

2-2

Solving Equations by Multiplying or Dividing

Objective

Solve one-step equations in one variable by using multiplication or division.

Who uses this?

Pilots can make quick calculations by solving one-step equations. (See Example 4.)



Inverse Operations

Multiply by x . \leftrightarrow Divide by x .

Solving an equation that contains multiplication or division is similar to solving an equation that contains addition or subtraction. Use inverse operations to undo the operations on the variable.

Remember that an equation is like a balanced scale. To keep the balance, whatever you do on one side of the equation, you must also do on the other side.

EXAMPLE 1 Solving Equations by Using Multiplication

Solve each equation. Check your answer.

$$\begin{aligned} \text{A} \quad -4 &= \frac{k}{-5} \\ (-5)(-4) &= (-5)\left(\frac{k}{-5}\right) \\ 20 &= k \end{aligned}$$

Since k is divided by -5 , multiply both sides by -5 to undo the division.

$$\begin{array}{r} \text{Check} \quad -4 = \frac{k}{-5} \\ -4 \quad \left| \quad \frac{20}{-5} \\ -4 \quad \left| \quad -4 \quad \checkmark \end{array}$$

To check your solution, substitute 20 for k in the original equation.

$$\begin{aligned} \text{B} \quad \frac{m}{3} &= 1.5 \\ (3)\left(\frac{m}{3}\right) &= (3)(1.5) \\ m &= 4.5 \end{aligned}$$

Since m is divided by 3, multiply both sides by 3 to undo the division.

$$\begin{array}{r} \text{Check} \quad \frac{m}{3} = 1.5 \\ \frac{4.5}{3} \quad \left| \quad 1.5 \\ 1.5 \quad \left| \quad 1.5 \quad \checkmark \end{array}$$

To check your solution, substitute 1.5 for m in the original equation.



Solve each equation. Check your answer.

1a. $\frac{p}{5} = 10$

1b. $-13 = \frac{y}{3}$

1c. $\frac{c}{8} = 7$

EXAMPLE 2 Solving Equations by Using Division

Solve each equation. Check your answers.

A $7x = 56$

$$\frac{7x}{7} = \frac{56}{7}$$

$$x = 8$$

Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.

Check $7x = 56$

$$\begin{array}{r|l} 7(8) & 56 \\ \hline 56 & 56 \checkmark \end{array}$$

To check your solution, substitute 8 for x in the original equation.

B $13 = -2w$

$$\frac{13}{-2} = \frac{-2w}{-2}$$

$$-6.5 = w$$

Since w is multiplied by -2 , divide both sides by -2 to undo the multiplication.

Check $13 = -2w$

$$\begin{array}{r|l} 13 & -2(-6.5) \\ \hline 13 & 13 \checkmark \end{array}$$

To check your solution, substitute -6.5 for w in the original equation.



Solve each equation. Check your answer.

2a. $16 = 4c$

2b. $0.5y = -10$

2c. $15k = 75$

Remember that dividing is the same as multiplying by the reciprocal. When solving equations, you will sometimes find it easier to multiply by a reciprocal instead of dividing. This is often true when an equation contains fractions.

EXAMPLE 3 Solving Equations That Contain Fractions

Solve each equation.

A $\frac{5}{9}v = 35$

$$\left(\frac{9}{5}\right)\frac{5}{9}v = \left(\frac{9}{5}\right)35$$

$$v = 63$$

The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$. Since v is multiplied by $\frac{5}{9}$, multiply both sides by $\frac{9}{5}$.

B $\frac{5}{2} = \frac{4y}{3}$

$$\frac{5}{2} = \frac{4y}{3}$$

$$\frac{5}{2} = \frac{4}{3}y$$

$\frac{4y}{3}$ is the same as $\frac{4}{3}y$.

$$\left(\frac{3}{4}\right)\frac{5}{2} = \left(\frac{3}{4}\right)\frac{4}{3}y$$

$$\frac{15}{8} = y$$

The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$. Since y is multiplied by $\frac{4}{3}$, multiply both sides by $\frac{3}{4}$.



