

# 2-11

## Percent Increase and Decrease



### Objective

Find percent increase and decrease.

### Vocabulary

percent change  
percent increase  
percent decrease  
discount  
markup

### Who uses this?

Consumers can use percent change to determine how much money they can save. (See Example 3.)

A **percent change** is an increase or decrease given as a percent of the original amount. **Percent increase** describes an amount that has grown and **percent decrease** describes an amount that has been reduced.



### Percent Change

percent change =  $\frac{\text{amount of increase or decrease}}{\text{original amount}}$ , expressed as a percent

### EXAMPLE 1 Finding Percent Increase or Decrease

Find each percent change. Tell whether it is a percent increase or decrease.

**A** from 25 to 49

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{49 - 25}{25} \\ &= \frac{24}{25} && \text{Simplify the numerator.} \\ &= 0.96 \\ &= 96\% && \text{Write the answer as a percent.} \end{aligned}$$

25 to 49 is an increase, so a change from 25 to 49 is a 96% increase.

**B** from 50 to 45

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{50 - 45}{50} \\ &= \frac{5}{50} && \text{Simplify the numerator.} \\ &= \frac{1}{10} && \text{Simplify the fraction.} \\ &= 10\% && \text{Write the answer as a percent.} \end{aligned}$$

50 to 45 is a decrease, so a change from 50 to 45 is a 10% decrease.

### Helpful Hint

Before solving, decide what is a reasonable answer. For Example 1A, 25 to 50 would be a 100% increase. So 25 to 49 should be slightly less than 100%.



Find each percent change. Tell whether it is a percent increase or decrease.

- 1a. from 200 to 110    1b. from 25 to 30    1c. from 80 to 115

## EXAMPLE 2 Finding the Result of a Percent Increase or Decrease

**A** Find the result when 30 is increased by 20%.

$$0.20(30) = 6 \quad \textit{Find 20\% of 30. This is the amount of the increase.}$$

$$30 + 6 = 36 \quad \textit{It is a percent increase, so add 6 to the original amount.}$$

30 increased by 20% is 36.

**B** Find the result when 65 is decreased by 80%.

$$0.80(65) = 52 \quad \textit{Find 80\% of 65. This is the amount of the decrease.}$$

$$65 - 52 = 13 \quad \textit{It is a percent decrease, so subtract 52 from 65.}$$

65 decreased by 80% is 13.



**2a.** Find the result when 72 is increased by 25%.

**2b.** Find the result when 10 is decreased by 40%.

Common applications of percent change are *discounts* and *markups*.

A **discount** is an amount by which an original price is reduced.

$$\text{discount} = \% \text{ of original price}$$

$$\text{final price} = \text{original price} - \text{discount}$$

A **markup** is an amount by which a wholesale cost is increased.

$$\text{markup} = \% \text{ of wholesale cost}$$

$$\text{final price} = \text{wholesale cost} + \text{markup}$$

## EXAMPLE 3 Discounts

### Helpful Hint

Before solving, decide what is a reasonable answer. For Example 3A, a 25% discount is \$2 off. So a 15% discount will be less than \$2 off.

**A** Admission to the museum is \$8. Students receive a 15% discount. How much is the discount? How much do students pay?

**Method 1** A discount is a percent decrease. So find \$8 decreased by 15%.

$$0.15(8) = 1.20 \quad \textit{Find 15\% of 8. This is the amount of the discount.}$$

$$8 - 1.20 = 6.80 \quad \textit{Subtract 1.20 from 8. This is the student price.}$$

**Method 2** Subtract percent discount from 100%.

$$100\% - 15\% = 85\% \quad \textit{Students pay 85\% of the regular price, \$8.}$$

$$0.85(8) = 6.80 \quad \textit{Find 85\% of 8. This is the student price.}$$

$$8 - 6.80 = 1.20 \quad \textit{Subtract 6.80 from 8. This is the amount of the discount.}$$

By either method, the discount is \$1.20. Students pay \$6.80.

**B** Christo used a coupon and paid \$7.35 for a pizza that normally costs \$10.50. Find the percent discount.

$$\$10.50 - \$7.35 = \$3.15$$

$$3.15 = x(10.50)$$

$$\frac{3.15}{10.50} = \frac{x \cdot 10.50}{10.50}$$

$$0.3 = x$$

$$30\% = x$$

*Think: 3.15 is what percent of 10.50? Let  $x$  represent the percent.*

*Since  $x$  is multiplied by 10.50, divide both sides by 10.50 to undo the multiplication.*

*Write the answer as a percent.*

The discount is 30%.



**3a.** A \$220 bicycle was on sale for 60% off. Find the sale price.

**3b.** Ray paid \$12 for a \$15 T-shirt. What was the percent discount?

## EXAMPLE 4 Markups

- A** Kaleb buys necklaces at a wholesale cost of \$48 each. He then marks up the price by 75% and sells the necklaces. What is the amount of the markup? What is the selling price?

### Method 1

A markup is a percent increase. So find \$48 increased by 75%.

$$0.75(48) = 36 \quad \textit{Find 75\% of 48. This is the amount of the markup.}$$

$$48 + 36 = 84 \quad \textit{Add to 48. This is the selling price.}$$

### Method 2

Add percent markup to 100%.

$$100\% + 75\% = 175\% \quad \textit{The selling price is 175\% of the wholesale price, \$48.}$$

$$1.75(48) = 84 \quad \textit{Find 175\% of 48. This is the selling price.}$$

$$84 - 48 = 36 \quad \textit{Subtract from 84. This is the amount of the markup.}$$

By either method, the amount of the markup is \$36. The selling price is \$84.

- B** Lars purchased a daily planner for \$32. The wholesale cost was \$25. What was the percent markup?

$$32 - 25 = 7 \quad \textit{Find the amount of the markup.}$$

$$7 = x(25) \quad \textit{Think: 7 is what percent of 25? Let } x \text{ represent the percent.}$$

$$\frac{7}{25} = \frac{25x}{25} \quad \textit{Since } x \text{ is multiplied by 25, divide both sides by 25 to undo the multiplication.}$$

$$0.28 = x$$

$$28\% = x \quad \textit{Write the answer as a percent.}$$

The markup was 28%.



- 4a.** A video game has a 70% markup. The wholesale cost is \$9. What is the selling price?

- 4b.** What is the percent markup on a car selling for \$21,850 that had a wholesale cost of \$9500?

## THINK AND DISCUSS

- 80% of a number is the same as a ?% decrease from that number. A 30% increase from a number is the same as ?% of that number.
- A markup of 200% will result in a final cost that is how many times the wholesale cost?
- What information would you need to find the percent change in your school's population over the last ten years?
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write and solve an example of the given type of problem.



Percent Increase	Percent Decrease	Discount	Markup

### GUIDED PRACTICE

1. **Vocabulary** Compare *percent increase* and *percent decrease*.

