CHAPTER 11

Exponential and Radical Functions

Chapter Focus

- Graph and use exponential functions to model real-world problems.
- Simplify radical expressions.
- Use radical equations to solve real-world problems.

Population Explosion

The concepts in this chapter are used to model many real-world phenomena, such as changes in wildlife populations.

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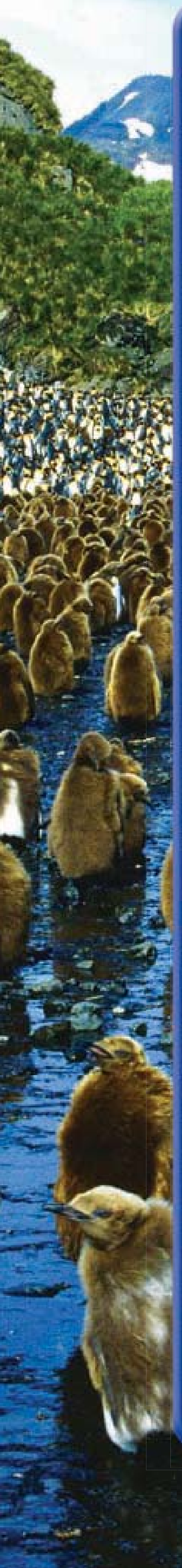


ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. like terms
2. square root
3. domain
4. perfect square
5. exponent

A. the set of second elements of a relation
B. terms that contain the same variables raised to the same powers
C. the set of first elements of a relation
D. a number that tells how many times a base is used as a factor
E. a number whose positive square root is a whole number
F. one of two equal factors of a number

Evaluate Powers

Find the value of each expression.

6. 2^4 **7.** 5^0 **8.** $7 \cdot 3^2$ **9.** $3 \cdot 5^3$
10. 3^5 **11.** $-6^2 + 8^1$ **12.** $40 \cdot 2^3$ **13.** $7^2 \cdot 3^1$

Graph Functions

Graph each function.

14. $y = 8$ **15.** $y = x + 3$ **16.** $y = x^2 - 4$ **17.** $y = x^2 + 2$

Fractions, Decimals, and Percents

Write each percent as a decimal.

18. 50% **19.** 25% **20.** 15.2% **21.** 200%
22. 1.9% **23.** 0.3% **24.** 0.1% **25.** 1.04%

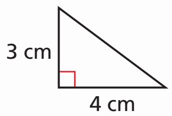
Squares and Square Roots

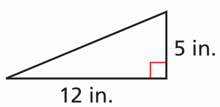
Find each square root.

26. $\sqrt{36}$ **27.** $\sqrt{81}$ **28.** $\sqrt{25}$ **29.** $\sqrt{64}$

Pythagorean Theorem

Find the length of the hypotenuse in each right triangle.

30.


31.


32.


Multiply Monomials and Polynomials

Multiply.

33. $5(2m - 3)$ **34.** $3x(8x + 9)$ **35.** $2t(3t - 1)$ **36.** $4r(4r - 5)$

CHAPTER 11

Study Guide: Preview

Where You've Been

Previously, you

- identified and extended arithmetic sequences.
- identified and graphed linear functions and quadratic functions.
- solved linear and quadratic equations.

In This Chapter

You will study

- another type of sequence—geometric sequences.
- two more types of functions—exponential functions and square-root functions.
- radical equations.

Where You're Going

You can use the skills in this chapter

- to analyze more complicated functions in later math courses, such as Calculus.
- to explore exponential growth and decay models that are used in science.
- to make informed decisions about finances.

