

Data Analysis and Probability

10A Data Analysis

- 10-1 Organizing and Displaying Data
- 10-2 Frequency and Histograms
- 10-3 Data Distributions
- Lab Use Technology to Make Graphs
- 10-4 Misleading Graphs and Statistics

10B Probability

- Lab Simulations
- 10-5 Experimental Probability
- Lab Use Random Numbers
- 10-6 Theoretical Probability
- 10-7 Independent and Dependent Events
- Lab Compound Events
- 10-8 Combinations and Permutations
- Ext Matrices

Chapter Focus

- Organize and display data to answer questions.
- Use descriptive statistics to summarize data sets.
- Understand experimental probability and theoretical probability.
- Use probability to make appropriate predictions.

You're the Designer

Research studies are designed to gather and analyze data in order to answer questions. The results are displayed in tables and graphs.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

- | | |
|--------------------|---|
| 1. difference | A. the result of an addition |
| 2. factor | B. a whole number that is multiplied by another whole number to get a product |
| 3. natural numbers | C. numbers that can be expressed in the form $\frac{a}{b}$, where a and b are both integers and $b \neq 0$ |
| 4. ratio | D. the result of a subtraction |
| 5. sum | E. a comparison of two quantities by division |
| | F. the counting numbers: 1, 2, 3, ... |

Solve Proportions

Solve each proportion.

6. $\frac{3}{4} = \frac{x}{12}$	7. $\frac{15}{9} = \frac{3}{x}$	8. $\frac{10}{20} = \frac{x}{100}$	9. $\frac{250}{1500} = \frac{x}{100}$
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Compare and Order Real Numbers

Compare. Write $<$, $>$, or $=$.

10. $20 \square 13$	11. $\frac{2}{3} \square \frac{1}{2}$	12. $\frac{3}{4} \square \frac{7}{9}$	13. $0.75 \square \frac{9}{12}$
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Order the numbers from least to greatest.

14. $\frac{1}{2}, \frac{4}{5}, \frac{1}{8}, \frac{3}{4}, \frac{2}{3}$	15. $0.12, \frac{2}{5}, \frac{3}{4}, 0.3, \frac{1}{3}$
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Multiply Decimals

Multiply.

16. 0.25×300	17. 0.5×4000	18. 0.05×200	19. 0.125×9600
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Divide Decimals

Divide.

20. $435 \div 10$	21. $32 \div 100$	22. $777 \div 1000$	23. $295 \div 10,000$
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Fractions, Decimals, and Percents

Write the equivalent decimal.

24. $\frac{3}{5}$	25. 45%	26. $\frac{3}{4}$	27. 8%
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Write the equivalent percent.

28. $\frac{1}{4}$	29. 0.2	30. 0.36	31. $\frac{1}{10}$
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Where You've Been

Previously, you

- read information from tables and graphs.
- added, subtracted, multiplied, and divided real numbers.
- worked with ratios and percents.

In This Chapter

You will study

- how to organize data in tables, graphs, and plots.
- how to find the central tendency of a data set by calculating mean, median, and mode.
- writing experimental and theoretical probability as ratios, percents, and decimals.
- combinations, permutations, and factorials as extensions of multiplication.

Where You're Going

You can use the skills in this chapter

- to present your findings from science laboratory experiments in an appropriate and accurate graphical form.
- to be more informed about statistical information in the news and not to be misled by how it is presented.

Key Vocabulary/Vocabulario

combination	combinación
compound event	suceso compuesto
dependent events	sucesos dependientes
experimental probability	probabilidad experimental
frequency	frecuencia
independent events	sucesos independientes
median	mediana
outlier	valor extremo
permutation	permutación
probability	probabilidad
quartile	cuartil
theoretical probability	probabilidad teórica