SEE EXAMPLE 1

p. 144

Find each percent change. Tell whether it is a percent increase or decrease

2. 25 to 45                      3. 10 to 8                      4. 400 to 300  
 5. 16 to 18                      6. 40 to 50                      7. 50 to 40

SEE EXAMPLE 2

p. 144

8. Find the result when 40 is increased by 85%.  
 9. Find the result when 60 is increased by 3%.  
 10. Find the result when 350 is decreased by 10%.  
 11. Find the result when 16 is decreased by 50%.

SEE EXAMPLE 3

p. 145

12. What is the final price on a \$185 leather jacket that is on sale for 40% off?  
 13. Neal bought a book on sale for \$3.60. It was originally priced at \$12. What was Neal's discount as a percent?

SEE EXAMPLE 4

p. 146

14. Yolanda bought a video that was priced at a 65% markup over the manufacturer's cost of \$12. What was Yolanda's cost?  
 15. Randy sells hats for \$12.35. The wholesale cost of each hat is \$6.50. What is Randy's markup as a percent?

### PRACTICE AND PROBLEM SOLVING

#### Independent Practice

For Exercises	See Example
16–27	1
28–31	2
32–33	3
34–35	4

Find each percent change. Tell whether it is a percent increase or decrease.

16. 50 to 60                      17. 4 to 3                      18. 96 to 84                      19. 9 to 45  
 20. 32 to 30                      21. 15 to 19.5                      22. 150 to 180                      23. 17 to 14.45  
 24. 20 to 15                      25. 265 to 318                      26. 35 to 105                      27. 300 to 275  
 28. Find the result when 24 is increased by 75%.

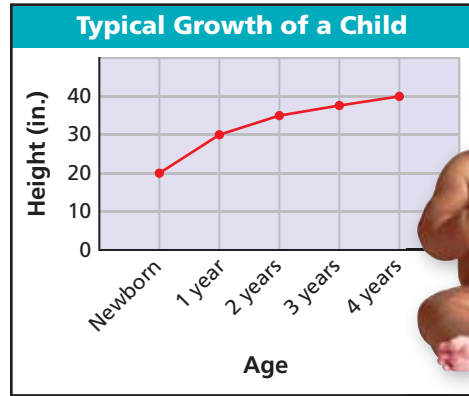
29. Find the result when 240 is increased by 5%.  
 30. Find the result when 30 is decreased by 85%.  
 31. Find the result when 8 is decreased by 5%.  
 32. The cost of Hisako's school supplies was \$49.80. She had a coupon for 30% off the entire purchase. What was the final price?  
 33. With the purchase of 10 greeting cards, Addie received a discount. She paid \$26.35 for the cards that would normally have cost \$31. What percent discount did Addie receive?  
 34. Irma sells boxing gloves in her sporting goods store for a 9% markup over the manufacturer's cost of \$40. What is the selling price of the gloves?  
 35. Bottled water in a certain vending machine costs \$1.50. This price is a markup from the wholesale cost of \$0.20. What is the markup as a percent?  
 36. **Critical Thinking** Is the percent increase from 50 to 80 the same as the percent decrease from 80 to 50? Why or why not?

#### Extra Practice

Skills Practice p. S7

Application Practice p. S29

**Multi-Step** The graph shows the average height of a child from birth to age 4. Use the graph for Exercises 37–39.



Source: www.kidshealth.org

37. By what percent does a child's height increase from birth to age 1 year?
38. By what percent does a child's height increase from birth to age 4 years?
39. **Estimation** Estimate the amount and percent of increase in a child's height from age 2 to age 3. Show that your estimate is reasonable.
40. **Employment** Last summer, Duncan charged \$20 to mow a lawn in his neighborhood. This summer, he'll charge \$23. What is the percent increase in Duncan's price? Show that your answer is reasonable.

Copy and complete the table.

	Original Amount	New Amount	Percent Change
41.	12	■	50% increase
42.	\$48	\$42.24	■
43.	$4\frac{1}{2}$	$13\frac{1}{2}$	■
44.	8525	■	20% decrease

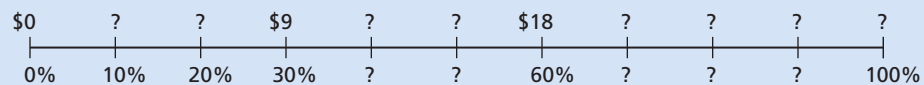
Find each missing number.

45. 20 increased by ■% is 24.
46. 80 decreased by ■% is 76.
47. 120 decreased by 50% is ■.
48. 200 increased by ■% is 210.
49. **Nutrition** A can of soup had 480 mg of sodium per serving. The sodium was reduced to 360 mg per serving so that the soup could be advertised as "low sodium." What was the percent change in sodium content?
50. **Write About It** Dana is shopping for shoes at a store advertising "everything 45% off." Describe a method she could use to estimate the discount and final price on a pair of shoes.

**MULTI-STEP  
TEST PREP**



51. This problem will prepare you for the Multi-Step Test Prep on page 152.
- Robert finds a shirt on a sale rack. All items on the rack are 40% off. The price on his shirt is missing. When the clerk scans the bar code, he tells Robert that the sale price of the shirt is \$18. What percent of the original price is \$18?
  - Set up a proportion to find the original price of the shirt.
  - Copy and complete the model below and explain how it helps you to solve this problem in another way.





52. Lucia gets film developed at Photo King, where 24 prints cost \$7.80. This week, Photo King is having a sale, and 24 prints cost \$6.63. What percent of the regular cost will Lucia save?
- (A) 15%      (B) 17%      (C) 25%      (D) 85%
53. Which of these does NOT represent “200 decreased by 45%”?
- (F)  $200(0.55)$       (G) 110      (H)  $200 - 0.45$       (J)  $200(1 - 0.45)$
54. Which of these represents a 15% increase?
- (A) A price is marked up from \$12.50 to \$15.  
(B) Joanna’s bank account balance grew from \$127.50 to \$150.  
(C) A baseball card’s value rose from \$6 to \$6.15.  
(D) Luis’s hourly wage was raised from \$8 to \$9.20.
55. The original price of an item was \$199. During a sale, the price was reduced by 45%. Then, during a clearance sale, the price was reduced an additional 20%. What was the final price?
- (F) \$71.91      (G) \$87.56      (H) \$98.97      (J) \$101.89
56. **Gridded Response** A skateboard that sells for \$65 is on sale for 15% off. What is the sale price in dollars?

## CHALLENGE AND EXTEND

Find each missing number.