Key Vocabulary/Vocabulario

common ratio	razón común
compound interest	interés compuesto
exponential decay	decrecimiento exponencial
exponential function	función exponencial
exponential growth	crecimiento exponencial
extraneous solution	solución extraña
geometric sequence	sucesión geométrica
like radicals	radicales semejantes
radical equation	ecuación radical
radical expression	expresión radical
radicand	radicando
square-root function	función de raíz cuadrada

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What does it mean when several items have something "in common"? What is a ratio? What do you think **common ratio** means?
2. In the division problem $2\overline{)50}$ with quotient 25, the *dividend* is 50. If a *radicand* is similar to a dividend, then what is the **radicand** in $\sqrt{16} = 4$?
3. A square-root sign is also known as a *radical*. Use this knowledge to define **radical expression** and **radical equation**.
4. The root word of *extraneous* is *extra*. *Extraneous* means *irrelevant* or *unrelated*. Use this information to define **extraneous solution**.

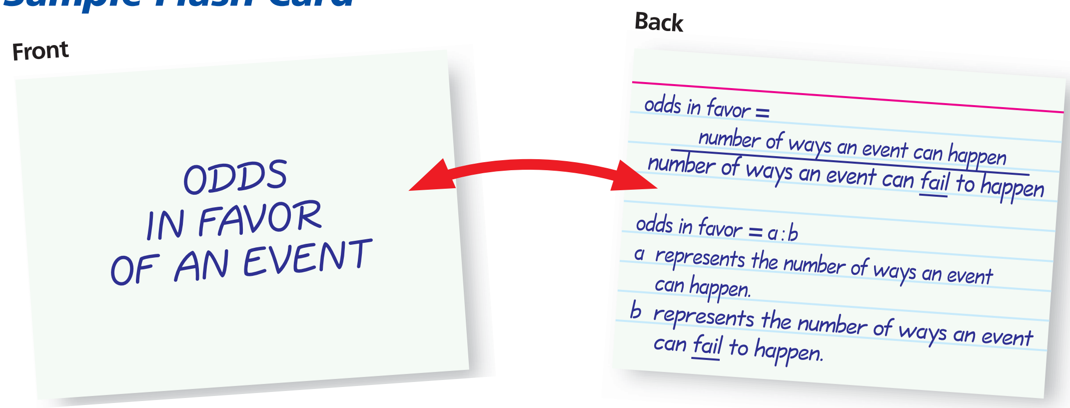
Reading and Writing Math

Study Strategy: Remember Formulas

In math, there are many formulas, properties, and rules that you should commit to memory.

To memorize a formula, create flash cards. Write the name of the formula on one side of a card. Write the formula on the other side of the card. You might also include a diagram or an example if helpful. Study your flash cards on a regular basis.

Sample Flash Card



Knowing when and how to apply a mathematical formula is as important as memorizing the formula itself.

To know what formula to apply, read the problem carefully and look for key words.

From Lesson 10-6

> The **probability** of choosing an ace from a deck of cards is $\frac{1}{13}$.
> What are the **odds of choosing an ace**?

The key words have been highlighted. The probability is given, and you are asked to find the odds. You should use the formula for *odds in favor of an event.*

Try This

Read each problem. Then write the formula(s) needed to solve it. What key words helped you identify the formula?

1. A manufacturer inspects 450 computer chips and finds that 22 are defective. What is the experimental probability that a chip chosen at random is defective?

2. The area of a rectangular pool is 120 square feet. The length is 1 foot less than twice the width. What is the perimeter of the pool?

11-1 Geometric Sequences

Objectives

Recognize and extend geometric sequences.

Find the *n*th term of a geometric sequence.

Vocabulary

geometric sequence

common ratio

Who uses this?

Bungee jumpers can use geometric sequences to calculate how high they will bounce.

The table shows the heights of a bungee jumper's bounces.

Bounce	1	2	3
Height (ft)	200	80	32

The height of the bounces shown in the table form a *geometric sequence*. In a **geometric sequence**, the ratio of successive terms is the same number r, called the **common ratio**.

Writing Math

The variable a is often used to represent terms in a sequence. The variable a_4 (read "a sub 4") is the fourth term in a sequence.

Geometric sequences can be thought of as functions. The term number, or position in the sequence, is the input, and the term itself is the output.