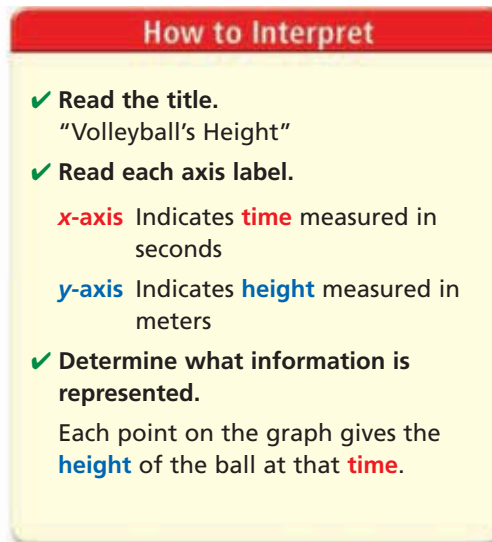
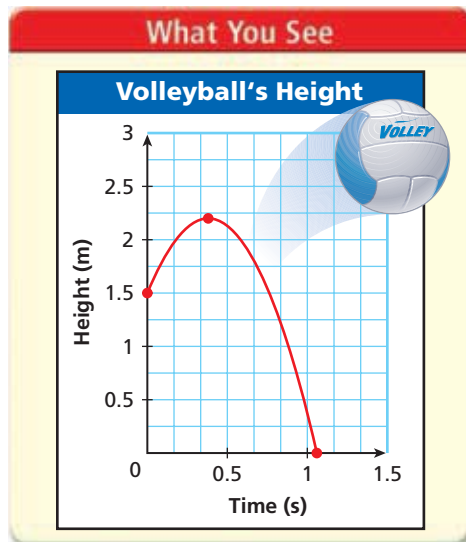
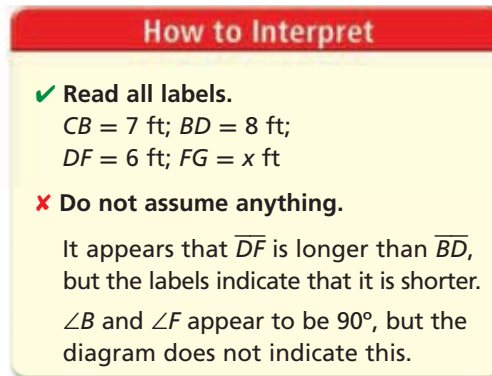
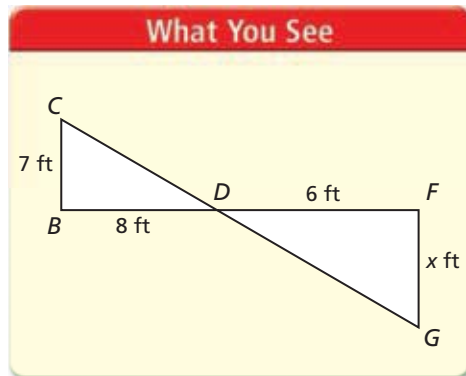
Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The *median strip* is the middle region that divides a highway in half. Use this knowledge to define **median** as it relates to a set of data.
2. The word **quartile** starts with the prefix *quart-*. What are some other words that start with the prefix *quart-*? What do they all have in common?
3. **Probability** is the chance something will happen. Based on your understanding of the words *experiment* and *theory*, compare and contrast the terms **experimental probability** and **theoretical probability**.
4. A *compound word* is made up of two or more words. What do you think makes up a **compound event**?

Reading Strategy: Read and Interpret Graphics

Knowing how to interpret figures, diagrams, charts, and graphs will help you gather the information you need to get the correct answer.



Try This

Look up each exercise in the text and answer the corresponding questions.

1. Lesson 2-7 Exercise 42: What is the title of the table? What is the record for the 200-meter run?
2. Lesson 7-8 Exercise 69: What does x represent in the diagram? Make a list of the information you know from the diagram.
3. Lesson 8-3 Exercise 51: What is represented by the y -axis? What is the y -value when x equals 3.5? Is the graph linear? How do you know?
4. Lesson 9-8 Exercise 33: What does the dashed line represent? What facts about the parallelogram are given from the diagram?



Bar and Circle Graphs

Data displayed in bar graphs and circle graphs can be used to solve equations. In these problems, parts of the graphs are missing.

Example 1

The top part of this graph was torn off. If Warren received 15% of the votes, how many votes did Adams receive?

Step 1 Find the total number of votes. Let t represent the total.

$$\begin{array}{ccccccc} 15\% & \text{of} & \text{the total votes} & \text{is} & 42 \text{ votes.} \\ 0.15 & \cdot & t & = & 42 \end{array}$$

$$0.15t = 42$$

$$t = 280 \text{ votes}$$

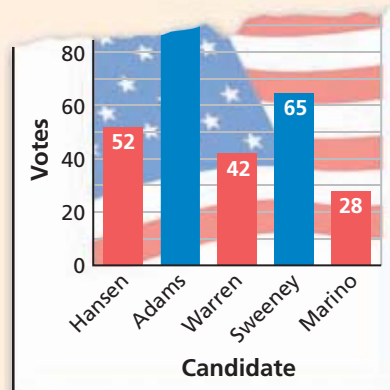
Step 2 Find the number of votes Adams received.

