8-4 Factoring $ax^2 + bx + c$

Objective
Factor quadratic trinomials of the form $ax^2 + bx + c$.

Why learn this?
The height of a football that has been kicked can be modeled by a factored polynomial. (See Exercise 69.)

In the previous lesson you factored trinomials of the form $x^2 + bx + c$. Now you will factor trinomials of the form $ax^2 + bx + c$, where $a \neq 0$ or 1.

When you multiply $(3x + 2)(2x + 5)$, the coefficient of the x^2-term is the product of the coefficients of the x-terms. Also, the constant term in the trinomial is the product of the constants in the binomials.

$$(3x+2)(2x+5) = 6x^2 + 19x + 10$$

To factor a trinomial like $ax^2 + bx + c$ into its binomial factors, write two sets of parentheses: $(\square x + \square)(\square x + \square)$.

Write two numbers that are factors of a next to the x's and two numbers that are factors of c in the other blanks. Then multiply to see if the product is the original trinomial. If there are not two such integers, the trinomial is unfactorable.

EXAMPLE 1 Factoring $ax^2 + bx + c$ by Guess and Check

Factor $4x^2 + 16x + 15$ by guess and check.

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(\square x + \square)(\square x + \square)$ *The first term is $4x^2$, so at least one variable term has a coefficient other than 1.*

The coefficient of the x^2-term is 4. The constant term in the trinomial is 15.

$(1x + 15)(4x + 1) = 4x^2 + 61x + 15$ ✗

$(1x + 5)(4x + 3) = 4x^2 + 23x + 15$ ✗

$(1x + 3)(4x + 5) = 4x^2 + 17x + 15$ ✗

$(1x + 1)(4x + 15) = 4x^2 + 19x + 15$ ✗

$(2x + 15)(2x + 1) = 4x^2 + 32x + 15$ ✗

$(2x + 5)(2x + 3) = 4x^2 + 16x + 15$ ✓

Try factors of 4 for the coefficients and factors of 15 for the constant terms.

The factors of $4x^2 + 16x + 15$ are $(2x + 5)$ and $(2x + 3)$.

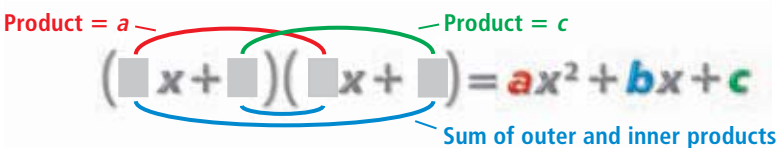
$4x^2 + 16x + 15 = (2x + 5)(2x + 3)$

CHECK IT OUT!

Factor each trinomial by guess and check.

1a. $6x^2 + 11x + 3$ **1b.** $3x^2 - 2x - 8$

So, to factor $ax^2 + bx + c$, check the factors of a and the factors of c in the binomials. The sum of the products of the outer and inner terms should be b.



Since you need to check all the factors of a and all the factors of c, it may be helpful to make a table. Then check the products of the outer and inner terms to see if the sum is b. You can multiply the binomials to check your answer.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When c Is Positive

Factor each trinomial. Check your answer.

A $2x^2 + 11x + 12$

$(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = 12$; Outer + Inner = 11*

Factors of 2	Factors of 12	Outer + Inner	
1 and 2	1 and 12	$1(12) + 2(1) = 14$	✗
1 and 2	12 and 1	$1(1) + 2(12) = 25$	✗
1 and 2	2 and 6	$1(6) + 2(2) = 10$	✗
1 and 2	6 and 2	$1(2) + 2(6) = 14$	✗
1 and 2	3 and 4	$1(4) + 2(3) = 10$	✗
1 and 2	4 and 3	$1(3) + 2(4) = 11$	✓

$(x + 4)(2x + 3)$

Check $(x + 4)(2x + 3) = 2x^2 + 3x + 8x + 12$ *Use the FOIL method.*

$= 2x^2 + 11x + 12$ ✓

B $5x^2 - 14x + 8$

$(\square x + \square)(\square x + \square)$ *$a = 5$ and $c = 8$; Outer + Inner = −14*

Factors of 5	Factors of 8	Outer + Inner	
1 and 5	−1 and −8	$1(-8) + 5(-1) = -13$	✗
1 and 5	−8 and −1	$1(-1) + 5(-8) = -41$	✗
1 and 5	−2 and −4	$1(-4) + 5(-2) = -14$	✓

$(x - 2)(5x - 4)$

Check $(x - 2)(5x - 4) = 5x^2 - 4x - 10x + 8$ *Use the FOIL method.*

$= 5x^2 - 14x + 8$ ✓

Remember!