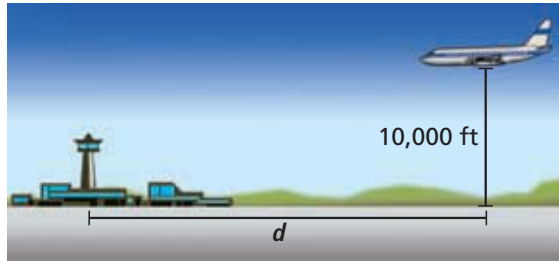
Solve each equation. Check your answer.

3a. $-\frac{1}{4} = \frac{1}{5}b$

3b. $\frac{4j}{6} = \frac{2}{3}$

3c. $\frac{1}{6}w = 102$

EXAMPLE 4 Aviation Application



The distance in miles from the airport that a plane should begin descending, divided by 3, equals the plane's height above the ground in thousands of feet. If a plane is 10,000 feet above the ground, write and solve an equation to find the distance at which the pilot should begin descending.

Caution!

The equation uses the plane's height above the ground in *thousands* of feet. So substitute 10 for h , not 10,000.

Distance divided by 3 equals height in thousands of feet.

$$\frac{d}{3} = h$$

Write an equation to represent the relationship.

$$\frac{d}{3} = 10$$

Substitute 10 for h . Since d is divided by 3, multiply both sides by 3 to undo the division.

$$(3)\frac{d}{3} = (3)10$$

$$d = 30$$

The pilot should begin descending 30 miles from the airport.



4. **What if...?** A plane began descending 45 miles from the airport. Use the equation above to find how high the plane was flying when the descent began.

You have now used four properties of equality to solve equations. These properties are summarized in the box below.



Properties of Equality

WORDS	NUMBERS	ALGEBRA
Addition Property of Equality You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$
Multiplication Property of Equality You can multiply both sides of an equation by the same number, and the statement will still be true.	$6 = 6$ $6(3) = 6(3)$ $18 = 18$	$a = b$ $ac = bc$
Division Property of Equality You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$8 = 8$ $\frac{8}{4} = \frac{8}{4}$ $2 = 2$	$a = b$ $(c \neq 0)$ $\frac{a}{c} = \frac{b}{c}$

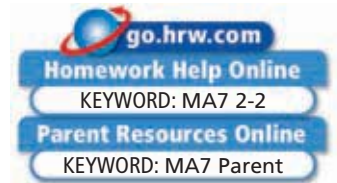
THINK AND DISCUSS

1. Tell how the Multiplication and Division Properties of Equality are similar to the Addition and Subtraction Properties of Equality.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.



2-2

Exercises



GUIDED PRACTICE

Solve each equation. Check your answer.

SEE EXAMPLE 1
p. 84

1. $\frac{k}{4} = 8$

2. $\frac{z}{3} = -9$

3. $-2 = \frac{w}{-7}$

4. $6 = \frac{t}{-5}$

5. $\frac{g}{1.9} = 10$

6. $2.4 = \frac{b}{5}$

SEE EXAMPLE 2
p. 85

7. $4x = 28$

8. $-64 = 8c$

9. $-9j = -45$

10. $84 = -12a$

11. $4m = 10$

12. $2.8 = -2h$

SEE EXAMPLE 3
p. 85

13. $\frac{1}{2}d = 7$

14. $15 = \frac{5}{6}f$

15. $\frac{2}{3}s = -6$

16. $9 = -\frac{3}{8}r$

17. $\frac{1}{10} = \frac{4}{5}y$

18. $\frac{1}{4}v = -\frac{3}{4}$

SEE EXAMPLE 4
p. 86

19. **Recreation** The Baseball Birthday Batter Package at a minor league ballpark costs \$192. The package includes tickets, drinks, and cake for a group of 16 children. Write and solve an equation to find the cost per child.

