

Inequalities

3A Simple Inequalities

- 3-1 Graphing and Writing Inequalities
- 3-2 Solving Inequalities by Adding or Subtracting
- 3-3 Solving Inequalities by Multiplying or Dividing

3B Multi-Step and Compound Inequalities

- 3-4 Solving Two-Step and Multi-Step Inequalities
- 3-5 Solving Inequalities with Variables on Both Sides
- Lab Truth Tables and Compound Statements
- 3-6 Solving Compound Inequalities
- 3-7 Solving Absolute-Value Inequalities

Chapter Focus

- Solve multi-step inequalities.
- Write and solve inequalities to solve problems.

For a Good Cause

You can use the concepts in this chapter to plan for a fund-raising event. Inequalities help you determine how to reach your fund-raising goals.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

- | | |
|----------------------------|--|
| 1. equation | A. mathematical phrase that contains operations, numbers, and/or variables |
| 2. evaluate | B. mathematical statement that two expressions are equivalent |
| 3. inverse operations | C. value of a variable that makes a statement true |
| 4. like terms | D. terms that contain the same variables raised to the same powers |
| 5. solution of an equation | E. to find the value of an expression |
| | F. operations that undo each other |

Evaluate Expressions

Evaluate each expression for $a = 2$ and $b = 6$.

6. $b - a$ 7. ab 8. $b \div a$ 9. $a + b$

Compare and Order Real Numbers

Compare. Write $<$, $>$, or $=$.

10. $10 \blacksquare 21$ 11. $5.27 \blacksquare 5.23$ 12. $20\% \blacksquare 0.2$ 13. $\frac{1}{3} \blacksquare \frac{2}{5}$

Combine Like Terms

Simplify each expression by combining like terms.

14. $6x + x$ 15. $-8a + 3a$
16. $9x^2 - 15x^2$ 17. $2.1x + 4.3x$

Distributive Property

Simplify each expression.

18. $2(x + 3)$ 19. $(3 - d)5$
20. $4(r - 1)$ 21. $3(4 + m)$

Solve One-Step Equations

Solve.

22. $s - 3 = 8$ 23. $-7x = 21$ 24. $y + 11 = 2$ 25. $\frac{h}{2} = 6$
26. $t + 2 = -2$ 27. $6x = 42$ 28. $r - 8 = -13$ 29. $\frac{y}{3} = -12$

Where You've Been

Previously, you

- learned the properties of equality.
- solved equations by using inverse operations.
- solved equations with variables on both sides.

In This Chapter

You will study

- the properties of inequality.
- how to solve inequalities by using inverse operations.
- how to solve inequalities with variables on both sides.
- how to solve compound inequalities.

Where You're Going

You can use the skills in this chapter

- in all your future math classes, including Geometry.
- in other classes, such as Health, Chemistry, Physics, and Economics.
- in the real world to plan a budget, to find cost-efficient services, and to set financial goals.

Key Vocabulary/Vocabulario

compound inequality	desigualdad compuesta
inequality	desigualdad
intersection	intersección
solution of an inequality	solución de una desigualdad
union	unión