57. ■ increased by 15% is 230.      58. ■ increased by 50% is 48.  
59. ■ decreased by 20% is 500.      60. ■ decreased by 70% is 4.35.
61. The label on a bottle of orange juice says “now 25% more.” The bottle has 80 fluid ounces of juice. What was the original volume? Show that your answer is reasonable.
62. Angelina paid \$21 for a backpack that was 30% off. What was the original price? Show that your answer is reasonable.
63. **Multi-Step** Mr. Hansen owns a bookstore. He buys used books at 25% of the cover price and sells them at a 45% markup of what he paid. Jerry sold Mr. Hansen three books with cover prices of \$7.95, \$5.95, and \$12.10. If Jerry bought back his books, how much would he pay?

## SPIRAL REVIEW

Find the complement and the supplement of each angle. (*Previous course*)

64.  $65^\circ$       65.  $10^\circ$       66.  $45^\circ$       67.  $30^\circ$

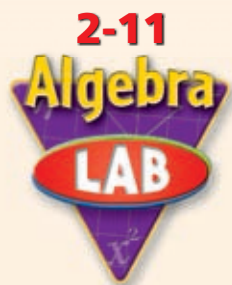
Solve each equation. Check your answer. (*Lessons 2-1 and 2-2*)

68.  $-15 + x = -3$       69.  $x + 16 = -4$       70.  $r - 3 = 6$       71.  $n - (-10) = 67$   
72.  $98 = 7z$       73.  $\frac{x}{3} = 12$       74.  $-x = 4$       75.  $\frac{x}{5} = -20$

Estimate each amount. (*Lesson 2-10*)

76. the tip on a \$60.65 check using a tip rate of 15%  
77. the tax on a \$70 DVD player when the sales tax is 6%





# Explore Changes in Population

You can use percents to describe changes in populations. A population may grow by a certain percent or decrease by a certain percent. Explore changing populations in these activities.

Use with Lesson 2-11

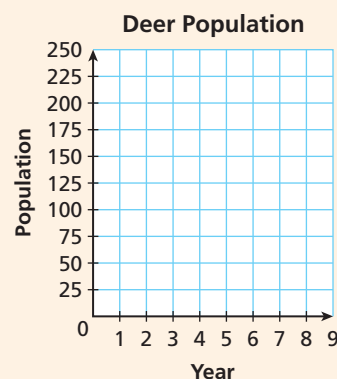


## Activity 1

A team of biologists is studying a population of deer. There are 32 deer in the first year of the study. Due to a lack of predators, the biologists find that the herd grows by 50% every year.

1 Copy and complete the table. The first two rows have been completed for you.

Year	Percent Increase	Amount of Increase	Population
1			32
2	50%	$0.50 \cdot 32 = 16$	$32 + 16 = 48$
3	50%	■	■
4	50%	■	■
5	50%	■	■
6	50%	■	■



- Describe the percent increase from year to year.
- Describe the amount of increase from year to year.
- Copy the grid above onto graph paper. Plot the year and the population of deer on the graph as six ordered pairs (year, population). Connect the points with a smooth curve.
- Describe the shape of your graph.

## Try This

A researcher places 10 bacteria on a dish. This species increases by 100% every hour.

1. Copy and complete the table below.

Hour	0	1	2	3	4	5
Amount of Increase		■	■	■	■	■
Bacteria	10	■	■	■	■	■

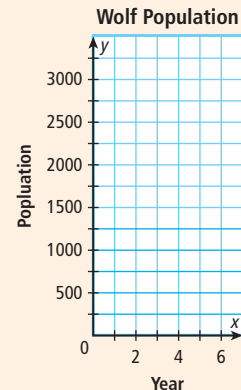
- Graph points from the table as (hour, bacteria). Connect the points with a smooth curve.
- Compare this graph with the graph of the deer population.
- Why does the amount of increase change when the percent of increase stays the same?

## Activity 2

A second team of biologists is studying a population of wolves. There are 3125 wolves in the first year. The biologists find that this population decreases by 40% every year.

- 1 **Make a Prediction** Based on your results in Activity 1, what do you think will happen to the amount of decrease each year?
- 2 Copy and complete the table below. The first two rows have been completed for you.

Year	Percent Decrease	Amount of Decrease	Population
1			3125
2	40%	$0.40 \cdot 3125 = 1250$	$3125 - 1250 = 1875$
3	40%	■	■
4	40%	■	■
5	40%	■	■
6	40%	■	■



- 3 What happens to the amount of decrease in the wolf population from year to year? Was your prediction from Problem 1 correct?
- 4 **Make a Prediction** Copy the grid above onto graph paper. Based on your results in Activity 1, what do you think the graph of ordered pairs (year, population) will look like?
- 5 Plot the year and the population of wolves on the graph as six ordered pairs (year, population). Connect the points with a smooth curve.
- 6 Describe the shape of your graph. Was your prediction from Problem 4 correct?

## Try This

A half-life is the amount of time it takes half of an amount of radioactive substance to decay into another substance. Tritium is a radioactive form of hydrogen with a half-life of 12.3 years. In other words, after one half-life of 12.3 years, an amount of tritium will have decreased by 50%.

5. Suppose you start with 128 grams of tritium. Copy and complete the table below.
6. Make a graph that shows how much tritium is left after 0, 1, 2, 3, 4, and 5 half-lives.
7. Compare this graph with the graph of the wolf population.
8. Describe the graph of a population that increases by a fixed percent. Why does the graph have this shape?
9. Describe the graph of a population that decreases by a fixed percent. Why does the graph have this shape?

Half-lives	0	1	2	3	4	5
Percent Decrease	0	50%	50%	50%	50%	50%
Amount of Decrease (g)		■	■	■	■	■
Tritium Remaining (g)	128	■	■	■	■	■



# MULTI-STEP TEST PREP



## Percentages

**Bargain Hunters** Maria is on her high school's lacrosse team, and her friend Paula is on the softball team. The girls notice an advertisement in the newspaper for a clearance sale at their favorite sporting goods store. The ad shows an additional  $\frac{1}{4}$  off the already reduced prices of 60% off. Maria and Paula head to the store to shop for bargains.

1. Maria finds a lacrosse stick with a regular price of \$65. Find the sale price of the lacrosse stick prior to the additional  $\frac{1}{4}$  off.
2. Find the sale price of Maria's lacrosse stick with the additional  $\frac{1}{4}$  off.
3. Paula says that with the extra  $\frac{1}{4}$  off, the total discount is 85% off. Maria thinks the discount is less than that. Who is correct? Explain your reasoning.
4. Paula finds a softball glove with a price tag that is not readable. The sales clerk scans the bar code and says the sale price, including the extra  $\frac{1}{4}$  off, is \$16.50. What was the original price of the softball glove? Show your reasoning.
5. Sales tax is 7.8%. Find the total amount that the girls will pay for the lacrosse stick and the softball glove together, including tax.



