CHAPTER 2

Equations

2A Equations and Formulas

2B Proportion and Percent

Chapter Focus

- Use properties of equality.
- Choose procedures to solve equations efficiently.
- Write and solve equations to solve problems.

All in *Proportion*

A common use of equations and proportional relationships is the construction of scale models.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. constant
2. expression
3. order of operations
4. variable

A. a mathematical phrase that contains operations, numbers, and/or variables

B. a mathematical statement that two expressions are equivalent

C. a process for evaluating expressions

D. a symbol used to represent a quantity that can change

E. a value that does not change

Order of Operations

Simplify each expression.

5. $(7 - 3) \div 2$
6. $4 \cdot 6 \div 3$
7. $12 - 3 + 1$
8. $2 \cdot 10 \div 5$
9. $125 \div 5^2$
10. $7 \cdot 6 + 5 \cdot 4$

Add and Subtract Integers

Add or subtract.

11. $-15 + 19$
12. $-6 - (-18)$
13. $6 + (-8)$
14. $-12 + (-3)$

Add and Subtract Fractions

Perform each indicated operation. Give your answer in the simplest form.

15. $\frac{1}{4} + \frac{2}{3}$
16. $1\frac{1}{2} - \frac{3}{4}$
17. $\frac{3}{8} + \frac{2}{3}$
18. $\frac{3}{2} - \frac{2}{3}$

Evaluate Expressions

Evaluate each expression for the given value of the variable.

19. $2x + 3$ for $x = 7$
20. $3n - 5$ for $n = 7$
21. $13 - 4a$ for $a = 2$
22. $3y + 5$ for $y = 5$

Connect Words and Algebra

23. Janie bought 4 apples and 6 bananas. Each apple cost \$0.75, and each banana cost \$0.60. Write an expression representing the total cost.
24. A rectangle has a width of 13 inches and a length of ℓ inches. Write an expression representing the area of the rectangle.
25. Write a phrase that could be modeled by the expression $n + 2n$.

CHAPTER 2

Study Guide: Preview

Where You've Been

Previously, you

- practiced using operations in algebra.
- used variables to represent quantities.
- wrote expressions to represent situations.
- simplified and evaluated expressions.

In This Chapter

You will study

- how to use inverse operations to solve equations containing variables.
- writing equations to represent situations.
- simplifying equations before solving.

Where You're Going

You can use the skills in this chapter

- to compare unit prices for consumer products.
- to calculate percentages in taxes, tips, interest, and commissions.
- to create or interpret scale models and drawings.
- to solve problems in science courses and all future math courses.

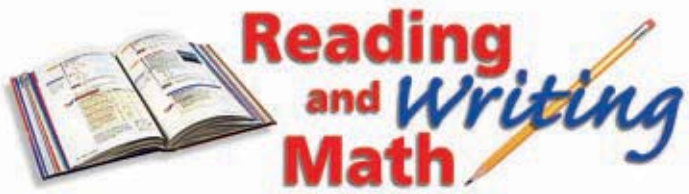
Key Vocabulary/Vocabulario

equation	ecuación
formula	fórmula
identity	identidad
indirect measurement	medición indirecta
literal equation	ecuación literal
percent	porcentaje
percent change	porcentaje de cambio
proportion	proporción
ratio	razón
unit rate	tasa unitaria

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word **equation** begins with the root *equa-*. List some other words that begin with *equa-*. What do all these words have in common?
2. The word *literal* means "of letters." How might a **literal equation** be different from an equation like $3 + 5 = 8$?
3. One definition of **identity** is "exact sameness." An equation consists of two expressions. If an equation is an *identity,* what do you think is true about the expressions?
4. The word *per* means "for each," and the word *cent* means "hundred." How can you use these meanings to understand the term **percent**?



CHAPTER 2

Study Strategy: Use Your Own Words

Explaining a concept using your own words will help you better understand it. For example, learning to solve equations might seem difficult if the textbook doesn't use the same words that you would use.

As you work through each lesson:

- Identify the important ideas from the explanation in the book.
- Use your own words to explain the important ideas you identified.