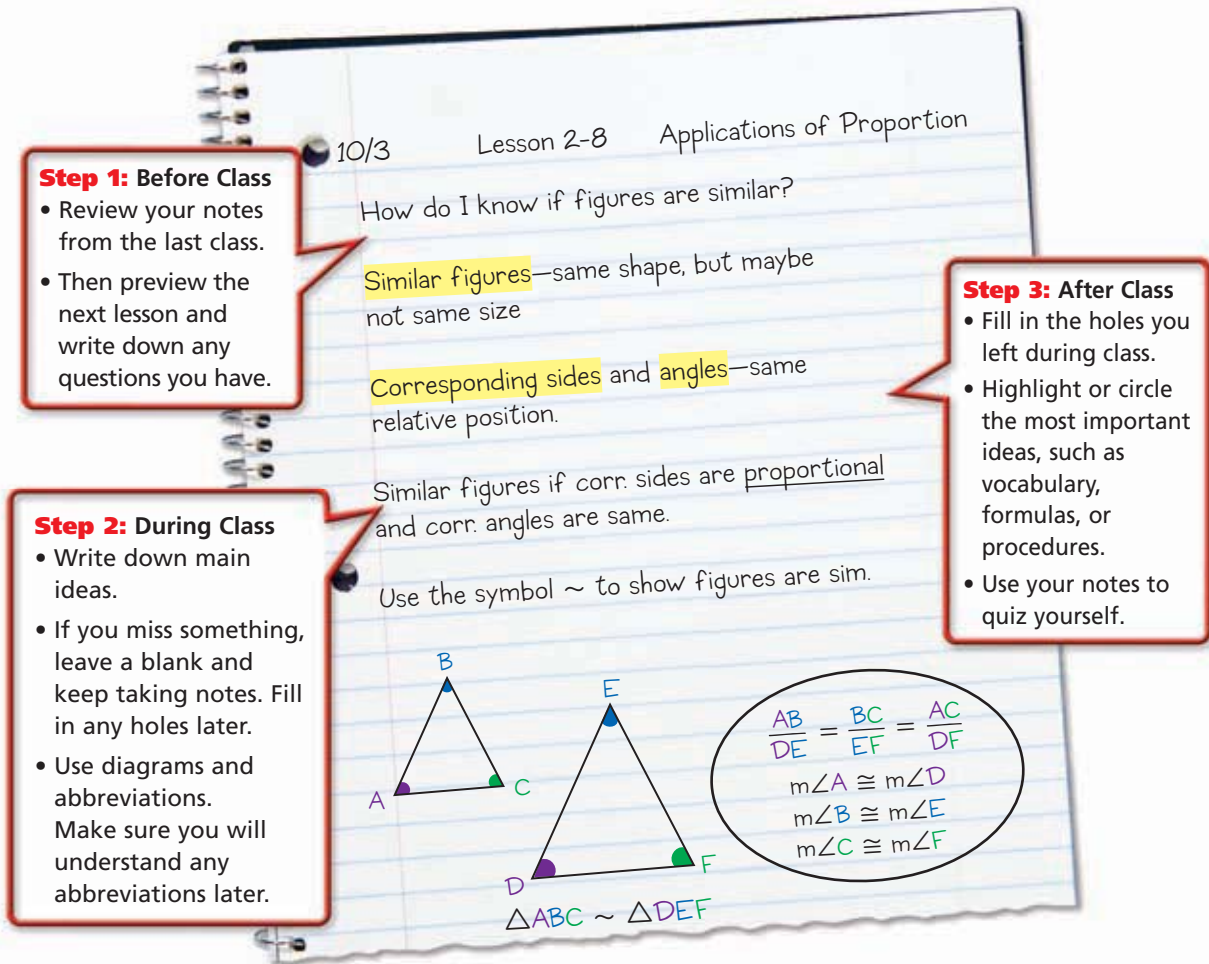
Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The prefix *in-* means “not.” An *equality* states that two things are equal. Use these meanings to write your own definition for the word **inequality**.
2. The word *compound* means “consisting of two or more parts.” What do you think a **compound inequality** might be?
3. The **intersection** of two roads is the place where the two roads overlap. What do you think the *intersection* of two graphs would be?
4. The word **union** begins with the root *uni-*. List some other words that begin with *uni-*. What do all of these words have in common?

Study Strategy: Use Your Notes Effectively

Taking notes helps you arrange, organize, and process information from your textbook and class lectures. In addition to taking notes, you need to use your notes before and after class effectively.



10/3 Lesson 2-8 Applications of Proportion

How do I know if figures are similar?

Similar figures—same shape, but maybe not same size

Corresponding sides and angles—same relative position.