## Quiz for Lessons 2-7 Through 2-11

### 2-7 Rates, Ratios, and Proportions

- Last week, the ratio of laptops to desktops sold at a computer store was 2:3. Eighteen desktop models were sold. How many laptop models were sold?
- Anita read 150 pages in 5 hours. What is her reading rate in pages per minute?
- Twenty-six crackers contain 156 Calories. Find the unit rate in Calories per cracker.
- A store developed 1024 photographs in 8 hours. Find the unit rate in photographs per hour.

Solve each proportion.

5.  $\frac{-18}{n} = \frac{9}{2}$

6.  $\frac{d}{5} = \frac{2}{4}$

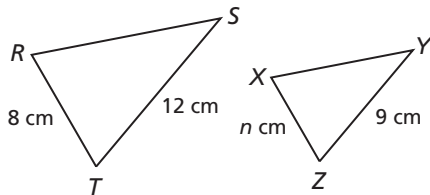
7.  $\frac{4}{12} = \frac{r+2}{16}$

8.  $\frac{-3}{7} = \frac{6}{x+6}$

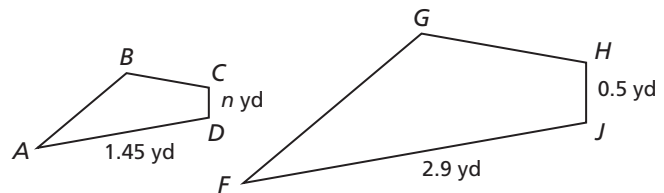
### 2-8 Applications of Proportions

Find the value of  $n$  in each diagram.

9.  $\triangle RST \sim \triangle XYZ$



10.  $ABCD \sim FGHI$



### 2-9 Percents

- Find 40% of 25.
- Find 130% of 9.
- 35 is what percent of 70?
- What percent of 400 is 640?
- 16 is 80% of what number?
- 200% of what number is 28?
- A volunteer at the zoo is responsible for feeding the animals in 15 exhibits in the reptile house. This represents 20% of the total exhibits in the reptile house. How many exhibits are in the reptile house?

### 2-10 Applications of Percents

- Peter earns \$32,000 per year plus a 2.5% commission on his jewelry sales. Find Peter's total salary for the year when his sales are valued at \$420,000.
- Estimate the tax on a \$21,899 car when the tax rate is 5%.

### 2-11 Percent Increase and Decrease

Find each percent change. Tell whether it is a percent increase or decrease.

- from 60 to 66
- from 48 to 12
- from 200 to 80
- from 9.8 to 14.7
- Andrea purchased a picture frame for \$14.56. This price was a 30% markup from the wholesale cost. What was the wholesale cost?

**Vocabulary**

commission . . . . .	139	interest . . . . .	139	sales tax . . . . .	140
conversion factor . . . . .	121	literal equation . . . . .	108	scale . . . . .	122
corresponding angles . . . . .	127	markup . . . . .	145	scale drawing . . . . .	122
corresponding sides . . . . .	127	percent . . . . .	133	scale factor . . . . .	129
cross products . . . . .	121	percent change . . . . .	144	scale model . . . . .	122
dimensional analysis . . . . .	121	percent decrease . . . . .	144	similar . . . . .	127
discount . . . . .	145	percent increase . . . . .	144	solution of an equation . . . . .	77
equation . . . . .	77	principal . . . . .	139	tip . . . . .	140
formula . . . . .	107	proportion . . . . .	120	unit rate . . . . .	120
identity . . . . .	101	rate . . . . .	120		
indirect measurement . . . . .	128	ratio . . . . .	120		

Complete the sentences below with vocabulary words from the list above.

- A formula is a type of a(n) \_\_\_\_\_?
- A(n) \_\_\_\_\_ is used to compare two quantities by division.

**2-1 Solving Equations by Adding or Subtracting (pp. 77–82)****EXAMPLES**

Solve each equation. Check your answer.

$$\blacksquare x - 12 = -8.3 \quad \blacksquare -7.8 = 5 + t$$

$$\begin{array}{r} +12 \quad +12 \\ x - 12 = -8.3 \\ \hline x = 3.7 \end{array}$$

$$\begin{array}{r} -5 \quad -5 \\ -7.8 = 5 + t \\ \hline -12.8 = t \end{array}$$

Check  $x - 12 = -8.3$       Check  $-7.8 = 5 + t$

$$\begin{array}{r|l} 3.7 - 12 & -8.3 \\ -8.3 & -8.3 \checkmark \end{array} \quad \begin{array}{r|l} -7.8 & 5 + (-12.8) \\ -7.8 & -7.8 \checkmark \end{array}$$

**EXERCISES**

Solve each equation. Check your answer.

- $b - 16 = 20$
- $4 + x = 2$
- $9 + a = -12$
- $-7 + y = 11$
- $z - \frac{1}{4} = \frac{7}{8}$
- $w + \frac{2}{3} = 3$

- Robin needs 108 signatures for her petition. So far, she has 27. Write and solve an equation to determine how many more signatures she needs.

**2-2 Solving Equations by Multiplying or Dividing (pp. 84–90)****EXAMPLES**

Solve each equation.

$$\blacksquare \frac{z}{2.4} = 12 \quad \blacksquare -8x = 148$$

$$\begin{array}{r} (2.4) \frac{z}{2.4} = (2.4) 12 \\ z = 28.8 \end{array}$$

$$\begin{array}{r} -8x = 148 \\ -8 \quad -8 \\ \hline x = -18.5 \end{array}$$

**EXERCISES**

Solve each equation. Check your answer.

- $35 = 5x$
- $-3n = 10$
- $-30 = \frac{n}{3}$
- $\frac{x}{-5} = -2.6$
- $5y = 0$
- $-4.6r = 9.2$

## 2-3 Solving Two-Step and Multi-Step Equations (pp. 92–98)

### EXAMPLE

■ Solve  $\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$ .

$$\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$$

$$20\left(\frac{3x}{5} - \frac{x}{4} + \frac{1}{2}\right) = 20\left(\frac{6}{5}\right)$$

$$12x - 5x + 10 = 24$$

$$7x + 10 = 24$$

$$\begin{array}{r} -10 \\ \hline 7x \end{array} = \begin{array}{r} -10 \\ \hline 14 \end{array}$$

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

Multiply by  
the LCD.

Combine like  
terms.

### EXERCISES

Solve each equation. Check your answer.

16.  $4t - 13 = 57$

17.  $5 - 2y = 15$

18.  $\frac{k}{5} - 6 = 2$

19.  $\frac{5}{6}f - \frac{3}{4}f + \frac{3}{4} = \frac{1}{2}$

20.  $7x - 19x = 6$

21.  $4 + 3a - 6 = 43$

22. If  $8n + 22 = 70$ , find the value of  $3n$ .

23. If  $0 = 6n - 36$ , find the value of  $n - 5$ .

24. The sum of the measures of two angles is  $180^\circ$ . One angle measures  $3a$  and the other angle measures  $2a - 25$ . Find  $a$ . Then find the measure of each angle.