When b is negative and c is positive, the factors of c are both negative.

Factor each trinomial. Check your answer.

2a. $6x^2 + 17x + 5$ **2b.** $9x^2 - 15x + 4$ **2c.** $3x^2 + 13x + 12$

When c is negative, one factor of c will be positive and the other factor will be negative. Only some of the factors are shown in the examples, but you may need to check all of the possibilities.

EXAMPLE 3 Factoring $ax^2 + bx + c$ When c Is Negative

Factor each trinomial. Check your answer.

A $4y^2 + 7y - 2$

$(\square y + \square)(\square y + \square)$ *a = 4 and c = −2; Outer + Inner = 7*

Factors of 4	Factors of −2	Outer + Inner	
1 and 4	1 and −2	$1(-2) + 4(1) = 2$	✗
1 and 4	−1 and 2	$1(2) + 4(-1) = -2$	✗
1 and 4	2 and −1	$1(-1) + 4(2) = 7$	✓

$(y + 2)(4y - 1)$

Check $(y + 2)(4y - 1) = 4y^2 - y + 8y - 2$ *Use the FOIL method.*

$= 4y^2 + 7y - 2$ ✓

B $4x^2 + 19x - 5$

$(\square x + \square)(\square x + \square)$ *a = 4 and c = −5; Outer + Inner = 19*

Factors of 4	Factors of −5	Outer + Inner	
1 and 4	1 and −5	$1(-5) + 4(1) = -1$	✗
1 and 4	−1 and 5	$1(5) + 4(-1) = 1$	✗
1 and 4	5 and −1	$1(-1) + 4(5) = 19$	✓

$(x + 5)(4x - 1)$

Check $(x + 5)(4x - 1) = 4x^2 - x + 20x - 5$ *Use the FOIL method.*

$= 4x^2 + 19x - 5$ ✓

C $2x^2 - 7x - 15$

$(\square x + \square)(\square x + \square)$ *a = 2 and c = −15; Outer + Inner = −7*

Factors of 2	Factors of −15	Outer + Inner	
1 and 2	1 and −15	$1(-15) + 2(1) = -13$	✗
1 and 2	−1 and 15	$1(15) + 2(-1) = 13$	✗
1 and 2	3 and −5	$1(-5) + 2(3) = 1$	✗
1 and 2	−3 and 5	$1(5) + 2(-3) = -1$	✗
1 and 2	5 and −3	$1(-3) + 2(5) = 7$	✗
1 and 2	−5 and 3	$1(3) + 2(-5) = -7$	✓

$(x - 5)(2x + 3)$

Check $(x - 5)(2x + 3) = 2x^2 + 3x - 10x - 15$ *Use the FOIL method.*

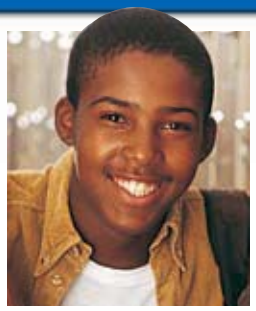
$= 2x^2 - 7x - 15$ ✓

Factor each trinomial. Check your answer.

3a. $6x^2 + 7x - 3$ **3b.** $4n^2 - n - 3$

Student to Student

Factoring $ax^2 + bx + c$



Reggie Wilson
Franklin High School

When a, b, and c are positive, I like to use a box to help me factor. I look for factors of ac that add to b. Then I arrange the terms in a box and factor.

To factor $6x^2 + 7x + 2$, first I find the factors I need.

$ac = 2(6) = 12 \quad b = 7$

Factors of 12	Sum
1 and 12	13
2 and 6	8
3 and 4	7

Then I rewrite the trinomial as $6x^2 + 3x + 4x + 2$.

Now I arrange $6x^2 + 3x + 4x + 2$ in a box and factor out the common factors from each row and column.

$6x^2$	$3x$	→ $3x$
$4x$	2	→ 2
$2x$	1	

The factors are $(2x + 1)$ and $(3x + 2)$.

