

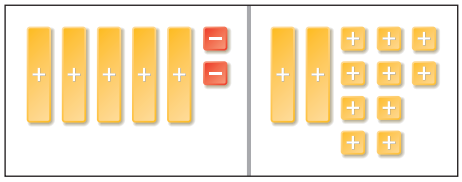
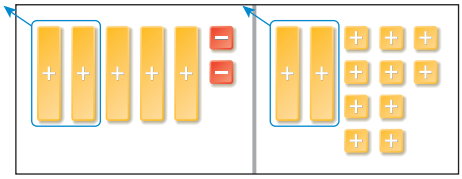
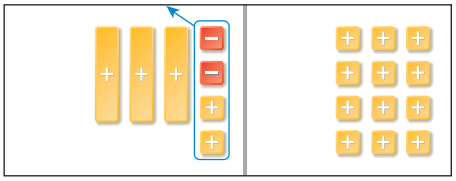
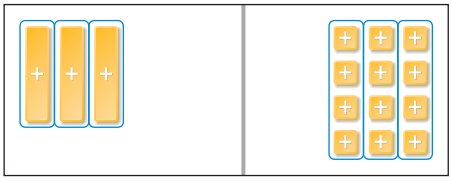


REMEMBER

go.hrw.com
Lab Resources Online
KEYWORD: MA11 LAB2

Activity

Use algebra tiles to model and solve $5x - 2 = 2x + 10$.

MODEL		ALGEBRA
	*Model $5x - 2$ on the left side of the mat and $2x + 10$ on the right side.* *Remember that $5x - 2$ is the same as $5x + (-2)$.*	$5x - 2 = 2x + 10$
	*Remove 2 x-tiles from both sides. This represents subtracting $2x$ from both sides of the equation.*	$5x - 2 - 2x = 2x - 2x + 10$ $3x - 2 = 10$
	*Place 2 yellow tiles on both sides. This represents adding 2 to both sides of the equation.* *Remove zero pairs.*	$3x - 2 + 2 = 10 + 2$ $3x = 12$
	*Separate each side into 3 equal groups.* *Each group is $\frac{1}{3}$ of the side.* *One x-tile is equivalent to 4 yellow tiles.*	$\frac{1}{3}(3x) = \frac{1}{3}(12)$ $x = 4$

Try This

Use algebra tiles to model and solve each equation.

1. $3x + 2 = 2x + 5$ **2.** $5x + 12 = 2x + 3$ **3.** $9x - 5 = 6x + 13$ **4.** $x = -2x + 9$