## 2-4 Solving Equations with Variables on Both Sides (pp. 100–106)

### EXAMPLE

■ Solve  $x + 7 = 12 + 3x - 7x$ .

$$x + 7 = 12 + 3x - 7x$$

$$x + 7 = 12 - 4x$$

$$\begin{array}{r} +4x \\ \hline 5x + 7 = 12 \end{array}$$

$$5x + 7 = 12$$

$$\begin{array}{r} -7 \\ \hline 5x = 5 \end{array}$$

$$5x = 5$$

$$\frac{5x}{5} = \frac{5}{5}$$

$$x = 1$$

Combine like  
terms.

### EXERCISES

Solve each equation. Check your answer.

25.  $4x + 2 = 3x$

26.  $-3r - 8 = -5r - 12$

27.  $-a - 3 + 7 = 3a$

28.  $-(x - 4) = 2x + 6$

29.  $\frac{2}{3}n = 4n - \frac{10}{3}n - \frac{1}{2}$

30.  $0.2(7 + 2t) = 0.4t + 1.4$

31. One photo shop charges \$0.36 per print. Another photo shop charges \$2.52 plus \$0.08 per print. Juan finds that the cost of printing his photos is the same at either shop. How many photos does Juan have to print?

## 2-5 Solving for a Variable (pp. 107–111)

### EXAMPLE

■ Solve  $A = P + Prt$  for  $r$ .

$$A = P + Prt$$

$$\begin{array}{r} -P \\ \hline A - P = \end{array} \quad Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A - P}{Pt} = r$$

### EXERCISES

Solve for the indicated variable.

32.  $C = \frac{360}{n}$  for  $n$

33.  $S = \frac{n}{2}(a + \ell)$  for  $a$

34. The formula  $a = \frac{d}{g}$  gives the average gas mileage  $a$  of a vehicle that uses  $g$  gallons of gas to travel  $d$  miles. Use the formula to find how many gallons of gas a vehicle with an average gas mileage of 20.2 miles per gallon will use to travel 75 miles. Round your answer to the nearest tenth.

## 2-6 Solving Absolute-Value Equations (pp. 112–117)

### EXAMPLES

- Solve  $3|y + 4| = 30$ .

$$\frac{3|y + 4|}{3} = \frac{|30|}{3} \quad \text{Divide both sides by 3.}$$

$$|y + 4| = 10$$

Case 1

$$y + 4 = 10$$

$$\frac{-4}{y} = \frac{-4}{6}$$

Case 2

$$y + 4 = -10$$

$$\frac{-4}{y} = \frac{-4}{-14}$$

### EXERCISES

Solve each equation. Check your answer.

35.  $|x + 6| = 21$                       36.  $7|y - 5| = 14$
37.  $3|y| + 4 = 31$                     38.  $12 = |x - 5.4|$
39.  $|g + 6| + 12 = 14$               40.  $|x| = \frac{5}{7}$
41. Jason is driving his car at 55 mi/h. He needs to keep his car within 5 mi/h of his current speed. Write and solve an absolute-value equation to find Jason's maximum and minimum speeds.

## 2-7 Rates, Ratios, and Proportions (pp. 120–126)

- The ratio of skateboarders to bikers in an extreme sports contest is 7:2. There are 91 skateboarders. How many bikers are there?

$$\frac{\text{skateboarders}}{\text{bikers}} \rightarrow \frac{7}{2} \quad \text{Write a proportion.}$$

$$\frac{7}{2} \times \frac{91}{x} \quad \text{Let } x \text{ be the number of bikers.}$$

$$7 \cdot x = 2 \cdot 91 \quad \text{Use cross products.}$$

$$7x = 182$$

$$\frac{7x}{7} = \frac{182}{7} \quad \text{Solve for } x.$$

$$x = 26$$

There are 26 bikers.

42. A recipe for a casserole calls for 2 cups of rice. The recipe makes 6 servings of casserole. How many cups of rice will you need to make 10 servings of casserole?

Use dimensional analysis to convert each rate. Round your answer to the nearest hundredth if necessary.

43. 30 cm/s to m/h                      44. 75 ft/s to mi/min

Solve each proportion. Check your answer.

45.  $\frac{n}{8} = \frac{2}{10}$

46.  $\frac{2}{9} = \frac{12}{x}$

47.  $\frac{3}{k} = \frac{9}{15}$

48.  $\frac{1}{3} = \frac{x}{x - 6}$

## 2-8 Applications of Proportions (pp. 127–132)

- When Janelle stood next to the Washington Monument, she cast a 1.2-foot-long shadow, and the monument cast a 111-foot-long shadow. Janelle is 6 feet tall. How tall is the monument?

$$\frac{x}{111} = \frac{6}{1.2} \quad \text{Write a proportion.}$$

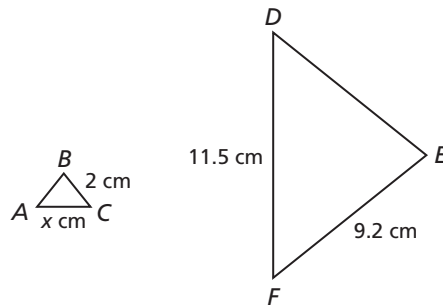
$$1.2x = 666 \quad \text{Use cross products.}$$

$$\frac{1.2x}{1.2} = \frac{666}{1.2} \quad \text{Solve for } x.$$

The monument is 555 feet tall.

49. Find the value of  $x$  in the diagram.

$$\triangle ABC \sim \triangle DEF$$



50. A tree casts a shadow that is 14 ft long at the same time that a nearby 2-foot-tall pole casts a shadow that is 1.75 ft long. How tall is the tree?
51. A circle has a radius of 9 inches. The radius is multiplied by  $\frac{2}{3}$  to form a second circle. How is the ratio of the areas related to the ratio of the radii?



## 2-9 Percents (pp. 133–138)

### EXAMPLES

- Earth's surface area is about 197 million square miles. About 58 million square miles is land. What percent is water?

197 million – 58 million = 139 million

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

$$\frac{139}{197} = \frac{n}{100}$$

$$197n = 13,900 \quad \text{Use cross products.}$$

$$n = 70.56 \quad \text{Solve for } n.$$

About 71% of Earth's surface area is water.

### EXERCISES

- Find 2.3% of 230.
- Find 115% of 2700.
- What percent of 18 is 12? Round your answer to the nearest tenth of a percent.
- What percent of 14 is 56?
- 90% of what number is 120? Round the number to the nearest tenth.
- 90 is 37.5% of what number?
- A student answered 32 questions correctly and 8 incorrectly. What percent of the questions were answered correctly?