Let a represent the number of votes received by Adams.
Let h , w , s , and m represent the number of votes received by Hansen, Warren, Sweeney, and Marino.

$$\begin{array}{r} t = a + h + w + s + m \\ 280 = a + 52 + 42 + 65 + 28 \\ 280 = a + 187 \\ \underline{-187} \quad \underline{-187} \\ 93 = a \end{array}$$

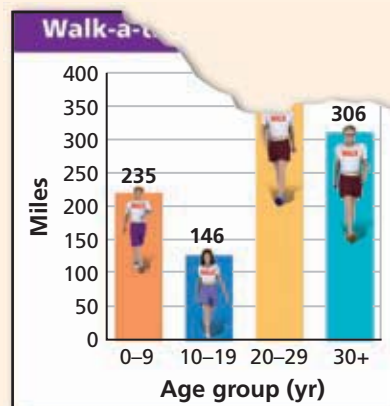
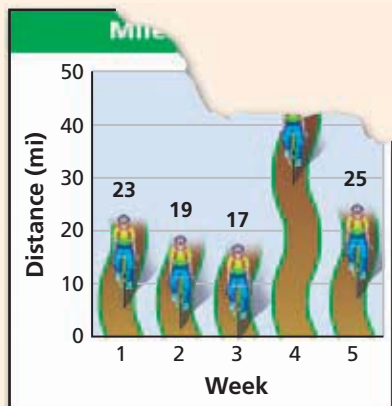
*Substitute the numbers shown on the graph.
Simplify the right side of the equation.
Subtract 187 from both sides.*

Adams received 93 votes.



Try This

- The missing bar is twice as tall as the bar for week 2. How many total miles did Kim bike in these five weeks?
- People aged 20–29 years walked 275 more miles than the oldest age group. Find the total miles walked by all age groups.



Remember that a circle graph represents all the data in a data set. The percent represented by each section is a part of the whole data set, so the sum of all the percents must be 100%.

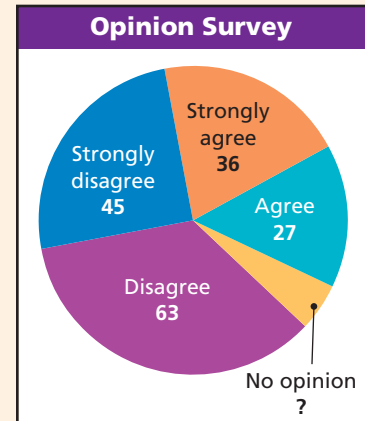
Example 2

A survey asked people in a neighborhood to agree or disagree with the following statement:

“We need a traffic light at Jefferson Avenue and Third Street.”

If 35% of the people disagreed with the statement, how many people had no opinion?

The number of people who answered “no opinion” is missing from the graph.



Step 1 Find the total number of people who answered the survey.

Let t represent the total number of people.

$$0.35 \text{ of } t \text{ is } 63 \text{ people.}$$

$$0.35 \cdot t = 63$$

$$0.35t = 63$$

$$t = 180 \text{ people}$$

Step 2 Find the number of people who answered “no opinion.”

Let n represent the number of “no opinion” answers. Let d , s , g , and a represent the number of “disagree,” “strongly disagree,” “strongly agree,” and “agree” answers.

$$t = n + d + s + g + a$$

$$180 = n + 63 + 45 + 36 + 27$$

$$180 = n + 171$$

$$\underline{-171} \quad \underline{-171}$$

$$9 = n$$

Substitute the numbers shown on the graph.

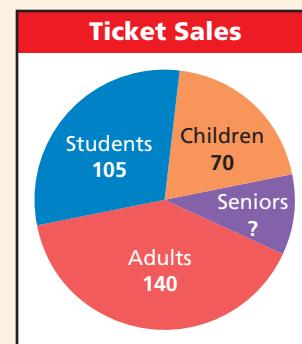
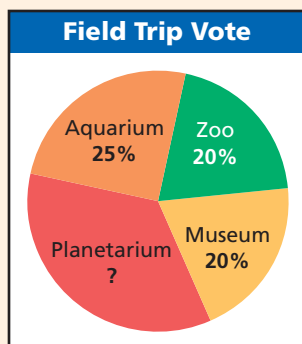
Simplify the right side of the equation.

Subtract 171 from both sides.

There were 9 people who had no opinion.

Try This

- The students in a junior high school voted on their choice for a field trip. Sixteen students voted for the natural history museum. How many students voted for the winning choice?
- At the fall dance recital, 40% of the tickets were sold to adults. What percent of the sales were to seniors?



10-1

Organizing and Displaying Data



Objectives

Organize data in tables and graphs.

Choose a table or graph to display data.

Vocabulary

bar graph
line graph
circle graph

Who uses this?

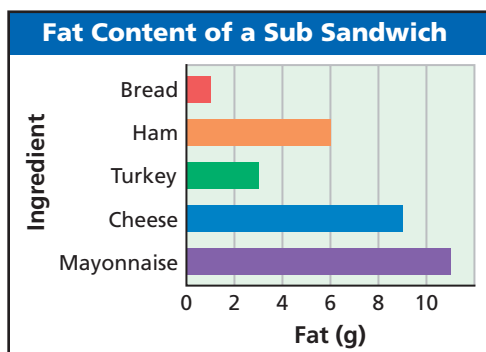
Nutritionists can display health information about food in bar graphs.