When the leading coefficient is negative, factor out -1 from each term before using other factoring methods.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-2x^2 - 15x - 7$.

$-1(2x^2 + 15x + 7)$ *Factor out -1.*

$-1(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = 7$; Outer + Inner = 15*

Factors of 2	Factors of 7	Outer + Inner	
1 and 2	1 and 7	$1(7) + 2(1) = 9$	✗
1 and 2	7 and 1	$1(1) + 2(7) = 15$	✓

$(x + 7)(2x + 1)$

$-1(x + 7)(2x + 1)$

Caution!

When you factor out -1 in an early step, you must carry it through the rest of the steps.

CHECK IT OUT! **Factor each trinomial. Check your answer.**

4a. $-6x^2 - 17x - 12$ **4b.** $-3x^2 - 17x - 10$

THINK AND DISCUSS

1. Let a, b, and c be positive. If $ax^2 + bx + c$ is the product of two binomials, what do you know about the signs of the numbers in the binomials?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Write each of the following trinomials in the appropriate box and factor each one.

$3x^2 + 10x - 8$ $3x^2 + 10x + 8$
$3x^2 - 10x + 8$ $3x^2 - 10x - 8$

Factoring $ax^2 + bx + c$	
$c > 0$	
$b > 0$	$b < 0$
$c < 0$	
$b < 0$	$b > 0$

8-4 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 568

Factor each trinomial by guess and check.

1. $2x^2 + 9x + 10$
2. $5x^2 + 31x + 6$
3. $5x^2 + 7x - 6$
4. $6x^2 + 37x + 6$
5. $3x^2 - 14x - 24$
6. $6x^2 + x - 2$

Factor each trinomial. Check your answer.

SEE EXAMPLE 2 p. 569

7. $5x^2 + 11x + 2$
8. $2x^2 + 11x + 5$
9. $4x^2 - 9x + 5$
10. $2y^2 - 11y + 14$
11. $5x^2 + 9x + 4$
12. $3x^2 + 7x + 2$

SEE EXAMPLE 3 p. 570

13. $4a^2 + 8a - 5$
14. $15x^2 + 4x - 3$
15. $2x^2 + x - 6$
16. $6n^2 - 11n - 10$
17. $10x^2 - 9x - 1$
18. $7x^2 - 3x - 10$

SEE EXAMPLE 4 p. 571

19. $-2x^2 + 5x + 12$
20. $-4n^2 - 16n + 9$
21. $-5x^2 + 7x + 6$
22. $-6x^2 + 13x - 2$
23. $-4x^2 - 8x + 5$
24. $-5x^2 + x + 18$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
25–33	1
34–42	2
43–48	3
49–51	4

Extra Practice

Skills Practice p. S19
Application Practice p. S35

Factor each trinomial by guess and check.

25. $9x^2 + 9x + 2$
26. $2x^2 + 7x + 5$
27. $3n^2 + 8n + 4$
28. $10d^2 + 17d + 7$
29. $4c^2 - 17c + 15$
30. $6x^2 + 14x + 4$
31. $8x^2 + 22x + 5$
32. $6x^2 - 13x + 6$
33. $5x^2 + 9x - 18$

Factor each trinomial. Check your answer.

34. $6x^2 + 23x + 7$
35. $10n^2 - 17n + 7$
36. $3x^2 + 11x + 6$
37. $7x^2 + 15x + 2$
38. $3n^2 + 4n + 1$
39. $3x^2 - 19x + 20$
40. $6x^2 + 11x + 4$
41. $4x^2 - 31x + 21$
42. $10x^2 + 31x + 15$
43. $12y^2 + 17y - 5$
44. $3x^2 + 10x - 8$
45. $4x^2 + 4x - 3$
46. $2n^2 - 7n - 4$
47. $3x^2 - 4x - 15$
48. $3n^2 - n - 4$
49. $-4x^2 - 4x + 15$
50. $-3x^2 + 16x - 16$
51. $-3x^2 - x + 2$

Geometry For Exercises 52–54, write the polynomial modeled and then factor.

52.

$12x^2$	$24x$
$3x$	6

53.

$2x^2$	$-x$
$-4x$	2

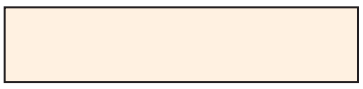
54.

$5x^2$	$-4x$
$35x$	-28

Factor each trinomial, if possible.