1	2	3	4	← Position
↓	↓	↓	↓	
3	6	12	24	← Term
a_1	a_2	a_3	a_4	

To find a term in a geometric sequence, multiply the previous term by r.

Finding a Term of a Geometric Sequence

The nth term of a geometric sequence with common ratio r is

$$a_n = a_{n-1}r$$

EXAMPLE 1 Extending Geometric Sequences

Find the next three terms in each geometric sequence.

A 1, 3, 9, 27, ...

Step 1 Find the value of r by dividing each term by the one before it.

1 3 9 27

$\frac{3}{1} = 3$ $\frac{9}{3} = 3$ $\frac{27}{9} = 3$ ← The value of r is 3.

Step 2 Multiply each term by 3 to find the next three terms.

27 81 243 729

$\times 3$ $\times 3$ $\times 3$ $a_n = a_{n-1}r$

The next three terms are 81, 243, and 729.

Helpful Hint

When the terms in a geometric sequence alternate between positive and negative, the value of r is negative.

B $-16, 4, -1, \frac{1}{4}, \ldots$

Step 1 Find the value of r by dividing each term by the one before it.

$$-16 \quad 4 \quad -1 \quad \frac{1}{4}$$

$$\frac{4}{-16} = -\frac{1}{4} \qquad \frac{-1}{4} = -\frac{1}{4} \qquad \frac{\frac{1}{4}}{-1} = -\frac{1}{4}$$

← The value of r is $-\frac{1}{4}$.

Step 2 Multiply each term by $-\frac{1}{4}$ to find the next three terms.

$$\frac{1}{4} \quad -\frac{1}{16} \quad \frac{1}{64} \quad -\frac{1}{256}$$

$$\times\left(-\tfrac{1}{4}\right) \qquad \times\left(-\tfrac{1}{4}\right) \qquad \times\left(-\tfrac{1}{4}\right) \qquad a_n = a_{n-1}r$$

The next three terms are $-\frac{1}{16}$, $\frac{1}{64}$, and $-\frac{1}{256}$.

CHECK IT OUT! **Find the next three terms in each geometric sequence.**

1a. 5, −10, 20, −40, … **1b.** 512, 384, 288, …

To find the output a_n of a geometric sequence when n is a large number, you need an equation, or function rule.

The pattern in the table shows that to get the nth term, multiply the first term by the common ratio raised to the power $n - 1$.

Words	Numbers	Algebra
1st term	3	a_1
2nd term	$3 \cdot 2^1 = 6$	$a_1 \cdot r^1$
3rd term	$3 \cdot 2^2 = 12$	$a_1 \cdot r^2$
4th term	$3 \cdot 2^3 = 24$	$a_1 \cdot r^3$
nth term	$3 \cdot 2^{n-1}$	$a_1 \cdot r^{n-1}$

If the first term of a geometric sequence is a_1, the nth term is a_n, and the common ratio is r, then

$$a_n = a_1 r^{n-1}$$

nth term 1st term Common ratio

EXAMPLE 2

Finding the *n*th Term of a Geometric Sequence

A **The first term of a geometric sequence is 128, and the common ratio is 0.5. What is the 10th term of the sequence?**

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_{10} = 128(0.5)^{10-1}$ *Substitute 128 for a_1, 10 for n, and 0.5 for r.*

$= 128(0.5)^9$ *Simplify the exponent.*

$= 0.25$ *Use a calculator.*

B **For a geometric sequence, $a_1 = 8$ and $r = 3$. Find the 5th term of this sequence.**

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_5 = 8(3)^{5-1}$ *Substitute 8 for a_1, 5 for n, and 3 for r.*

$= 8(3)^4$ *Simplify the exponent.*

$= 648$ *Use a calculator.*

Caution!

When writing a function rule for a sequence with a negative common ratio, remember to enclose r in parentheses.
$-2^{12} \neq (-2)^{12}$

C What is the 13th term of the geometric sequence 8, −16, 32, −64, … ?