Bar graphs, line graphs, and circle graphs can be used to present data in a visual way.

A **bar graph** displays data with vertical or horizontal bars. Bar graphs are a good way to display data that can be organized into categories. Using a bar graph, you can quickly compare the categories.

EXAMPLE 1 Reading and Interpreting Bar Graphs

Use the graph to answer each question.



- A** Which ingredient contains the most fat?
mayonnaise *The bar for mayonnaise is the longest.*
- B** How many more grams of fat are in ham than in turkey?
 $6 - 3 = 3$ *There are 6 grams of fat in ham and 3 grams of fat in turkey.*
- C** How many total fat grams are in this sandwich?
 $1 + 6 + 3 + 9 + 11 = 30$ *Add the number of fat grams for each ingredient.*
- D** What percent of the total fat grams in this sandwich are from turkey?
 $\frac{3}{30} = \frac{1}{10} = 10\%$ *Out of 30 total fat grams, 3 fat grams are from turkey.*



Use the graph to answer each question.

- 1a. Which ingredient contains the least amount of fat?
1b. Which ingredients contain at least 8 grams of fat?

A double-bar graph can be used to compare two data sets. A double-bar graph has a key to distinguish between the two sets of data.

EXAMPLE 2 Reading and Interpreting Double Bar Graphs

Use the graph to answer each question.

- A** In which year did State College have the greatest average attendance for basketball?

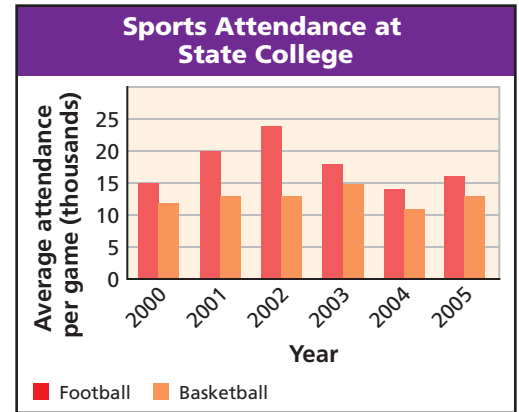
2003

Find the tallest orange bar.

- B** On average, how many more people attended a football game than a basketball game in 2001?

$20,000 - 13,000 = 7,000$

Find the height of each bar for 2001 and subtract.

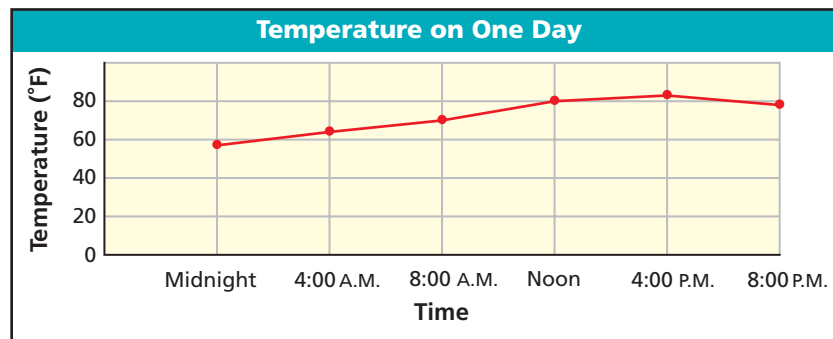


2. Use the graph to determine which years had the same average basketball attendance. What was the average attendance for those years?

A **line graph** displays data using line segments. Line graphs are a good way to display data that changes over a period of time.

EXAMPLE 3 Reading and Interpreting Line Graphs

Use the graph to answer each question.



- A** At what time was the temperature the warmest?

4:00 P.M.

Identify the highest point.

- B** During which 4-hour time period did the temperature increase the most?

From 8:00 A.M. to noon

Look for the segment with the greatest positive slope.

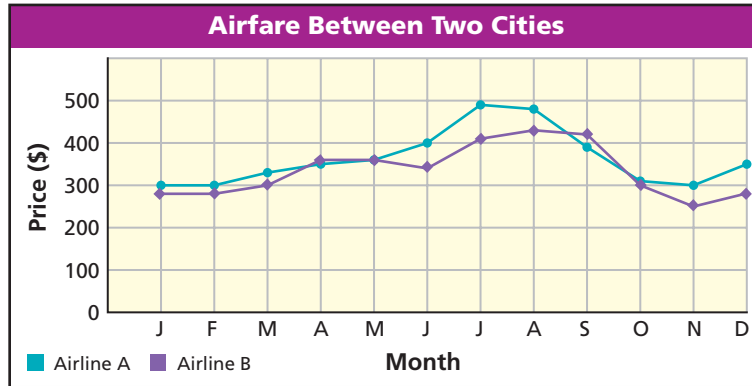


3. Use the graph to estimate the difference in temperature between 4:00 A.M. and noon.

A double-line graph can be used to compare how two related data sets change over time. A double-line graph has a key to distinguish between the two sets of data.