55. $9n^2 + 17n + 8$
56. $2x^2 - 7x - 4$
57. $4x^2 - 12x + 5$
58. $5x^2 - 4x + 12$
59. $3x^2 + 14x + 16$
60. $-3x^2 - 11x + 4$
61. $6x^2 - x - 12$
62. $10a^2 + 11a + 3$
63. $4x^2 - 12x + 9$

64. Geometry The area of a rectangle is $(6x^2 + 11x + 5)$ cm^2. The width is $(x + 1)$ cm. What is the length of the rectangle?



65. Write About It Write a paragraph describing how to factor $6x^2 + 13x + 6$. Show each step you would take and explain your steps.

Complete each factorization.

66.

$8x^2 + 18x - 5$

$8x^2 + 20x - 2x - 5$

$(8x^2 + 20x) - (2x + 5)$

$\square(\square + \square) - \square(2x + 5)$

$(\square - \square)(2x + 5)$

67.

$4x^2 + 9x + 2$

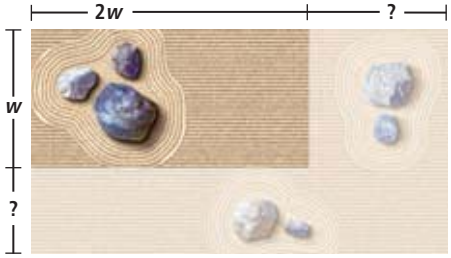
$4x^2 + 8x + x + 2$

$(4x^2 + 8x) + (x + 2)$

$\square(\square + \square) + \square(x + 2)$

$(\square + \square)(x + 2)$

68. Gardening The length of Rebecca's rectangular garden was two times the width w. Rebecca increased the length and width of the garden so that the area of the new garden is $(2w^2 + 7w + 6)$ square yards. By how much did Rebecca increase the length and the width?



69. Physics The height of a football that has been thrown or kicked can be described by the expression $-16t^2 + vt + h$ where t is the time in seconds, v is the initial upward velocity, and h is the initial height in feet.

a. Write an expression for the height of a football at time t when the initial upward velocity is 20 feet per second and the initial height is 6 feet.

b. Factor your expression from part **a.**

c. Find the height of the football after 1 second.

70. ERROR ANALYSIS A student attempted to factor $2x^2 + 11x + 12$ as shown. Find and explain the error.

$2x^2 + 11x + 12$

Factors of 12	Sum	
1 and 12	13	✓
2 and 6	8	x
3 and 4	7	x

$(2x + 1)(x + 12)$

MULTI-STEP TEST PREP

71. This problem will prepare you for the Multi-Step Test Prep on page 576.
The equation $d = 2t^2$ gives the distance from the start point of a toy boat that starts at rest and accelerates at 4 cm/s^2. The equation $d = 10t - 8$ gives the distance from the start point of a second boat that starts at rest 8 cm behind the first boat and travels at a constant rate of 10 cm/s.

a. By setting the equations equal to each other, you can determine when the cars are the same distance from the start point: $2t^2 = 10t - 8$. Use properties of algebra to collect all terms on the left side of the equation, leaving 0 on the right side.

b. Factor the expression on the left side of the equation.

c. The boats are the same distance from the start point at $t = 1$ and $t = 4$. Explain how the factors you found in part **b** were used to find these two times.

Match each trinomial with its correct factorization.

72. $6x^2 - 29x - 5$ **A.** $(x + 5)(6x + 1)$

73. $6x^2 - 31x + 5$ **B.** $(x - 5)(6x - 1)$

74. $6x^2 + 31x + 5$ **C.** $(x + 5)(6x - 1)$

75. $6x^2 + 29x - 5$ **D.** $(x - 5)(6x + 1)$

76. **Critical Thinking** The quadratic trinomial $ax^2 + bx + c$ has $a > 0$ and can be factored into the product of two binomials.

a. Explain what you know about the signs of the constants in the factors if $c > 0$.

b. Explain what you know about the signs of the constants in the factors if $c < 0$.