What Arturo Reads	What Arturo Writes
To evaluate an expression is to find its value.	*Evaluate an expression—find the value.*
To evaluate an algebraic expression, substitute numbers for the variables in the expression and then simplify the expression.	*Substitute a number for each variable (letter), and find the answer.*
A replacement set is a set of numbers that can be substituted for a variable.	*Replacement set—numbers that can be substituted for a letter.*

Try This

Rewrite each paragraph in your own words.

1. Two numbers are opposites if their sum is 0. A number and its opposite are on opposite sides of zero on a number line, but are the same distance from zero.
2. The Commutative and Associative Properties of Addition and Multiplication allow you to rearrange an expression to simplify it.
3. The terms of an expression are the parts to be added or subtracted. Like terms are terms that contain the same variables raised to the same powers. Constants are also like terms.

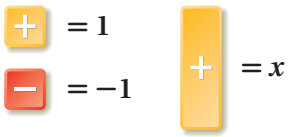
2-1 Algebra LAB

Model One-Step Equations

You can use algebra tiles and an equation mat to model and solve equations. To find the value of the variable, place or remove tiles to get the x-tile by itself on one side of the mat. You must place or remove the same number of yellow tiles or the same number of red tiles on both sides.

Use with Lesson 2-1

KEY



REMEMBER

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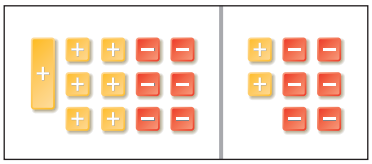
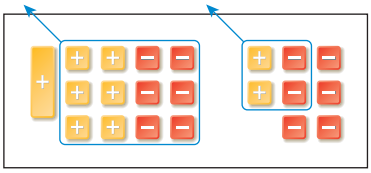
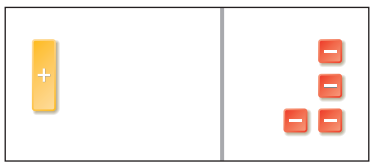
Activity

Use algebra tiles to model and solve $x + 6 = 2$.

MODEL		ALGEBRA
	*Model x + 6 on the left side of the mat and 2 on the right side of the mat.*	$x + 6 = 2$
	*Place 6 red tiles on both sides of the mat. This represents adding −6 to both sides of the equation.*	$x + 6 + (-6) = 2 + (-6)$
	*Remove zero pairs from both sides of the mat.*	$x + 0 = 0 + (-4)$
	*One x-tile is equivalent to 4 red tiles.*	$x = -4$

Try This

Use algebra tiles to model and solve each equation.

1. $x + 2 = 5$
2. $x - 7 = 8$
3. $x - 5 = 9$
4. $x + 4 = 7$

2-1 Solving Equations by Adding or Subtracting

Objective
Solve one-step equations in one variable by using addition or subtraction.

Vocabulary
equation
solution of an equation

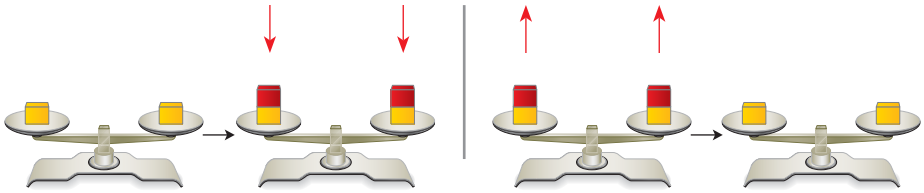
Who uses this?
Athletes can use an equation to estimate their maximum heart rates. (See Example 4.)

An **equation** is a mathematical statement that two expressions are equal. A **solution of an equation** is a value of the variable that makes the equation true.

To find solutions, *isolate the variable*. A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side. Isolate a variable by using inverse operations, which "undo" operations on the variable.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

Inverse Operations
Add *x*. ⟷ Subtract *x*.



EXAMPLE 1 **Solving Equations by Using Addition**

Solve each equation.