EXAMPLE 4 Reading and Interpreting Double-Line Graphs

Use the graph to answer each question.



- A** In which month(s) did airline B charge more than airline A?
 April and September *Identify the points when the purple line is higher than the blue line.*
- B** During which month(s) did the airlines charge the same airfare?
 May *Look for the point where the data points overlap.*



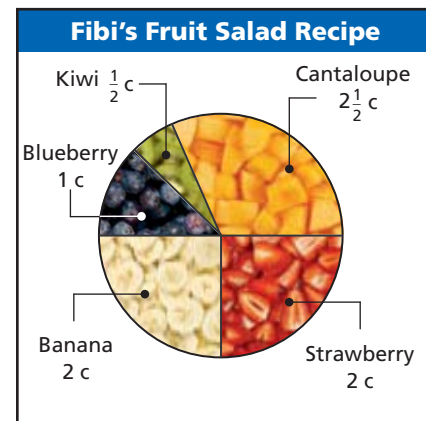
4. Use the graph to describe the general trend of the data.

A **circle graph** shows parts of a whole. The entire circle represents 100% of the data and each sector represents a percent of the total. Circle graphs are good for comparing each category of data to the whole set.

EXAMPLE 5 Reading and Interpreting Circle Graphs

Use the graph to answer each question.

- A** Which two fruits together make up half of the fruit salad?
 bananas and strawberries
Look for two fruits that together make up half of the circle.
- B** Which fruit is used more than any other?
 cantaloupe
Look for the largest sector of the graph.



5. Use the graph to determine what percent of the fruit salad is cantaloupe.

Reading Math

The sections of a circle graph are called **sectors**.

EXAMPLE

6

Choosing and Creating an Appropriate Display

Use the given data to make a graph. Explain why you chose that type of graph.

A

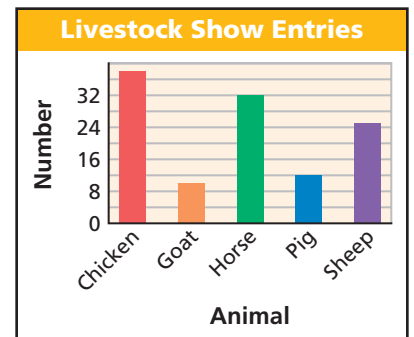
Livestock Show Entries	
Animal	Number
Chicken	38
Goat	10
Horse	32
Pig	12
Sheep	25

A bar graph is appropriate for this data because it will be a good way to compare categories.

Step 1 Determine an appropriate scale and interval. The scale must include all of the data values. The scale is separated into equal parts, called intervals.

Step 2 Use the data to determine the lengths of the bars. Draw bars of equal width. The bars should not touch.

Step 3 Title the graph and label the horizontal and vertical scales.



B

Division of Crops	
Crop	Area (acres)
Corn	70
Fallow	50
Mixed vegetables	10
Soybeans	40
Wheat	30

A circle graph is appropriate for this data because it shows categories as parts of a whole.

Step 1 Calculate the percent of the total represented by each category.

$$\text{Corn: } \frac{70}{200} = 0.35 = 35\%$$

$$\text{Soybeans: } \frac{40}{200} = 0.2 = 20\%$$

$$\text{Fallow: } \frac{50}{200} = 0.25 = 25\%$$

$$\text{Wheat: } \frac{30}{200} = 0.15 = 15\%$$

$$\text{Mixed vegetables: } \frac{10}{200} = 0.05 = 5\%$$

Step 2 Find the angle measure for each sector of the graph. Since there are 360° in a circle, multiply each percent by 360° .

$$\text{Corn: } 0.35 \times 360^\circ = 126^\circ$$

$$\text{Fallow: } 0.25 \times 360^\circ = 90^\circ$$

$$\text{Mixed vegetables: } 0.05 \times 360^\circ = 18^\circ$$

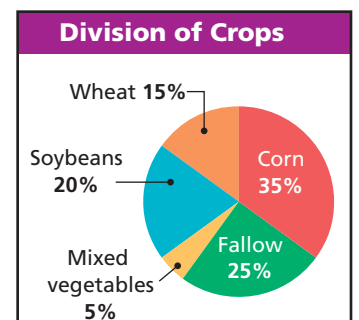
$$\text{Soybeans: } 0.2 \times 360^\circ = 72^\circ$$

$$\text{Wheat: } 0.15 \times 360^\circ = 54^\circ$$

Step 3 Use a compass to draw a circle.