77. What value of b would make $3x^2 + bx - 8$ factorable?

Ⓐ 3 Ⓑ 10 Ⓒ 11 Ⓓ 25

78. Which product of binomials is represented by the model?

$5x^2$	$4x$
$15x$	12

Ⓕ $(x + 4)(3x + 5)$ Ⓗ $(x + 3)(5x + 4)$

Ⓖ $(x + 4)(5x + 3)$ Ⓙ $(x + 5)(3x + 4)$

79. Which binomial is a factor of $24x^2 - 49x + 2$?

Ⓐ $x - 2$ Ⓑ $x - 1$ Ⓒ $x + 1$ Ⓓ $x + 2$

80. Which value of c would make $2x^2 + x + c$ NOT factorable?

Ⓕ -15 Ⓖ -9 Ⓗ -6 Ⓙ -1

CHALLENGE AND EXTEND

Factor each trinomial. Check your answer.

81. $1 + 4x + 4x^2$ **82.** $1 - 14x + 49x^2$ **83.** $1 + 18x + 81x^2$

84. $25 + 30x + 9x^2$ **85.** $4 + 20x + 25x^2$ **86.** $4 - 12x + 9x^2$

87. Find all possible values of b such that $3x^2 + bx + 2$ can be factored.

88. Find all possible values of b such that $3x^2 + bx - 2$ can be factored.

89. Find all possible values of b such that $5x^2 + bx + 1$ can be factored.

SPIRAL REVIEW

90. Archie makes \$12 per hour. The function $f(x) = 12x$ gives the amount of money that Archie makes in x hours. Graph this function and give its domain and range. *(Lesson 5-1)*

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions. *(Lesson 6-6)*

91. $\begin{cases} y < -2x + 1 \\ y > 3x - 5 \end{cases}$ **92.** $\begin{cases} y \geq -x + 2 \\ y \leq x - 3 \end{cases}$ **93.** $\begin{cases} y \leq -4x \\ y > 2x - 6 \end{cases}$

Factor each trinomial. Check your answer. *(Lesson 8-3)*

94. $x^2 + 6x + 8$ **95.** $x^2 - 8x - 9$ **96.** $x^2 - 8x + 12$

8-4 Technology LAB

Use a Graph to Factor Polynomials

You can use a graphing calculator to help factor polynomials.

Use with Lesson 8-4

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Lab Resources Online
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Activity

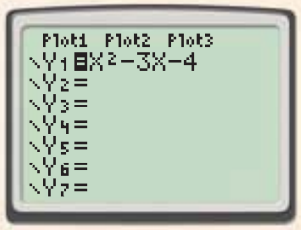
Factor $x^2 - 3x - 4$ using algebra and check your factorization using a graphing calculator.

1. $x^2 - 3x - 4$

   $(x + \square)(x + \square)$ *b = –3 and c = –4; look for factors of –4 whose sum is –3.*

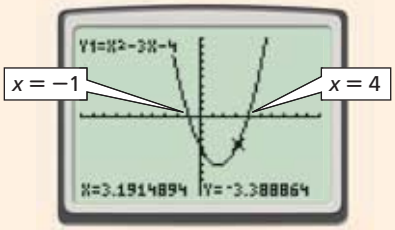
   $(x - 4)(x + 1)$ *–4(1) = –4; –4 + 1 = –3*

2. Press Y= and enter $x^2 - 3x - 4$ for **Y1.**



3. Press GRAPH to view the graph of the equation.

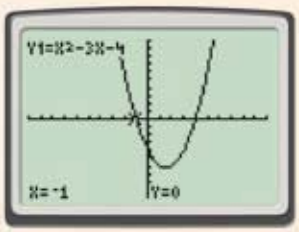
4. Press TRACE and use the left and right buttons to move the cursor along the graph. The graph appears to cross the x-axis at $x = -1$ and $x = 4$.



5. To find the value of y at $x = -1$, enter -1 and press ENTER while in *Trace* mode. The calculator gives you a value for y. Then enter 4 to find the value of y at $x = 4$.

   The calculator tells you that $y = 0$ at $x = -1$ and at $x = 4$.

   Notice that for a function with a binomial factor of the form $(x - a)$, it appears that a is an x-intercept.



Try This

Graph each trinomial and use the graph to predict the factors. Then factor each trinomial using algebra.