A $x - 10 = 4$

$x - 10 = 4$
$+10 \quad +10$
$x = 14$

Since 10 is subtracted from x, add 10 to both sides to undo the subtraction.

Check

$x - 10 = 4$	
$14 - 10$	4
4	4 ✓

To check your solution, substitute 14 for x in the original equation.

B $\frac{2}{5} = m - \frac{1}{5}$

$\frac{2}{5} = m - \frac{1}{5}$
$+\frac{1}{5} \quad +\frac{1}{5}$
$\frac{3}{5} = m$

Since $\frac{1}{5}$ is subtracted from m, add $\frac{1}{5}$ to both sides to undo the subtraction.

Writing Math
Solutions are sometimes written in a *solution set*. For Example 1A, the solution set is {14}. For Example 1B, the solution set is $\left\{\frac{3}{5}\right\}$.

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $n - 3.2 = 5.6$ **1b.** $-6 = k - 6$ **1c.** $16 = m - 9$

EXAMPLE 2 Solving Equations by Using Subtraction

Solve each equation. Check your answer.

A $x + 7 = 9$

$x + 7 = 9$
$-7 \quad -7$
$x = 2$

Since 7 is added to x, subtract 7 from both sides to undo the addition.

Check

$x + 7 = 9$
$2 + 7 \mid 9$
$9 \mid 9$ ✓

To check your solution, substitute 2 for x in the original equation.

B $0.7 = r + 0.4$

$0.7 = r + 0.4$
$-0.4 \quad -0.4$
$0.3 = r$

Since 0.4 is added to r, subtract 0.4 from both sides to undo the addition.

Check

$0.7 = r + 0.4$
$0.7 \mid 0.3 + 0.4$
$0.7 \mid 0.7$ ✓

To check your solution, substitute 0.3 for r in the original equation.

Solve each equation. Check your answer.

2a. $d + \frac{1}{2} = 1$ **2b.** $-5 = k + 5$ **2c.** $6 + t = 14$

Remember that subtracting is the same as adding the opposite. When solving equations, you will sometimes find it easier to add an opposite to both sides instead of subtracting. For example, this method may be useful when the equation contains negative numbers.

EXAMPLE 3 Solving Equations by Adding the Opposite

Solve $-8 + b = 2$.

$-8 + b = 2$
$+8 \quad +8$
$b = 10$

Since –8 is added to b, add 8 to both sides.

Solve each equation. Check your answer.

3a. $-2.3 + m = 7$ **3b.** $-\frac{3}{4} + z = \frac{5}{4}$ **3c.** $-11 + x = 33$

Student to Student

Zero As a Solution

Ama Walker
Carson High School

I used to get confused when I got a solution of 0. But my teacher reminded me that 0 is a number just like any other number, so it can be a solution of an equation. Just check your answer and see if it works.

$x + 6 = 6$
$-6 \quad -6$
$x = 0$

Check

$x + 6 = 6$
$0 + 6 \mid 6$
$6 \mid 6$ ✓

EXAMPLE 4 *Fitness Application*

A person's maximum heart rate is the highest rate, in beats per minute, that the person's heart should reach. One method to estimate maximum heart rate states that your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find the maximum heart rate of a 15-year-old.

Age	added to	maximum heart rate	is	220.
a	$+$	r	$=$	220

$a + r = 220$ *Write an equation to represent the relationship.*

$15 + r = 220$ *Substitute 15 for a. Since 15 is added to r, subtract 15 from both sides to undo the addition.*

$-15 \quad -15$

$r = 205$

The maximum heart rate for a 15-year-old is 205 beats per minute. Since age added to maximum heart rate is 220, the answer should be less than 220. So 205 is a reasonable answer.

CHECK IT OUT!

4. What if...? Use the method above to find a person's age if the person's maximum heart rate is 185 beats per minute.

The properties of equality allow you to perform inverse operations, as in the previous examples. These properties say that you can perform the same operation on both sides of an equation.