Mark the center and use a straightedge to draw one radius. Then use a protractor to draw each central angle.

Step 4 Title the graph and label each sector.



Use the given data to make a graph. Explain why you chose that type of graph.

C

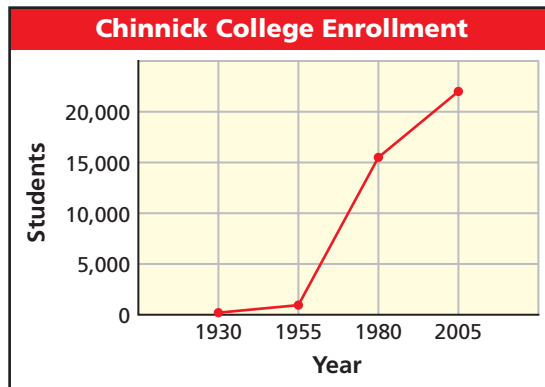
Chinnick College Enrollment	
Year	Students
1930	586
1955	2,361
1980	15,897
2005	21,650

A line graph is appropriate for this data because it will show the change in enrollment over a period of time.

Step 1 Determine the scale and interval for each set of data. Time should be plotted on the horizontal axis because it is independent.

Step 2 Plot a point for each pair of values. Connect the points using line segments.

Step 3 Title the graph and label the horizontal and vertical scales.



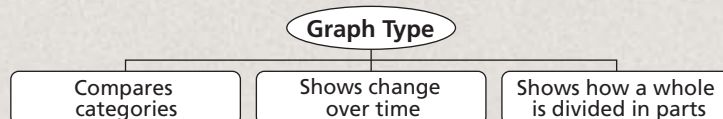
6. Use the given data to make a graph. Explain why you chose that type of graph.

The data below shows how Vera spends her time during a typical 5-day week during the school year.

Vera's Schedule						
Activity	Sleeping	Eating	School	Sports	Homework	Other
Time (h)	45	8	30	10	10	17

THINK AND DISCUSS

1. What are some comparisons you can make by looking at a bar graph?
2. Name some key components of a good line graph.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, tell which kind of graph is described.



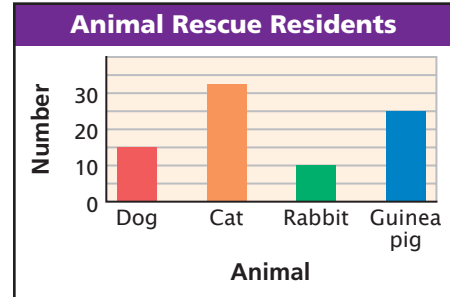
GUIDED PRACTICE

Vocabulary Use the vocabulary from this lesson to answer the following questions.

1. In a *circle graph*, what does each sector represent?
2. In a *line graph*, how does the slope of a line segment relate to the rate of change?

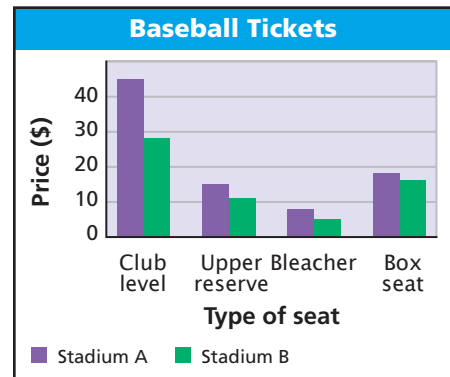
SEE EXAMPLE 1
 p. 700

- Use the bar graph for Exercises 3 and 4.
3. Estimate the total number of animals at the shelter.
 4. There are 3 times as many ___?___ as ___?___ at the animal shelter.



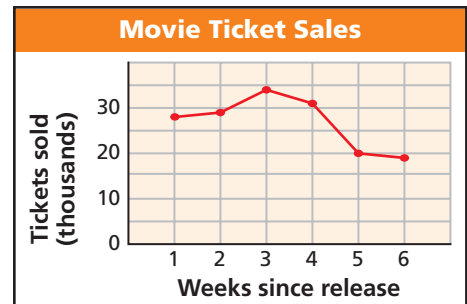
SEE EXAMPLE 2
 p. 701

- Use the double-bar graph for Exercises 5–7.
5. About how much more is a club level seat at stadium A than at stadium B?
 6. Which type of seat is the closest in price at the two stadiums?
 7. Describe one relationship between the ticket prices at stadium A and stadium B.