## 2-10 Applications of Percents (pp. 139–143)

- After 10 months, the simple interest earned on \$3000 was \$52.50. Find the interest rate.

197 million – 58 million = 139 million

$$I = Prt$$

$$52.5 = 3000(r)\left(\frac{10}{12}\right) \quad \text{Substitute.}$$

$$52.5 = 2500r \quad \text{Multiply.}$$

$$\frac{52.5}{2500} = \frac{2500r}{2500} \quad \text{Solve for } r.$$

$$0.021 = r$$

The interest rate is 2.1%. Write as a percent.

- A salesperson earns a base salary of \$36,000 plus  $2\frac{1}{2}\%$  commission on sales. His total sales for one year was \$500,000. Find the salesperson's total pay for that year.
- Find the simple interest paid for 10 years on a \$10,000 loan at 9% per year.
- The sales tax rate is 8%. Estimate the tax on a jacket that costs \$69.95.
- Estimate the tip on a \$10.75 check using a tip rate of 20%.

## 2-11 Percent Increase and Decrease (pp. 144–149)

Find each percent change. Tell whether it is a percent increase or decrease.

- from 50 to 56

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{56 - 50}{50} = \frac{6}{50} = 0.12 = 12\% \end{aligned}$$

The change is a 12% increase.

- from 120 to 72

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{120 - 72}{120} = \frac{48}{120} = 0.40 = 40\% \end{aligned}$$

The change is a 40% decrease.

Find each percent change. Tell whether it is a percent increase or decrease. Round your answer to the nearest percent.

- from 19 to 26
- from 42 to 28
- Find the result when 65 is increased by 40%.
- Find the result when 150 is decreased by 15%.
- Tom sells sunglasses that he buys wholesale for \$2.50 each. He then marks up the price 150%. What is the amount of the markup? What is the selling price?
- The original price of a shirt was \$79.99. It is on sale for \$49.99. What is the percent discount? Round to the nearest tenth.



Solve each equation.

1.  $y - 7 = 2$
2.  $x + 12 = 19$
3.  $-5 + z = 8$
4.  $9x = 72$
5.  $\frac{m}{-8} = -2.5$
6.  $\frac{7}{8}a = 42$
7.  $15 = 3 - 4x$
8.  $\frac{2a}{3} + \frac{1}{5} = \frac{7}{6}$
9.  $8 - (b - 2) = 11$
10.  $-2x + 4 = 5 - 3x$
11.  $3(q - 2) + 2 = 5q - 7 - 2q$
12.  $5z = -3(z + 7)$

Solve for the indicated variable.

13.  $r - 2s = 14$  for  $s$
14.  $V = \frac{1}{3}bh$  for  $b$
15.  $P = 2(\ell + w)$  for  $\ell$

Solve each equation.

16.  $|x - 14| = 21$
17.  $13 = |y + 2| - 3$
18.  $4|z| = 20$
19.  $3|x| + 5 = 8$
20.  $3|g + 1| + 5 = 7$
21.  $|2v| = 6$

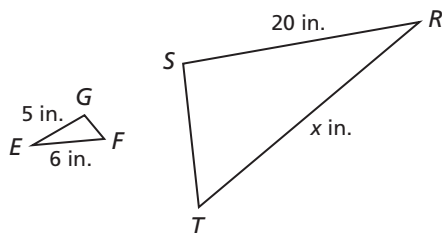
22. A store sells 3 videotapes for \$4.99. Find the unit rate in dollars per videotape. Round to the nearest cent.
23. Twenty-five students use 120 sheets of paper. Find the unit rate in sheets per student.
24. Nutritionists recommend that teenagers consume 1300 milligrams of calcium per day. Use dimensional analysis to convert this rate to grams per year.

Solve each proportion.

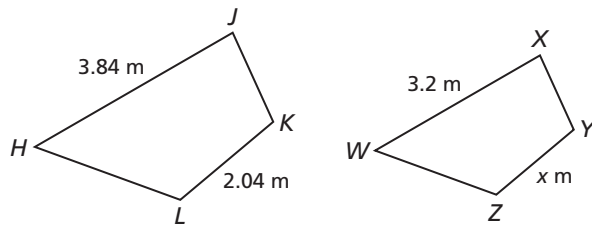
25.  $\frac{5}{4} = \frac{x}{12}$
26.  $\frac{8}{2z} = \frac{15}{60}$
27.  $\frac{x + 10}{10} = \frac{18}{12}$
28. The scale on a map is 1 inch : 500 miles. If two cities are 875 miles apart, how far apart are they on the map?

Find the value of  $x$  in each diagram. Round your answer to the nearest tenth.

29.  $\triangle EFG \sim \triangle RTS$



30.  $HJKL \sim WXYZ$



31. What is 23% of 46?
32. 37.5 is 60% of what number?
33. What percent of 175 is 35?
34. Find the simple interest earned after 5 years on an investment of \$2000 at 3.2% per year.
35. A lunch check is \$27.95. Estimate a 15% tip.

Find each percent change. Tell whether it is a percent increase or decrease.

36. from 180 to 234
37. from 12 to 48
38. from 56 to 21

# COLLEGE ENTRANCE EXAM PRACTICE



## FOCUS ON ACT

The ACT Mathematics Test is one of four tests in the ACT. You have 60 minutes to answer 60 multiple-choice questions. The questions cover material typically taught through the end of eleventh grade. You will need to know some basic formulas.



There is no penalty for incorrect answers on the ACT. If you are unsure of the correct answer, eliminate as many answer choices as possible. Then make your best guess. Be sure you have marked an answer for every question before time runs out.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. At a certain high school, the ratio of left-handed to right-handed basketball players is 1:4. If there are a total of 20 players on the team, how many players are right-handed?
- (A) 1  
(B) 4  
(C) 5  
(D) 12  
(E) 16

2. If  $y - 3 = \frac{2}{5}(x + 1)$ , then  $x = ?$

(F)  $\frac{5(y - 3) - 2}{2}$

(G)  $y - \frac{22}{5}$

(H)  $\frac{2(y - 3)}{5} - 1$

(J)  $\frac{2(y + 1) + 15}{5}$

(K)  $\frac{5}{2}y - 4$

3. What is  $\frac{1}{5}\%$  of 20?

(A) 0.004

(B) 0.04

(C) 0.4

(D) 4

(E) 100

4. If  $x - 3 = 4 - 2(x + 5)$ , then  $x = ?$

(F) -3

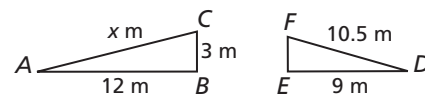
(G) -1

(H) 1

(J)  $\frac{3}{2}$

(K)  $\frac{11}{3}$

5. If  $\triangle ABC \sim \triangle DEF$ , what is the length of  $\overline{AC}$ ?



(A) 2.6 meters

(B) 3.5 meters

(C) 7 meters

(D) 14 meters

(E) 15 meters

6. A movie theater makes 30% of its revenue from concession sales. If concession sales were \$174,000, what was the total revenue?