8 −16 32 −64

$\frac{-16}{8} = -2$ $\frac{32}{-16} = -2$ $\frac{-64}{32} = -2$ *The value of r is −2.*

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_{13} = 8(-2)^{13-1}$ *Substitute 8 for a_1, 13 for n, and −2 for r.*

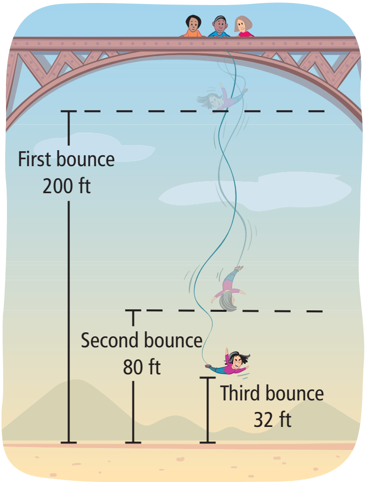
$= 8(-2)^{12}$ *Simplify the exponent.*

$= 32{,}768$ *Use a calculator.*

CHECK IT OUT! **2.** What is the 8th term of the sequence 1000, 500, 250, 125, … ?

EXAMPLE 3 *Sports Application*

A bungee jumper jumps from a bridge. The diagram shows the bungee jumper's height above the ground at the top of each bounce. The heights form a geometric sequence. What is the bungee jumper's height at the top of the 5th bounce?



200 80 32

$\frac{80}{200} = 0.4$ $\frac{32}{80} = 0.4$

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_5 = 200(0.4)^{5-1}$ *Substitute 200 for a_1, 5 for n, and 0.4 for r.*

$= 200(0.4)^4$ *Simplify the exponent.*

$= 5.12$ *Use a calculator.*

The height of the 5th bounce is 5.12 feet.

CHECK IT OUT! **3.** The table shows a car's value for 3 years after it is purchased. The values form a geometric sequence. How much will the car be worth in the 10th year?

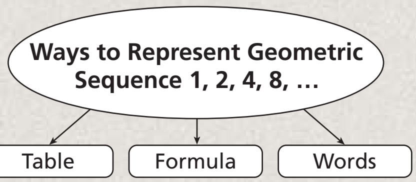
Year	Value ($)
1	10,000
2	8,000
3	6,400

THINK AND DISCUSS

1. How do you determine whether a sequence is geometric?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a way to represent the geometric sequence.



11-1 Exercises

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GUIDED PRACTICE

1. **Vocabulary** What is the *common ratio* of a geometric sequence?

SEE EXAMPLE 1 p. 790

Find the next three terms in each geometric sequence.

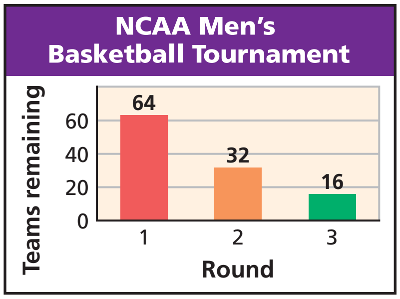
2. 2, 4, 8, 16, …
3. 400, 200, 100, 50, …
4. 4, −12, 36, −108, …

SEE EXAMPLE 2 p. 791

5. The first term of a geometric sequence is 1, and the common ratio is 10. What is the 10th term of the sequence?
6. What is the 11th term of the geometric sequence 3, 6, 12, 24, … ?

SEE EXAMPLE 3 p. 792

7. **Sports** In the NCAA men's basketball tournament, 64 teams compete in round 1. Fewer teams remain in each following round, as shown in the graph, until all but one team have been eliminated. The numbers of teams in each round form a geometric sequence. How many teams compete in round 5?



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–13	1
14–15	2
16	3

Extra Practice

Skills Practice p. S24
Application Practice p. S38

Find the next three terms in each geometric sequence.