20. **Nutrition** An orange contains about 80 milligrams of vitamin C, which is 10 times as much as an apple contains. Write and solve an equation to find the amount of vitamin C in an apple.

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

21. $\frac{x}{2} = 12$

22. $-40 = \frac{b}{5}$

23. $-\frac{j}{6} = 6$

24. $-\frac{n}{3} = -4$

25. $-\frac{q}{5} = 30$

26. $1.6 = \frac{d}{3}$

27. $\frac{v}{10} = 5.5$

28. $\frac{h}{8.1} = -4$

29. $5t = -15$

30. $49 = 7c$

31. $-12 = -12u$

32. $-7m = 63$

33. $-52 = -4c$

34. $11 = -2z$

35. $5f = 1.5$

36. $-8.4 = -4n$

Independent Practice

For Exercises	See Example
21–28	1
29–36	2
37–44	3
45	4

Extra Practice

Skills Practice p. S6

Application Practice p. S29

Solve each equation. Check your answer.

37. $\frac{5}{2}k = 5$ 38. $-9 = \frac{3}{4}d$ 39. $-\frac{5}{8}b = 10$ 40. $-\frac{4}{5}g = -12$
 41. $\frac{4}{7}t = -2$ 42. $-\frac{4}{5}p = \frac{2}{3}$ 43. $\frac{2}{3} = -\frac{1}{3}q$ 44. $-\frac{5}{8} = -\frac{3}{4}a$

45. **Finance** After taxes, Alexandra's take-home pay is $\frac{7}{10}$ of her salary before taxes. Write and solve an equation to find Alexandra's salary before taxes for the pay period that resulted in \$392 of take-home pay.
 46. **Earth Science** Your weight on the Moon is about $\frac{1}{6}$ of your weight on Earth. Write and solve an equation to show how much a person weighs on Earth if he weighs 16 pounds on the Moon. How could you check that your answer is reasonable?
 47. **/// ERROR ANALYSIS ///** For the equation $\frac{x}{3} = 15$, a student found the value of x to be 5. Explain the error. What is the correct answer?



Geometry The perimeter of a square is given. Write and solve an equation to find the length of each side of the square.

48. $P = 36$ in. 49. $P = 84$ in. 50. $P = 100$ yd 51. $P = 16.4$ cm



Statistics



American Robert P. Wadlow (1918–1940) holds the record for world's tallest man—8 ft 11.1 in. He also holds world records for the largest feet and hands.

Source: Guinness World Records 2005

Write an equation to represent each relationship. Then solve the equation.

52. Five times a number is 45.
 53. A number multiplied by negative 3 is 12.
 54. A number divided by 4 is equal to 10.
 55. The quotient of a number and 3 is negative 8.
 56. **Statistics** The mean height of the students in Marta's class is 60 in. There are 18 students in her class. Write and solve an equation to find the total measure of all students' heights. (*Hint:* The mean is found by dividing the sum of all data values by the number of data values.)
 57. **Finance** Lisa earned \$6.25 per hour at her after-school job. Each week she earned \$50. Write and solve an equation to show how many hours she worked each week.
 58. **Critical Thinking** Will the solution of $\frac{x}{2.1} = 4$ be greater than 4 or less than 4? Explain.
 59. **Consumer Economics** Dion's long-distance phone bill was \$13.80. His long-distance calls cost \$0.05 per minute. Write and solve an equation to find the number of minutes he was charged for. Show that your answer is reasonable.
 60. **Nutrition** An 8 oz cup of coffee has about 184 mg of caffeine. This is 5 times as much caffeine as in a 12 oz soft drink. Write and solve an equation to find about how much caffeine is in a 12 oz caffeinated soft drink. Round your answer to the nearest whole number. Show that your answer is reasonable.