(F) \$52,200

(G) \$121,800

(H) \$248,570

(J) \$580,000

(K) \$746,000



## Multiple Choice: Eliminate Answer Choices

You can answer some problems without doing many calculations. Use logic to eliminate answer choices and save time.

### EXAMPLE 1

Which number is the square of 123,765?

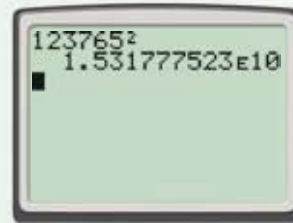
- (A) 15,317,775,225      (C) 15,317,775,230  
 (B) 15,317,775,233      (D) 15,317,775,227

*Your calculator will not help you on this question. Due to rounding, any of the answer choices are possible.*

*But you can use this fact to eliminate three of the answer choices:*

*The square of any number ending in 5 is also a number ending in 5.*

The only answer choice that ends in 5 is A, 15,317,775,225.



$$\begin{array}{r} \square \square \square \square 5 \\ \times \square \square \square \square 5 \\ \hline \square \square \square \square 5 \end{array}$$

### EXAMPLE 2

What is a possible area of the wooden triangle shown?

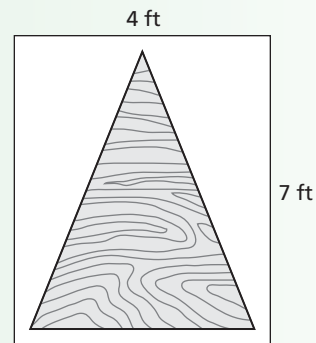
- (F) 11 square feet      (H) 14 square feet  
 (G) 20 square feet      (J) 24 square feet

*The triangle is inside a rectangle with an area of  $7 \times 4 = 28$  square feet.*

*If the triangle had the same base and height as the rectangle, its area would be half the area of the rectangle, 14 square feet.*

*However, the triangle fits inside the rectangle, so its area must be less than 14 square feet.*

The only answer choice that is less than 14 square feet is F, 11 square feet.





Try to eliminate unreasonable answer choices. Some choices may be too great or too small, have incorrect units, or not be divisible by a necessary number.

Read each test item and answer the questions that follow.

#### Item A

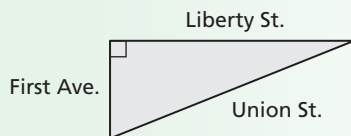
The top speed of a three-toed sloth is 0.12 miles per hour. About how many feet can a sloth travel in an hour?

- A 0.12 feet                       C 2.27 feet  
 B 600 feet                          D 7500 inches

1. Are there any answer choices you can eliminate immediately? If so, which choices and why?
2. Describe how you can use estimation to find the correct answer.

#### Item B

A city park is shaped like a triangle. The Liberty Street side of the park is 120 feet long, and the First Avenue side is 50 feet long.



What is the approximate length of the side of the park that faces Union Street?

- F 25 feet                             H 65 feet  
 G 110 inches                        J 130 feet

3. Can any of the answer choices be eliminated immediately? If so, which choices and why?
4. Are there any properties you can use to solve this problem? If so, what are they?
5. Describe how to find the correct answer without doing any calculations.

#### Item C

Approximately how long will the average 18-year-old have slept in his lifetime?

- A 6 weeks                             C 6 years  
 B 6 months                          D 6 decades

6. Which answer choice can be eliminated immediately? Why?
7. Explain how to use mental math to solve this problem.

#### Item D

Sheila's paychecks for February and March were equal. If she worked every day during both months, for which month was her daily pay lower?

- F February  
 G March  
 H Her daily pay did not change.  
 J Cannot be determined

8. What do you need to know to solve this problem?
9. Describe how you can find the correct answer.

#### Item E

Greg tripled the number of baseball cards he had last week. Which of these could be the number of cards Greg has now?

- A 100                                     C 150  
 B 200                                     D 250

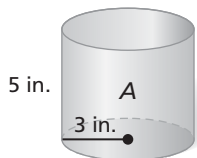
10. The number of cards that Greg has now must be divisible by what number? How can you tell if a number is divisible by this number?
11. Describe how to find the answer to this problem.



## CUMULATIVE ASSESSMENT, CHAPTERS 1–2

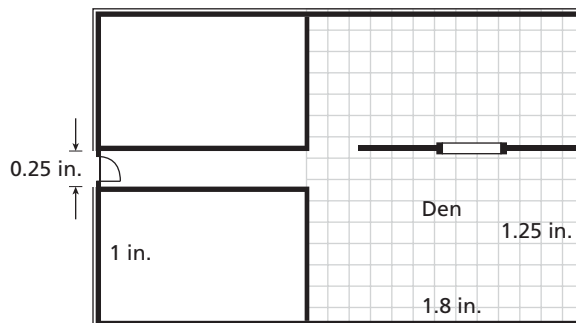
### Multiple Choice

- What operation does  $\diamond$  represent if  $x \diamond 2.2 = 4.5$  when  $x = 9.9$ ?
  - Addition
  - Subtraction
  - Multiplication
  - Division
- A couple earns \$4819.25 a month. They pay 9.5% of their monthly income as the monthly payment on their car. To the nearest dollar, how much does the couple pay for their monthly car payment?
  - \$458
  - \$507
  - \$4578
  - \$4810
- Every dimension of cylinder *A* is multiplied by 4 to make cylinder *B*. What is the ratio of the volume of cylinder *A* to the volume of cylinder *B*?



- $\frac{1}{64}$
  - $\frac{1}{16}$
  - $\frac{1}{4}$
  - $\frac{1}{3}$
- A clock loses 5 minutes every day. How much time will it lose in 2 hours?
    - 0.417 second
    - 25 seconds
    - 240 seconds
    - 600 seconds
  - A statue is 8 feet tall. The display case for a model of the statue is 18 inches tall. Which scale allows for the tallest model of the statue that will fit in the display case?
    - 1 inch:2 inches
    - 1 inch:7 inches
    - 1 inch:5 inches
    - 1 inch:10 inches
  - What is the value of  $-|6^2|$ ?
    - 36
    - 12
    - 8
    - 3

- Mr. Phillips wants to install hardwood flooring in his den. The flooring costs \$25.86 per square yard. The blueprint below shows his house. What other information do you need in order to find the total cost of the flooring?



- The lengths and widths of the adjoining rooms in the blueprint
  - The total area of the blueprint
  - The scale of inches in the blueprint to yards in the house
  - The width of the den
- What value of  $n$  makes the equation below have no solution?
 
$$2x + 2 = nx - 3$$
    - 2
    - 0
    - 2
    - 3
  - Which of the equations below represents the second step of the solution process?
 