Similar figures if corr. sides are proportional and corr. angles are same.

Use the symbol \sim to show figures are sim.

$\triangle ABC \sim \triangle DEF$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
 $m\angle A \cong m\angle D$
 $m\angle B \cong m\angle E$
 $m\angle C \cong m\angle F$

Step 1: Before Class

- Review your notes from the last class.
- Then preview the next lesson and write down any questions you have.

Step 2: During Class

- Write down main ideas.
- If you miss something, leave a blank and keep taking notes. Fill in any holes later.
- Use diagrams and abbreviations. Make sure you will understand any abbreviations later.

Step 3: After Class

- Fill in the holes you left during class.
- Highlight or circle the most important ideas, such as vocabulary, formulas, or procedures.
- Use your notes to quiz yourself.

Try This

1. Look at the next lesson in your textbook. Write down some questions you have about the material in that lesson. Leave space between each question so that you can write the answers during the next class.
2. Look at the notes you took during the last class. List three ways you can improve your note-taking skills.

3-1

Graphing and Writing Inequalities



Objectives

Identify solutions of inequalities in one variable.

Write and graph inequalities in one variable.

Vocabulary






inequality
solution of an inequality

Who uses this?

Members of a crew team can use inequalities to be sure they fall within a range of weights. (See Example 4.)

The athletes on a lightweight crew team must weigh 165 pounds or less. The acceptable weights for these athletes can be described using an *inequality*.

An **inequality** is a statement that two quantities are not equal. The quantities are compared by using one of the following signs:

				
$A < B$	$A > B$	$A \leq B$	$A \geq B$	$A \neq B$
A is less than B.	A is greater than B.	A is less than or equal to B.	A is greater than or equal to B.	A is not equal to B.

A **solution of an inequality** is any value of the variable that makes the inequality true.

EXAMPLE 1 Identifying Solutions of Inequalities

Describe the solutions of $3 + x < 9$ in words.

Test values of x that are positive, negative, and 0.

x	-2.75	0	5.99	6	6.01	6.1
$3 + x$	0.25	3	8.99	9	9.01	9.1
$3 + x \lessgtr 9$	$0.25 \lessgtr 9$	$3 \lessgtr 9$	$8.99 \lessgtr 9$	$9 \lessgtr 9$	$9.01 \lessgtr 9$	$9.1 \lessgtr 9$
Solution?	Yes	Yes	Yes	No	No	No

When the value of x is a number less than 6, the value of $3 + x$ is less than 9.

When the value of x is 6, the value of $3 + x$ is equal to 9.

When the value of x is a number greater than 6, the value of $3 + x$ is greater than 9.

The solutions of $3 + x < 9$ are numbers less than 6.



1. Describe the solutions of $2p > 8$ in words.

Writing Math

The solutions of the inequality in Example 1 can be written in *set-builder notation* as $\{x \mid x < 6\}$, which is read as "the set of all real numbers x such that x is less than 6."

An inequality like $3 + x < 9$ has too many solutions to list. You can use a graph on a number line to show all the solutions.



The solutions are shaded and an arrow shows that the solutions continue past those shown on the graph. To show that an endpoint is a solution, draw a solid circle at the number. To show that an endpoint is not a solution, draw an empty circle.



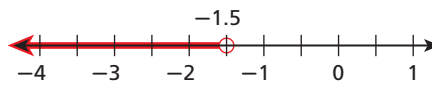
Graphing Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers less than 5	$x < 5$	
All real numbers greater than -1	$x > -1$	
All real numbers less than or equal to $\frac{1}{2}$	$x \leq \frac{1}{2}$	
All real numbers greater than or equal to 0	$x \geq 0$	

EXAMPLE 2 Graphing Inequalities

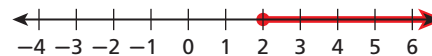
Graph each inequality.