8. −2, 10, −50, 250, …
9. 32, 48, 72, 108, …
10. 625, 500, 400, 320, …
11. 6, 42, 294, …
12. 6, −12, 24, −48, …
13. 40, 10, $\frac{5}{2}$, $\frac{5}{8}$, …
14. The first term of a geometric sequence is 18 and the common ratio is 3.5. What is the 5th term of the sequence?
15. What is the 14th term of the geometric sequence 1000, 100, 10, 1, … ?
16. **Physical Science** A ball is dropped from a height of 500 meters. The table shows the height of each bounce, and the heights form a geometric sequence. How high does the ball bounce on the 8th bounce? Round your answer to the nearest tenth of a meter.

Bounce	Height (m)
1	400
2	320
3	256

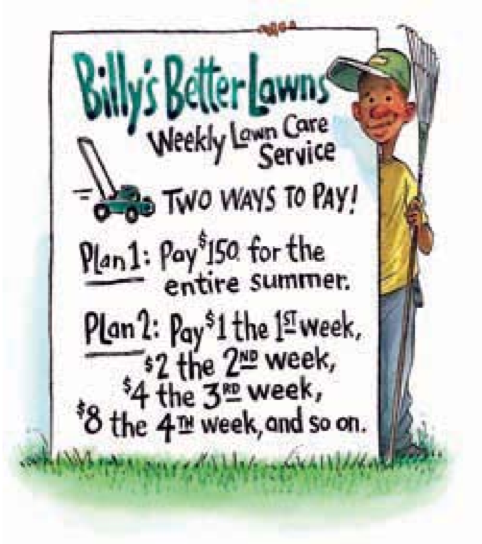
Find the missing term(s) in each geometric sequence.

17. 20, 40, ____, ____, …
18. ____, 6, 18, ____, …
19. 9, 3, 1, ____, …
20. 3, 12, ____, 192, ____, …
21. 7, 1, ____, ____, $\frac{1}{343}$, …
22. ____, 100, 25, ____, $\frac{25}{16}$, …
23. −3, ____, −12, 24, ____, …
24. ____, ____, 1, −3, 9, …
25. 1, 17, 289, ____, …

Determine whether each sequence could be geometric. If so, give the common ratio.

26. 2, 10, 50, 250, …
27. 15, 5, $\frac{5}{3}$, $\frac{5}{9}$, …
28. 6, 18, 24, 38, …
29. 9, 3, −1, −5, …
30. 7, 21, 63, 189, …
31. 4, 1, −2, −4, …

32. **Multi-Step** Billy earns money by mowing lawns for the summer. He offers two payment plans, as shown at right.



   a. Do the payments for plan 2 form a geometric sequence? Explain.
   b. If you were one of Billy's customers, which plan would you choose? (Assume that the summer is 10 weeks long.) Explain your choice.

33. **Measurement** When you fold a piece of paper in half, the thickness of the folded piece is twice the thickness of the original piece. A piece of copy paper is about 0.1 mm thick.
   a. How thick is a piece of copy paper that has been folded in half 7 times?
   b. Suppose that you could fold a piece of copy paper in half 12 times. How thick would it be? Write your answer in centimeters.

List the first four terms of each geometric sequence.

34. $a_1 = 3,\ a_n = 3(2)^{n-1}$
35. $a_1 = -2,\ a_n = -2(4)^{n-1}$
36. $a_1 = 5,\ a_n = 5(-2)^{n-1}$
37. $a_1 = 2,\ a_n = 2(2)^{n-1}$
38. $a_1 = 2,\ a_n = 2(5)^{n-1}$
39. $a_1 = 12,\ a_n = 12\left(\frac{1}{4}\right)^{n-1}$

40. **Critical Thinking** What happens to the terms of a geometric sequence when r is doubled? Use an example to support your answer.

41. **Geometry** The steps below describe how to make a geometric figure by repeating the same process over and over on a smaller and smaller scale.