Step 1:  $3(5x - 2) + 27 = -24$

Step 2:

Step 3:  $15x + 21 = -24$

Step 4:  $15x = -45$

Step 5:  $x = -3$

    - $3(5x + 27) - 2 = -24$
    - $3(5x + 25) = -24$
    - $15x - 2 + 27 = -24$
    - $15x - 6 + 27 = -24$



If you are stuck on a problem, skip it and come back later. Another problem might remind you of something that will help. If you feel yourself become tense, take a few deep breaths to relax.

10. Cass drove 3 miles to school, and then she drove  $m$  miles to a friend's house. The total mileage for these two trips was 8 miles. Which equation CANNOT be used to determine the number of miles Cass drove?

- (F)  $3 + m = 8$
- (G)  $3 - m = 8$
- (H)  $8 - 3 = m$
- (J)  $8 - m = 3$

11. If  $\frac{20}{x} = \frac{4}{x-5}$ , which of the following is a true statement?

- (A)  $x(x-5) = 80$
- (B)  $20x = 4(x-5)$
- (C)  $20(x-5) = 4x$
- (D)  $24 = 2x - 5$

### Gridded Response

12. Four times a number is two less than six times the same number minus ten. What is the number?
13. Melissa invested her savings in a retirement account that pays simple interest. A portion of her account record is shown below. What is the interest rate on Melissa's account? Write your answer as a decimal.

Date	Transaction	Amount	Balance
8/1	Beginning deposit	\$6000.00	\$6000.00
8/31	Interest payment	\$192.00	\$6192.00
9/1	Withdrawal	\$1000.00	\$5192.00
9/30	Interest payment	\$166.14	\$5358.14

14. At 2:45 P.M. you are 112 miles from Dallas. You want to be in Dallas at 4:30 P.M. What is the average number of miles per hour you must travel to be on time?
15. A cyclist travels 45 miles in 4 hours. How many feet does she travel in one second?
16. A bike rental shop charges a one-time charge of \$8 plus an hourly fee to rent a bike. Dan paid \$24.50 to rent a bike for  $5\frac{1}{2}$  hours. Find the bike shop's hourly fee in dollars.

### Short Response

17. Alex buys 5 calendars to give as gifts. Each calendar has the same price. When the cashier rings up Alex's calendars, the total cost before tax is \$58.75.
- a. Write and solve an equation to find the cost of each calendar.
  - b. The total cost of Alex's calendars after tax is \$63.45. Find the percent sales tax. Show your work and explain in words how you found your answer.
  - c. Alex's friend Keisha buys some calendars for the same price. She uses her 15% discount card. The total cost before tax is \$39.95. How many calendars did Keisha buy? Show your work and explain in words how you found your answer.
18. Is the percent increase from 50 to 100 the same as the percent decrease from 100 to 50? Explain your reasoning in words.

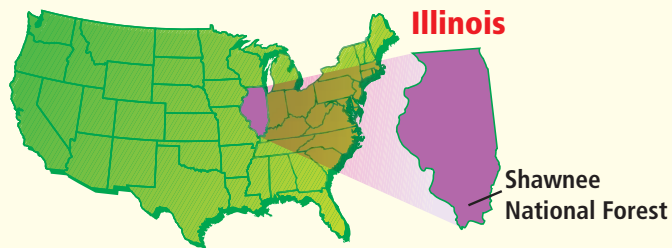
### Extended Response

19. Korena is putting a decorative border around her rectangular flower garden. The total perimeter of the garden is 200 feet.
- a. Draw three different rectangles that could represent Korena's flower garden. Label the dimensions of your rectangles.
  - b. Use the table to show the lengths and widths of five different rectangles that could represent Korena's flower garden. Do not use any of your rectangles from part a.

Possible Dimensions of Korena's Garden		
Length ( $\ell$ )	Width ( $w$ )	Perimeter ( $P$ )
■	■	■
■	■	■
■	■	■
■	■	■
■	■	■

- c. The length of Korena's garden is 4 times its width. Explain how to use the perimeter formula  $P = 2\ell + 2w$  to find the dimensions of Korena's garden.
- d. Find the dimensions of Korena's garden.





## ★ Bullfrogs

The bullfrog is the largest frog in Illinois. Adult bullfrogs are usually between  $3\frac{1}{2}$  and 6 inches in length, but some have been known to grow even larger. The largest recorded bullfrog found in Illinois was 8 inches long and weighed more than a pound!



Choose one or more strategies to solve each problem.

1. A bullfrog jumping up a hill is 8 feet from the top. Each time it jumps, it moves forward 18 inches. However, mud on the hill causes it to slide backward 2 inches after each jump. How many jumps will it take the frog to reach the top of the hill?
2. A certain pond is home to 120 bullfrog tadpoles. This is 90 more than 3 times the number of adult bullfrogs that live in a second pond. How many adult bullfrogs live in the second pond?

For 3–5, use the table.

3. How far could the largest recorded bullfrog in Illinois jump?
4. If a bullfrog is  $4\frac{1}{2}$  inches long, what is its maximum jumping distance?
5. A certain adult bullfrog can jump  $8\frac{3}{4}$  feet. This bullfrog must be at least how long?

Jumping Distances of Bullfrogs	
Length of Bullfrog (in.)	Maximum Jumping Distance (in.)
4	60
5	75
6	90



## ★ The Great Snake Migration

When temperatures drop, large numbers of snakes leave the swamps of southern Illinois. Some travel just a few hundred feet, while others travel several miles, to Snake Road, a 3-mile-long road on the western edge of the Shawnee Forest. In this area, you can see as many as 40 to 50 snakes in a day, including cottonmouth water moccasins, rattlesnakes, and copperheads.

Choose one or more strategies to solve each problem.

- In one hour, a researcher saw six snakes crossing Snake Road—three moccasins, a rattlesnake, a garter snake, and a diamondback water snake, though not in that order. The second snake to cross was a moccasin, and the other two moccasins crossed one after the other. The diamondback crossed before the garter snake. The first snake to cross was not the garter snake or the diamondback. A moccasin crossed last. In what order did the snakes cross Snake Road?



For 2, use the table.

In 2001–2002, several Illinois hospitals were asked how many vials of antivenom they had available to treat snake bites. The results are shown in the table. For example, 37 hospitals reported that they had between 1 and 4 vials of antivenom available.

Antivenom Survey Results						
Vials	0	1–4	5–9	10–14	15–19	20+
Hospitals	■	37	14	6	0	10

- The number of hospitals with no antivenom available to treat snake bites was 6 less than 3 times the number of hospitals that had 1–4 vials. How many hospitals had no antivenom with which to treat snake bites? What percent of the total number of hospitals surveyed does this represent? Round your answer to the nearest percent.