Know it! Note

Properties of Equality

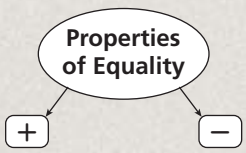
WORDS	NUMBERS	ALGEBRA
Addition Property of Equality		
You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality		
You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$

THINK AND DISCUSS

1. Describe how the Addition and Subtraction Properties of Equality are like a balanced scale.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.



2-1 Exercises

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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Will the *solution of an equation* such as $x - 3 = 9$ be a variable or a number? Explain.

Solve each equation. Check your answer.

SEE EXAMPLE 1 p. 77

2. $s - 5 = 3$
3. $17 = w - 4$
4. $k - 8 = -7$
5. $x - 3.9 = 12.4$
6. $8.4 = y - 4.6$
7. $\frac{3}{8} = t - \frac{1}{8}$

SEE EXAMPLE 2 p. 78

8. $t + 5 = -25$
9. $9 = s + 9$
10. $42 = m + 36$
11. $2.8 = z + 0.5$
12. $b + \frac{2}{3} = 2$
13. $n + 1.8 = 3$

SEE EXAMPLE 3 p. 78

14. $-10 + d = 7$
15. $20 = -12 + v$
16. $-46 + q = 5$
17. $2.8 = -0.9 + y$
18. $-\frac{2}{3} + c = \frac{2}{3}$
19. $-\frac{5}{6} + p = 2$

SEE EXAMPLE 4 p. 79

20. **Geology** In 1673, the Hope diamond was reduced from its original weight by about 45 carats, resulting in a diamond weighing about 67 carats. Write and solve an equation to find how many carats the original diamond weighed. Show that your answer is reasonable.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21–30	1
31–40	2
41–48	3
49	4

Extra Practice

Skills Practice p. S6
Application Practice p. S29

Solve each equation. Check your answer.

21. $1 = k - 8$
22. $u - 15 = -8$
23. $x - 7 = 10$
24. $-9 = p - 2$
25. $\frac{3}{7} = p - \frac{1}{7}$
26. $q - 0.5 = 1.5$
27. $6 = t - 4.5$
28. $4\frac{2}{3} = r - \frac{1}{3}$
29. $6 = x - 3$
30. $1.75 = k - 0.75$
31. $19 + a = 19$
32. $4 = 3.1 + y$
33. $m + 20 = 3$
34. $-12 = c + 3$
35. $v + 2300 = -800$
36. $b + 42 = 300$
37. $3.5 = n + 4$
38. $b + \frac{1}{2} = \frac{1}{2}$
39. $x + 5.34 = 5.39$
40. $2 = d + \frac{1}{4}$
41. $-12 + f = 3$
42. $-9 = -4 + g$
43. $-1200 + j = 345$
44. $90 = -22 + a$
45. $26 = -4 + y$
46. $1\frac{3}{4} = -\frac{1}{4} + w$
47. $-\frac{1}{6} + h = \frac{1}{6}$
48. $-5.2 + a = -8$

49. **Finance** Luis deposited $500 into his bank account. He now has $4732. Write and solve an equation to find how much was in his account before the deposit. Show that your answer is reasonable.

50. **ERROR ANALYSIS** Below are two possible solutions to $x + 12.5 = 21.6$. Which is incorrect? Explain the error.

A

$x + 12.5 = 21.6$
$-12.5 \quad -12.5$
$x = 9.1$

B

$x + 12.5 = 21.6$
$+12.5 \quad +12.5$
$x = 34.1$

Write an equation to represent each relationship. Then solve the equation.

51. Ten less than a number is equal to 12.

52. A number decreased by 13 is equal to 7.

53. Eight more than a number is 16.

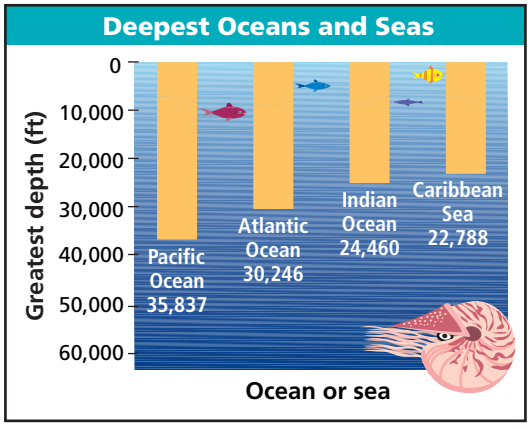
54. A number minus 3 is –8.