A $b < -1.5$



Draw an empty circle at -1.5 . Shade all the numbers less than -1.5 and draw an arrow pointing to the left.

B $r \geq 2$



Draw a solid circle at 2. Shade all the numbers greater than 2 and draw an arrow pointing to the right.



Graph each inequality.

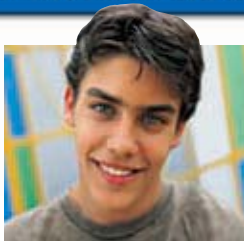
2a. $c > 2.5$

2b. $2^2 - 4 \geq w$

2c. $m \leq -3$

Student to Student

Graphing Inequalities

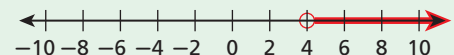


Victor Solomos
Palmer High School

To know which direction to shade a graph, I write inequalities with the variable on the left side of the inequality symbol. I know that the symbol has to point to the same number after I rewrite the inequality.

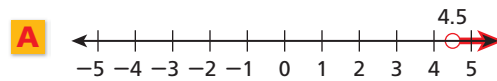
For example, I write $4 < y$ as $y > 4$.

Now the inequality symbol points in the direction that I should draw the shaded arrow on my graph.



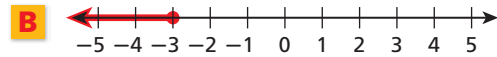
EXAMPLE 3 Writing an Inequality from a Graph

Write the inequality shown by each graph.



Use any variable. The arrow points to the right, so use either $>$ or \geq .
The empty circle at 4.5 means that 4.5 is not a solution, so use $>$.

$$h > 4.5$$

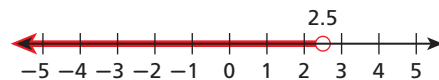


Use any variable. The arrow points to the left, so use either $<$ or \leq .
The solid circle at -3 means that -3 is a solution, so use \leq .

$$m \leq -3$$



3. Write the inequality shown by the graph.



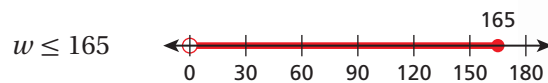
EXAMPLE 4 Sports Application

The members of a lightweight crew team can weigh no more than 165 pounds each. Define a variable and write an inequality for the acceptable weights of the team members. Graph the solutions.

Let w represent the weights that are allowed.

Athletes may weigh no more than 165 pounds.

$$w \leq 165$$



Stop the graph at 0 because a person's weight must be a positive number.



Reading Math

"No more than" means "less than or equal to."

"At least" means "greater than or equal to."

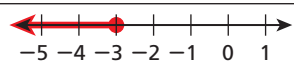


4. A store's employees earn at least \$8.25 per hour. Define a variable and write an inequality for the amount the employees may earn per hour. Graph the solutions.

THINK AND DISCUSS

1. Compare the solutions of $x > 2$ and $x \geq 2$.

2. **GET ORGANIZED** Copy and complete the graphic organizer. Draw a graph in the first row and write the correct inequality in the second row.

Inequality	Graph
$x > 1$	



GUIDED PRACTICE

1. **Vocabulary** How is a *solution of an inequality* like a solution of an equation?

SEE EXAMPLE 1 Describe the solutions of each inequality in words.

p. 170

2. $g - 5 \geq 6$

3. $-2 < h + 1$

4. $20 > 5t$

5. $5 - x \leq 2$

SEE EXAMPLE 2 Graph each inequality.