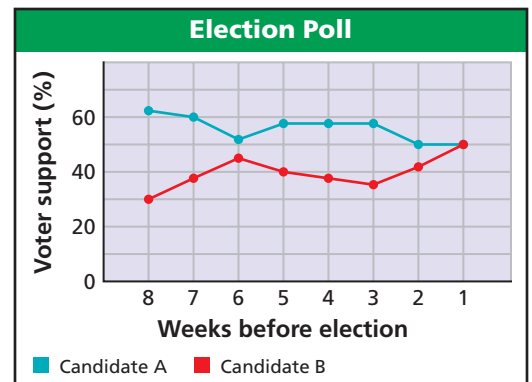
SEE EXAMPLE 3
 p. 701

- Use the line graph for Exercises 8 and 9.
8. Estimate the number of tickets sold during the week of the greatest sales.
 9. Which one-week period of time saw the greatest change in sales?



SEE EXAMPLE 4
 p. 702

- Use the double-line graph for Exercises 10–12.
10. When was the support for the two candidates closest?
 11. Estimate the difference in voter support for the two candidates five weeks before the election.
 12. Describe the general trend(s) of voter support for the two candidates.

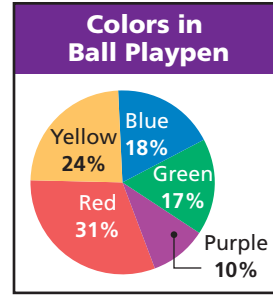


SEE EXAMPLE 5

p. 702

Use the circle graph for Exercises 13–15.

13. Which color is least represented in the ball playpen?
14. There are 500 balls in the playpen. How many are yellow?
15. Which two colors are approximately equally represented in the ball playpen?



SEE EXAMPLE 6

p. 703

16. The table shows the breakdown of Karim's monthly budget of \$100. Use the given data to make a graph. Explain why you chose that type of graph.

Item/Activity	Spending (\$)
Clothing	35
Food	25
Entertainment	25
Other	15

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–18	1
19–21	2
22–23	3
24–26	4
27–28	5
29	6

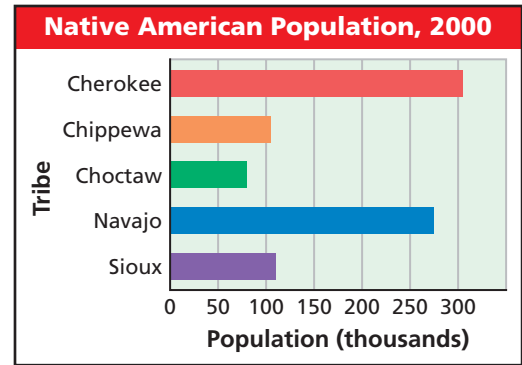
Extra Practice

Skills Practice p. S22

Application Practice p. S37

Use the bar graph for Exercises 17 and 18.

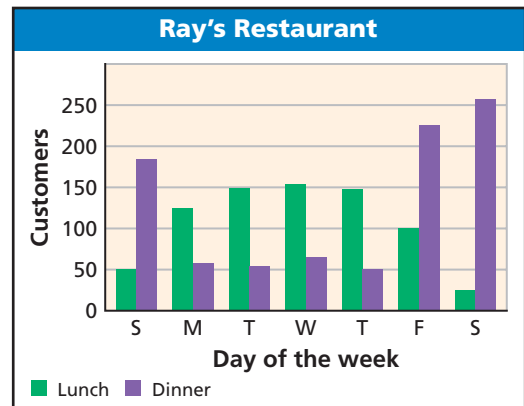
17. Estimate the difference in population between the tribes with the largest and the smallest population.
18. Approximately what percent of the total population shown in the table is Cherokee?



Source: U.S. Census Bureau

Use the double bar graph for Exercises 19–21.

19. On what day did Ray do the most overall business?
20. On what day did Ray have the busiest lunch?
21. On Sunday, about how many times as great was the number of dinner customers as the number of lunch customers?



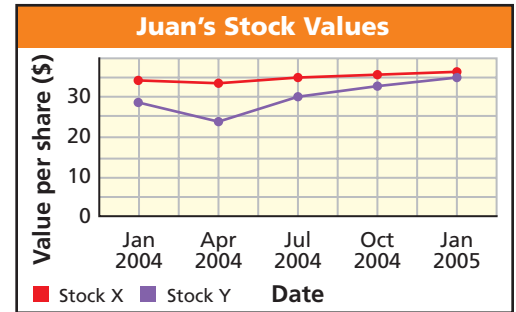
Use the line graph for Exercises 22 and 23.

22. Between which two games did Marlon's score increase the most?
23. Between which three games did Marlon's score increase by about the same amount?