   Step 1 (stage 0) Draw a large square.
   Step 2 (stage 1) Divide the square into four equal squares.
   Step 3 (stage 2) Divide each small square into four equal squares.
   Step 4 Repeat Step 3 indefinitely.

   a. Draw stages 0, 1, 2, and 3.
   b. How many small squares are in each stage? Organize your data relating stage and number of small squares in a table.
   c. Does the data in part **b** form a geometric sequence? Explain.
   d. Write a rule to find the number of small squares in stage n.

42. **Write About It** Write a series of steps for finding the nth term of a geometric sequence when you are given the first several terms.

MULTI-STEP TEST PREP

43. This problem will prepare you for the Multi-Step Test Prep on page 820.
   a. Three years ago, the annual tuition at a university was \$3000. The following year, the tuition was \$3300, and last year, the tuition was \$3630. If the tuition has continued to grow in the same manner, what is the tuition this year? What do you expect it to be next year?
   b. What is the common ratio?
   c. What would you predict the tuition was 4 years ago? How did you find that value?

TEST PREP

44. Which of the following is a geometric sequence?

Ⓐ $\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

Ⓑ $-2, -6, -10, -14, \ldots$

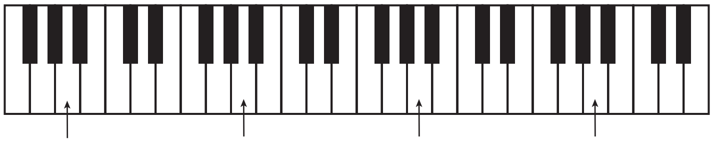
Ⓒ $3, 8, 13, 18, \ldots$

Ⓓ $5, 10, 20, 40, \ldots$

45. Which equation represents the nth term in the geometric sequence $2, -8, 32, -128, \ldots$?

Ⓕ $a_n = (-4)^n$ Ⓖ $a_n = (-4)^{n-1}$ Ⓗ $a_n = 2(-4)^n$ Ⓙ $a_n = 2(-4)^{n-1}$

46. The frequency of a musical note, measured in hertz (Hz), is called its pitch. The pitches of the A keys on a piano form a geometric sequence, as shown.



What is the frequency of A_7?

Ⓐ 880 Hz Ⓑ 1760 Hz Ⓒ 3520 Hz Ⓓ 7040 Hz

CHALLENGE AND EXTEND

Find the next three terms in each geometric sequence.

47. $x, x^2, x^3, \ldots$

48. $2x^2, 6x^3, 18x^4, \ldots$

49. $\frac{1}{y^3}, \frac{1}{y^2}, \frac{1}{y}, \ldots$

50. $\frac{1}{(x+1)^2}, \frac{1}{x+1}, 1, \ldots$

51. The 10th term of a geometric sequence is 0.78125. The common ratio is −0.5. Find the first term of the sequence.

52. The first term of a geometric sequence is 12 and the common ratio is $\frac{1}{2}$. Is 0 a term in this sequence? Explain.

53. A geometric sequence starts with 14 and has a common ration of 0.4. Colin finds that another number in the sequence is 0.057344. Which term in the sequence did Colin find?

54. The first three terms of a sequence are 1, 2, and 4. Susanna said the 8th term of this sequence is 128. Paul said the 8th term is 29. Explain how the students found their answers. Why could these both be considered correct answers?

SPIRAL REVIEW

Solve each inequality and graph the solutions. *(Lesson 3-2)*

55. $b - 4 > 6$

56. $-12 + x \leq -8$

57. $c + \frac{2}{3} < \frac{1}{3}$

Graph the solutions of each linear inequality. *(Lesson 6-5)*

58. $y < 2x - 4$

59. $3x + y > 6$

60. $-y \leq 2x + 1$

Write a function to describe each of the following graphs. *(Lesson 9-4)*

61. The graph of $f(x) = x^2 - 3$ translated 7 units up

62. The graph of $f(x) = 2x^2 + 6$ narrowed and translated 2 units down