p. 171

6. $x < -5$

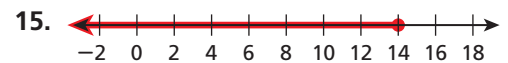
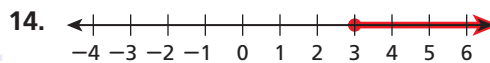
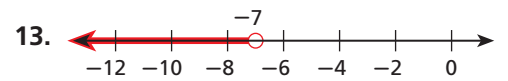
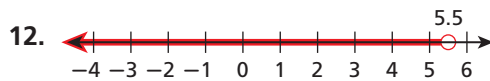
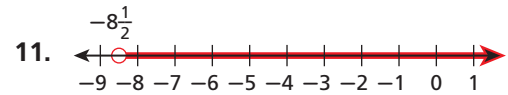
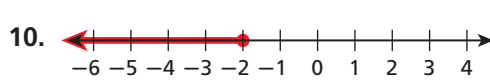
7. $c \geq 3\frac{1}{2}$

8. $(4 - 2)^3 > m$

9. $p \geq \sqrt{17 + 8}$

SEE EXAMPLE 3 Write the inequality shown by each graph.

p. 172



SEE EXAMPLE 4 Define a variable and write an inequality for each situation. Graph the solutions.

p. 172

16. There must be at least 20 club members present in order to hold a meeting.

17. A trainer advises an athlete to keep his heart rate under 140 beats per minute.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22–25	2
26–31	3
32–33	4

Describe the solutions of each inequality in words.

18. $-2t > -8$

19. $0 > w - 2$

20. $3k > 9$

21. $\frac{1}{2}b \leq 6$

Graph each inequality.

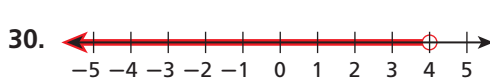
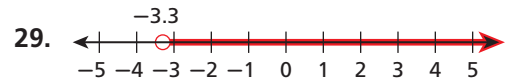
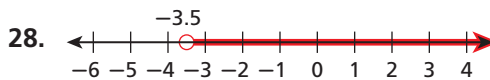
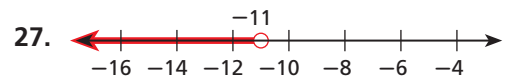
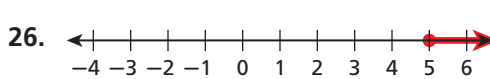
22. $7 < x$

23. $t \leq -\frac{1}{2}$

24. $d > 4(5 - 8)$

25. $t \leq 3^2 - 2^2$

Write the inequality shown by each graph.



Define a variable and write an inequality for each situation. Graph the solutions.

32. The maximum speed allowed on Main Street is 25 miles per hour.

33. Applicants must have at least 5 years of experience.

Extra Practice

Skills Practice p. S8

Application Practice p. S30

Write each inequality in words.

34. $x > 7$

35. $h < -5$

36. $d \leq 23$

37. $r \geq -2$

Write each inequality with the variable on the left. Graph the solutions.

38. $19 < g$

39. $17 \geq p$

40. $10 < e$

41. $0 < f$

Define a variable and write an inequality for each situation. Graph the solutions.

42. The highest temperature ever recorded on Earth was 135.9 °F at Al Aziziyah, Libya, on September 13, 1922.

43. Businesses with profits less than \$10,000 per year will be shut down.

44. You must be at least 46 inches tall to ride a roller coaster at an amusement park.

45. Due to a medical condition, a hiker can hike only in areas with an elevation no more than 5000 feet above sea level.



Write a real-world situation that could be described by each inequality.

46. $x \geq 0$

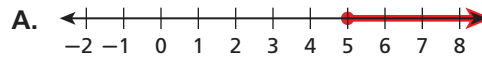
47. $x < 10$

48. $x \leq 12$

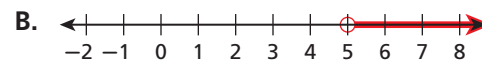
49. $x > 8.5$

Match each inequality with its graph.