1. $x^2 - x - 2$
2. $x^2 + 5x + 6$
3. $x^2 + x - 12$
4. $x^2 + 12x - 64$
5. $x^2 - 4x - 5$
6. $3x^2 + 16x - 12$

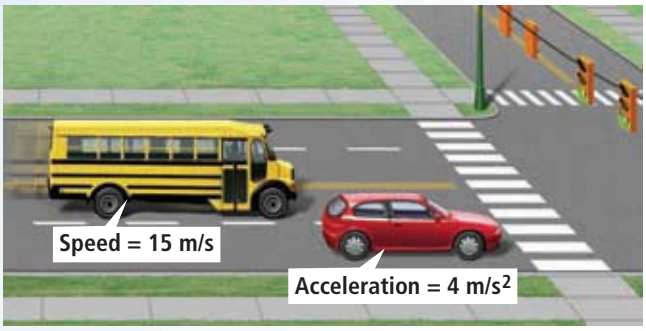
CHAPTER 8

MULTI-STEP TEST PREP

SECTION 8A

Factoring

Red Light, Green Light The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in meters, v is starting velocity in m/s, a is acceleration in m/s^2, and t is time in seconds.



1. A car is stopped at a traffic light. The light changes to green and the driver starts to drive, accelerating at a rate of 4 m/s^2. Write an equation for the distance the car travels in time t.

2. A bus is traveling at a speed of 15 m/s. The driver approaches the same traffic light in another traffic lane. He does not brake, and continues at the same speed. Write an equation for the distance the bus travels in time t. (*Hint:* At a constant speed, the acceleration is 0 m/s^2.)

3. Set the equations equal to each other so you can determine when the car and bus are the same distance from the intersection. Collect all the terms on the left side of this new equation, leaving 0 on the right side. Factor the expression on the left side of the equation.

4. Let $t = 0$ be the point at which the car is just starting to drive and the bus is even with the car. Find the other time when the vehicles will be the same distance from the intersection.

5. What distance will the two vehicles have traveled when they are again at the same distance from the intersection?

6. A truck traveling at 16 m/s is 24 meters behind the bus at $t = 0$. The equation $d = -24 + 16t$ gives the position of the truck. At what time will the truck be the same distance from the intersection as the bus? What will that distance be?

Quiz for Lessons 8-1 Through 8-4

8-1 Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 54 **2.** 42 **3.** 50 **4.** 120 **5.** 44 **6.** 78

Find the GCF of each pair of monomials.

7. $6p^3$ and $2p$

8. $12x^3$ and $18x^4$

9. -15 and $20s^4$

10. $3a$ and $4b^2$

11. Brent is making a wooden display case for his baseball collection. He has 24 balls from American League games and 30 balls from National League games. He wants to display the same number of baseballs in each row and does not want to put American League baseballs in the same row as National League baseballs. How many rows will Brent need in the display case to put the greatest number of baseballs possible in each row?

8-2 Factoring by GCF

Factor each polynomial. Check your answer.

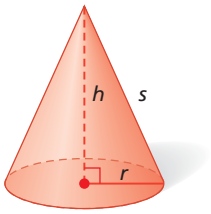
12. $2d^3 + 4d$

13. $m^2 - 8m^5$

14. $12x^4 - 8x^3 - 4x^2$

15. $3k^2 + 6k - 3$

16. The surface area of a cone can be found using the expression $s\pi r + \pi r^2$, where s represents the slant height and r represents the radius of the base. Factor this expression.



Factor each polynomial by grouping. Check your answer.

17. $w^3 - 4w^2 + w - 4$

18. $3x^3 + 6x^2 - 4x - 8$

19. $2p^3 - 6p^2 + 15 - 5p$

20. $n^3 - 6n^2 + 5n - 30$

8-3 Factoring $x^2 + bx + c$

Factor each trinomial. Check your answer.

21. $n^2 + 9n + 20$

22. $d^2 - 6d - 7$

23. $x^2 - 6x + 8$

24. $y^2 + 7y - 30$

25. $k^2 - 6k + 5$

26. $c^2 - 10c + 24$

27. Simplify and factor the expression $n(n + 3) - 4$. Show that the original expression and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

8-4 Factoring $ax^2 + bx + c$

Factor each trinomial. Check your answer.

28. $2x^2 + 11x + 5$

29. $3n^2 + 16n + 21$

30. $5y^2 - 7y - 6$

31. $4g^2 - 10g + 6$

32. $6p^2 - 18p - 24$

33. $12d^2 + 7d - 12$

34. The area of a rectangle is $(8x^2 + 8x + 2)$ cm^2. The width is $(2x + 1)$ cm. What is the length of the rectangle?