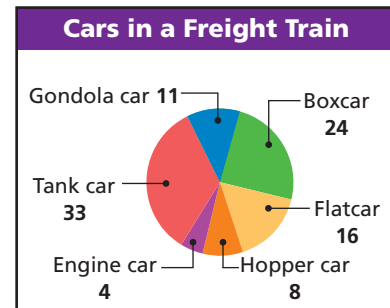
Use the double-line graph for Exercises 24–26.

24. What was the average value per share of Juan's two stocks in July 2004?
25. Which stock's value changed the most over any time period?
26. Describe the trend of the values of both stocks.



Use the circle graph for Exercises 27 and 28.

27. About what percent of the total number of cars are hopper cars?
28. About what percent of the total number of cars are gondola or tank cars?



29. The table shows the weight of twin babies at various times from birth to four weeks old. Use the given data to make a graph. Explain why you chose that type of graph.

Age (days)	Boy's Weight (lb)	Girl's Weight (lb)
1	5.3	5.7
3	5.0	5.2
7	5.5	5.9
14	6.2	6.8
28	7.9	7.5

Write *bar*, *double-bar*, *line*, *double-line*, or *circle* to indicate the type of graph that would best display the data described.

30. attendance at a carnival each year over a ten-year period
31. attendance at two different carnivals each year over a ten-year period
32. attendance at five different carnivals during the same year
33. attendance at a carnival by age group as it relates to total attendance
34. **Critical Thinking** Give an example of real-world data that would best be displayed by each type of graph: line graph, circle graph, double-bar graph.

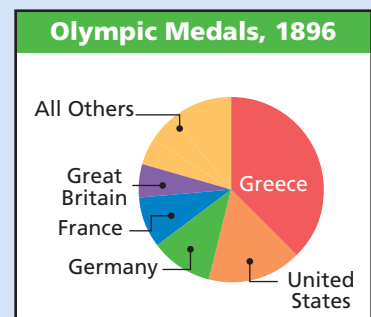
**MULTI-STEP
TEST PREP**



35. This problem will prepare you for the Multi-Step Test Prep on page 734.

The first modern Olympic Games took place in 1896 in Athens, Greece. The circle graph shows the total number of medals won by several countries at the Olympic Games of 1896.

- a. Which country won the most medals? Estimate the percent of the medals won by this country.
- b. Which country won the second most medals? Estimate the percent of the medals won by this country.





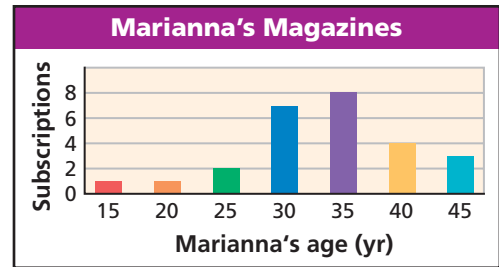
36. **Write About It** Explain how you could use a line graph to make predictions.



37. Which type of graph would best display the contribution of each high school basketball player to the team, in terms of points scored?
 (A) Bar graph (B) Line graph (C) Double-line graph (D) Circle graph

38. At what age did Marianna have 75% more magazine subscriptions than she did at age 40?

- (F) 25
- (G) 30
- (H) 35
- (J) 45



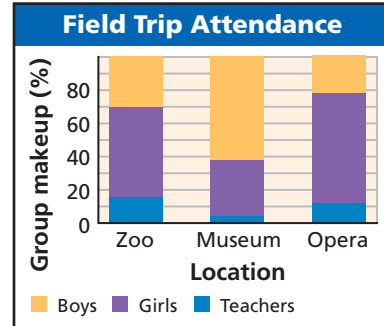
39. **Short Response** The table shows the number of students in each algebra class. Make a graph to display the data. Explain why you chose that type of graph.

Teacher	Students
Mr. Abrams	34
Ms. Belle	29
Mr. Marvin	25
Ms. Swanson	27

CHALLENGE AND EXTEND

Students and teachers at Lauren's school went on one of three field trips.

- 40. On which trip were there more boys than girls?
- 41. A total of 60 people went to the museum. Estimate the number of girls who went to the museum.
- 42. Explain why it is not possible to determine whether fewer teachers went to the museum than to the zoo or the opera.



SPIRAL REVIEW

Find the domain and range for each relation and tell whether the relation is a function. (Lesson 4-2)

43. $\{(-3, 3), (-1, 1), (0, 0), (1, 1), (3, 3)\}$

44.

x	1	2	3	4	5
y	2	4	6	8	10

45. Triangle ABC has vertices on a coordinate plane as follows:

$A = (0, 5)$, $B = (3, 0)$, $C = (8, 3)$. Show that $\triangle ABC$ is a right triangle. (Lesson 5-9)

Classify each polynomial according to its degree and number of terms. (Lesson 7-6)

46. $24y$

47. $3x^2 + 6$

48. $4m - 18m^2 - 45m^3 + 120$