50. $x \geq 5$



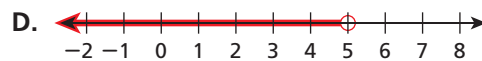
51. $x < 5$



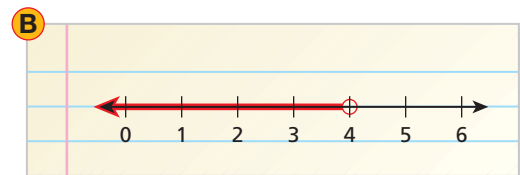
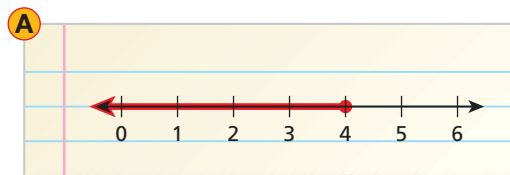
52. $x > 5$



53. $x \leq 5$



54. **///ERROR ANALYSIS///** Two students graphed the inequality $4 > b$. Which graph is incorrect? Explain the error.





**MULTI-STEP
TEST PREP**

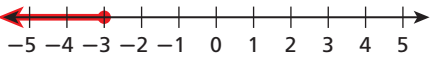


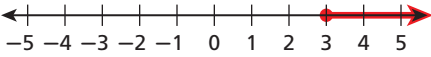


55. This problem will prepare you for the Multi-Step Test Prep on page 188.

- Mirna earned \$125 baby-sitting during the spring break. She needs to save \$90 for the German Club trip. She wants to spend the remainder of the money shopping. Write an inequality to show how much she can spend.
- Graph the inequality you wrote in part a.
- Mirna spends \$15 on a bracelet. Write an inequality to show how much money she has left to spend.

56. **Critical Thinking** Graph all positive integer solutions of the inequality $x < 5$.
-  57. **Write About It** Explain how to write an inequality that is modeled by a graph. What characteristics do you look for in the graph?
-  58. **Write About It** You were told in the lesson that the phrase “no more than” means “less than or equal to” and the phrase “at least” means “greater than or equal to.”
- What does the phrase “at most” mean?
 - What does the phrase “no less than” mean?



59. Which is NOT a solution of the inequality $5 - 2x \geq -3$?
- (A) 0 (B) 2 (C) 4 (D) 5
60. Which is NOT a solution of the inequality $3 - x < 2$?
- (F) 1 (G) 2 (H) 3 (J) 4
61. Which graph represents the solutions of $-2 \leq 1 - t$?
- (A)  (C) 
- (B)  (D) 

CHALLENGE AND EXTEND

Describe the values for x and y that make each inequality true.

62. $x + y \leq |x + y|$ 63. $x^2 < xy$ 64. $x - y \geq y - x$

Complete each statement. Write $<$ or $>$.

65. If $a > b$, then b \square a . 66. If $x > y$ and $y > z$, then x \square z .
67. Name a value of x that makes the statement $0.35 < x < 1.27$ true.
68. Is $\frac{5}{6}$ a solution of $x < 1$? How many solutions of $x < 1$ are between 0 and 1?

-  69. **Write About It** Explain how to graph all the solutions of $x \neq 5$.

SPIRAL REVIEW

Add or subtract. (Lesson 1-2)

70. $-7 + 5$ 71. $6 - (-4)$ 72. $8 - 13$ 73. $12 + (-5)$

Simplify each expression. (Lesson 1-7)

74. $x + 3x$ 75. $x + (x + 1) + (x + 2)$ 76. $5 + (x + 3) + 5 + 2(x + 3)$
77. There are twice as many girls in Sally’s class as boys. Write a rule for the number of girls in Sally’s class. Find the number of girls if there are 8 boys. (Lesson 1-8)
78. A video club charges a \$12 membership fee plus \$2.00 for each movie rental. Write a rule for the cost of renting x videos. Find the cost of renting 3, 7, and 15 videos. (Lesson 1-8)

Solve each equation. Check your answer. (Lesson 2-4)

79. $2b - 6 = b + 3$ 80. $-3(2 - x) = 5x + 2$ 81. $2(y + 1) = 2y + 1$