Use the equation $8y = 4x$ to find y for each value of x .

	x	$4x$	$8y = 4x$	y
61.	-4	$4(-4) = -16$	$8y = -16$	■
62.	-2	■	■	■
63.	0	■	■	■
64.	2	■	■	■



65. This problem will prepare you for the Multi-Step Test Prep on page 118.
- The formula for the mean of a data set is $\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$. One summer, there were 1926 wildfires in Arizona. Which value does this number represent in the formula?
 - The mean number of acres burned by each wildfire was 96.21. Which value does this number represent in the formula?
 - Use the formula and information given to find how many acres were burned by wildfires in Arizona that summer. Round your answer to the nearest acre. Show that your answer is reasonable.


Solve each equation. Check your answer.

66. $\frac{m}{6} = 1$ 67. $4x = 28$ 68. $1.2h = 14.4$ 69. $\frac{1}{5}x = 121$
 70. $2w = 26$ 71. $4b = \frac{3}{4}$ 72. $5y = 11$ 73. $\frac{n}{1.9} = 3$

Biology Use the table for Exercises 74 and 75.

Average Weight			
Animal	At Birth (g)	Adult Female (g)	Adult Male (g)
Hamster	2	130	110
Guinea pig	85	800	1050
Rat	5	275	480

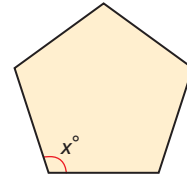


74. The mean weight of an adult male rat is 16 times the mean weight of an adult male mouse. Write and solve an equation to find the mean weight of an adult male mouse. Show that your answer is reasonable.
75. On average, a hamster at birth weighs $\frac{2}{3}$ the weight of a gerbil at birth. Write and solve an equation to find the average weight of a gerbil at birth. Show that your answer is reasonable.
-  76. **Write About It** Describe a real-world situation that can be modeled by $3x = 42$. Solve the equation and tell what the solution means in the context of your problem.



77. Which situation does NOT represent the equation $\frac{d}{2} = 10$?
- Leo bought a box of pencils. He gave half of them to his brother. They each got 10 pencils. How many pencils were in the box Leo bought?
 - Kasey evenly divided her money from baby-sitting into two bank accounts. She put \$10 in each account. How much did Kasey earn?
 - Gilbert cut a piece of ribbon into 2-inch strips. When he was done, he had ten 2-inch strips. How long was the ribbon to start?
 - Mattie had 2 more CDs than her sister Leona. If Leona had 10 CDs, how many CDs did Mattie have?
78. Which equation below shows a correct first step for solving $3x = -12$?
- $3x + 3 = -12 + 3$
 - $3(3x) = 3(-12)$
 - $3x - 3 = -12 - 3$
 - $\frac{3x}{3} = \frac{-12}{3}$

79. In a regular pentagon, all of the angles are equal in measure. The sum of the angle measures is 540° . Which of the following equations could be used to find the measure of each angle?



- (A) $\frac{x}{540} = 5$ (C) $540x = 5$
 (B) $5x = 540$ (D) $\frac{x}{5} = 540$
80. For which equation is $m = 10$ a solution?
 (F) $5 = 2m$ (G) $5m = 2$ (H) $\frac{m}{2} = 5$ (J) $\frac{m}{10} = 2$
81. **Short Response** Luisa bought 6 cans of cat food that each cost the same amount. She spent a total of \$4.80.
- Write an equation to determine the cost of one can of cat food. Tell what each part of your equation represents.
 - Solve your equation to find the cost of one can of cat food. Show each step.

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

82. $\left(3\frac{1}{5}\right)b = \frac{4}{5}$ 83. $\left(1\frac{1}{3}\right)x = 2\frac{2}{3}$ 84. $\left(5\frac{4}{5}\right)x = -52\frac{1}{5}$
 85. $\left(-2\frac{9}{10}\right)k = -26\frac{1}{10}$ 86. $\left(1\frac{2}{3}\right)w = 15\frac{1}{3}$ 87. $\left(2\frac{1}{4}\right)d = 4\frac{1}{2}$

Find each indicated value.