55. The sum of 5 and a number is 6.

56. Two less than a number is –5.

57. The difference of a number and 4 is 9.

58. **Geology** The sum of the Atlantic Ocean's average depth (in feet) and its greatest depth is 43,126. Use the information in the graph to write and solve an equation to find the average depth of the Atlantic Ocean. Show that your answer is reasonable.

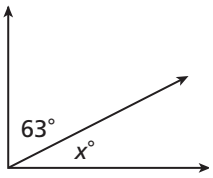


59. **School** Helene's marching band needs money to travel to a competition. Band members have raised $560. They need to raise a total of $1680. Write and solve an equation to find how much more they need. Show that your answer is reasonable.

60. **Economics** When you receive a loan to make a purchase, you often must make a down payment in cash. The amount of the loan is the purchase cost minus the down payment. Riva made a down payment of $1500 on a used car. She received a loan of $2600. Write and solve an equation to find the cost of the car. Show that your answer is reasonable.

Geometry **The angles in each pair are complementary. Write and solve an equation to find each value of *x*. (*Hint:* The measures of complementary angles add to 90°.)**

61.



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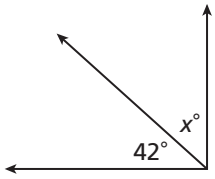
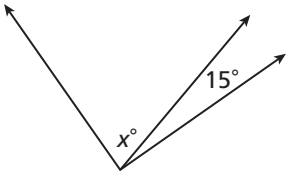


63.



LINK Geology

The ocean depths are home to many odd-looking creatures. The anglerfish pictured above, known as the common black devil, may appear menacing but reaches a maximum length of only about 5 inches.

MULTI-STEP TEST PREP

64. This problem will prepare you for the Multi-Step Test Prep on page 118.

Rates are often used to describe how quickly something is moving or changing.

a. A wildfire spreads at a rate of 1000 acres per day. How many acres will the fire cover in 2 days? Show that your answer is reasonable.

b. How many acres will the fire cover in 5 days? Explain how you found your answer.

c. Another wildfire spread for 7 days and covered a total of 780 square miles. How can you estimate the number of square miles the fire covered per day?

65. Statistics The range of a set of scores is 28, and the lowest score is 47. Write and solve an equation to find the highest score. (*Hint:* In a data set, the range is the difference between the highest and the lowest values.) Show that your answer is reasonable.

66. Write About It Describe a real-world situation that can be modeled by $x + 5 = 25$. Tell what the variable represents in your situation. Then solve the equation and tell what the solution means in the context of your problem.

67. Critical Thinking Without solving, tell whether the solution of $-3 + z = 10$ will be greater than 10 or less than 10. Explain.

TEST PREP

68. Which situation is best represented by $x - 32 = 8$?

- (A) Logan withdrew \$32 from her bank account. After her withdrawal, her balance was \$8. How much was originally in her account?
- (B) Daniel has 32 baseball cards. Joseph has 8 fewer baseball cards than Daniel. How many baseball cards does Joseph have?
- (C) Room A contains 32 desks. Room B has 8 fewer desks. How many desks are in Room B?
- (D) Janelle bought a bag of 32 craft sticks for a project. She used 8 craft sticks. How many craft sticks does she have left?

69. For which equation is $a = 8$ a solution?

(F) $15 - a = 10$ (G) $10 + a = 23$ (H) $a - 18 = 26$ (J) $a + 8 = 16$

70. Short Response Julianna used a gift card to pay for an \$18 haircut. The remaining balance on the card was \$22.

a. Write an equation that can be used to determine the original value of the card.

b. Solve your equation to find the original value of the card.

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

71. $3\frac{1}{5} + b = \frac{4}{5}$ **72.** $x - \frac{7}{4} = \frac{2}{3}$ **73.** $x + \frac{7}{4} = \frac{2}{3}$ **74.** $x - \frac{4}{9} = \frac{4}{9}$

75. If $p - 4 = 2$, find the value of $5p - 20$.

76. If $t + 6 = 21$, find the value of $-2t$.

77. If $x + 3 = 15$, find the value of $18 + 6x$.

78. If $2 + n = -11$, find the value of $6n$.

SPIRAL REVIEW

Multiply or divide. *(Lesson 1-3)*

79. $-63 \div (-7)$ **80.** $\frac{3}{7} \div \left(-\frac{4}{7}\right)$ **81.** $(-12)(-6)$

Give the side length of a square with the given area. *(Lesson 1-5)*

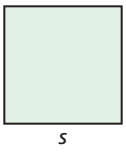
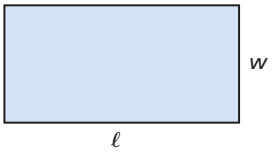
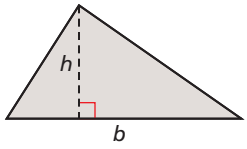
82. $225\ \text{m}^2$ **83.** $36\ \text{ft}^2$ **84.** $100\ \text{cm}^2$

Simplify each expression. *(Lesson 1-6)*

85. $8[-5 - (3 + 2)]$ **86.** $1 - [4^2 - (12 - 15)^2]$ **87.** $\frac{-12 + (-6)}{6}$

Area of Composite Figures

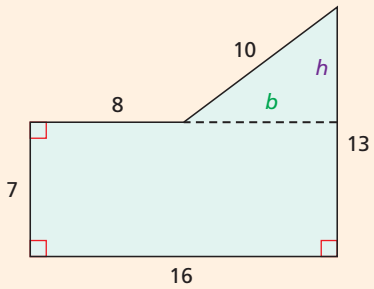
Review the area formulas for squares, rectangles, and triangles in the table below.

Squares	Rectangles	Triangles
		
$A = s^2$	$A = \ell w$	$A = \frac{1}{2}bh$

A *composite figure* is a figure that is composed of basic shapes. You can divide composite figures into combinations of squares, rectangles, and triangles to find their areas.

Example

Find the area of the figure shown.



Divide the figure into a rectangle and a right triangle. Notice that you do not know the base or the height of the triangle. Use b and h to represent these lengths.

The bottom of the rectangle is 16 units long; the top of the rectangle is 8 units long plus the base of the triangle. Use this information to write and solve an equation.

$$\begin{array}{rcl} b + 8 & = & 16 \\ \underline{-\,8} & & \underline{-\,8} \\ b & = & 8 \end{array}$$

The right side of the figure is 13 units long: 7 units from the rectangle plus the height of the triangle. Use this information to write and solve an equation.

$$\begin{array}{rcl} h + 7 & = & 13 \\ \underline{-\,7} & & \underline{-\,7} \\ h & = & 6 \end{array}$$

The area of the figure is the sum of the areas of the rectangle and the triangle.

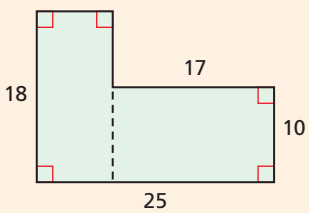
Area of rectangle
Area of triangle

$$A = \ell w + \frac{1}{2}bh$$
$$A = 16(7) + \frac{1}{2}(8)(6)$$
$$A = 112 + 24$$
$$A = 136 \text{ square units}$$

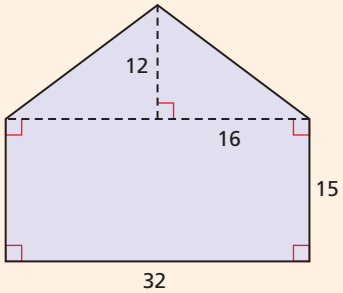
Try This

Find the area of each composite figure.

1.



2.



3.