88. If $2p = 4$, find the value of $6p + 10$. 89. If $6t = 24$, find the value of $-5t$.
 90. If $3x = 15$, find the value of $12 - 4x$. 91. If $\frac{n}{2} = -11$, find the value of $6n$.
 92. To isolate x in $ax = b$, what should you divide both sides by?
 93. To isolate x in $\frac{x}{a} = b$, what operation should you perform on both sides of the equation?
 94. **Travel** The formula $d = rt$ gives the distance d that is traveled at a rate r in time t .
 a. If $d = 400$ and $r = 25$, what is the value of t ?
 b. If $d = 400$ and $r = 50$, what is the value of t ?
 c. **What if...?** How did t change when r increased from 25 to 50?
 d. **What if...?** If r is doubled while d remains the same, what is the effect on t ?

SPIRAL REVIEW

Find each square root. (Lesson 1-5)

95. $\sqrt{144}$ 96. $\sqrt{196}$ 97. $\sqrt{625}$ 98. $-\sqrt{9}$

Write and solve an equation that could be used to answer each question. (Lesson 2-1)

99. Lisa's age plus Sean's age is 17. Sean is 11 years old. How old is Lisa?
 100. The length of a rectangle is 6 feet more than the width of the rectangle. The length is 32 feet. What is the width of the rectangle?

Solve each equation. Check your answer. (Lesson 2-1)

101. $2 = a - 4$ 102. $x - 12 = -3$ 103. $z - 5 = 11$ 104. $-4 = x + 5$



Solve Equations by Graphing

You can use graphs to solve equations. As you complete this activity, you will learn some of the connections between graphs and equations.

Use with Lesson 2-3

Activity

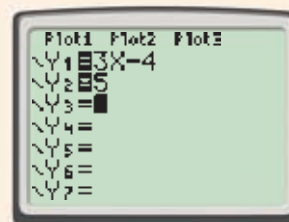
Solve $3x - 4 = 5$.

- 1 Press **Y=**. In Y_1 , enter the left side of the equation, $3x - 4$.

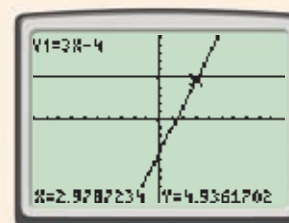
Y= 3 **X,T,θ,n** **-** 4 **ENTER**

In Y_2 , enter the right side of the equation, 5.

Y= 5 **ENTER**

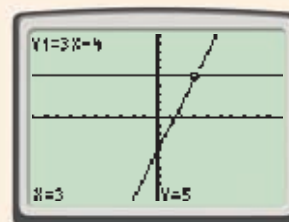


- 2 Press **GRAPH**. Press **TRACE**. The display will show the x - and y -values of a point on the first line. Press the right arrow key several times. Notice that the x - and y -values change.



- 3 Continue to trace as close as possible to the intersection of the two lines. The x -value of this point 2.9787..., is an approximation of the solution. The solution is about 3.

- 4 While still in trace mode, to check, press **3** **ENTER**. The display will show the y -value when $x = 3$. When $x = 3$, $y = 5$. So 3 is the solution. You can also check this solution by substituting 3 for x in the equation:



Check

$3x - 4 = 5$	
$3(3) - 4$	5
$9 - 4$	5
5	5 ✓

Try This

- Solve $3x - 4 = 2$, $3x - 4 = 17$, and $3x - 4 = -7$ by graphing.
- What does each line represent?
- Describe a procedure for finding the solution of $3x - 4 = y$ for any value of y .
- Solve $\frac{1}{2}x - 7 = -4$, $\frac{1}{2}x - 7 = 0$, and $\frac{1}{2}x - 7 = 2$ by graphing.

