8-2 Factoring by GCF

Objective
Factor polynomials by using the greatest common factor.

Why learn this?
You can determine the dimensions of a solar panel by factoring an expression representing the panel's area. (See Example 2.)

Recall that the Distributive Property states that $ab + ac = a(b + c)$. The Distributive Property allows you to factor out the GCF of the terms in a polynomial to write a factored form of the polynomial.

A polynomial is in its factored form when it is written as a product of monomials and polynomials that cannot be factored further. The expression $2(3x - 4x)$ is not fully factored because the terms in the parentheses have a common factor of x.

Writing Math

Aligning common factors can help you find the greatest common factor of two or more terms.

EXAMPLE 1 Factoring by Using the GCF

Factor each polynomial. Check your answer.

A $4x^2 - 3x$

$4x^2 = 2 \cdot 2 \cdot x \cdot x$
$3x = 3 \cdot x$
x

Find the GCF.

The GCF of $4x^2$ and $3x$ is x.

$4x(x) - 3(x)$

Write terms as products using the GCF as a factor.

$x(4x - 3)$

Use the Distributive Property to factor out the GCF.

Check $x(4x - 3)$
$4x^2 - 3x$ ✓

Multiply to check your answer.
The product is the original polynomial.

B $10y^3 + 20y^2 - 5y$

$10y^3 = 2 \cdot 5 \cdot y \cdot y \cdot y$
$20y^2 = 2 \cdot 2 \cdot 5 \cdot y \cdot y$
$5y = 5 \cdot y$
$5 \cdot y = 5y$

Find the GCF.

The GCF of $10y^3$, $20y^2$, and $5y$ is $5y$.

$2y^2(5y) + 4y(5y) - 1(5y)$

Write terms as products using the GCF as a factor.

$5y(2y^2 + 4y - 1)$

Use the Distributive Property to factor out the GCF.

Check $5y(2y^2 + 4y - 1)$
$10y^3 + 20y^2 - 5y$ ✓

Multiply to check your answer.
The product is the original polynomial.

Factor each polynomial. Check your answer.

C $-12x - 8x^2$

$-1(12x + 8x^2)$ *Both coefficients are negative. Factor out −1.*

$12x = 2 \cdot 2 \cdot 3 \cdot x$
$8x^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x$ *Find the GCF.*
$2 \cdot 2 \cdot x = 4x$ *The GCF of $12x$ and $8x^2$ is $4x$.*

$-1[3(4x) + 2x(4x)]$ *Write each term as a product using the GCF.*
$-1[4x(3 + 2x)]$ *Use the Distributive Property to factor out the GCF.*
$-1(4x)(3 + 2x)$
$-4x(3 + 2x)$

Check

$-4x(3 + 2x) = -12x - 8x^2$ ✓ *Multiply to check your answer.*

D $5x^2 + 7$

$5x^2 = 5 \cdot x \cdot x$ *Find the GCF.*
$7 = 7$
$5x^2 + 7$ *There are no common factors other than 1.*

The polynomial cannot be factored.

Caution!

When you factor out −1 as the first step, be sure to include it in all the other steps as well.

CHECK IT OUT!

Factor each polynomial. Check your answer.

1a. $5b + 9b^3$ **1b.** $9d^2 - 8^2$

1c. $-18y^3 - 7y^2$ **1d.** $8x^4 + 4x^3 - 2x^2$

To write expressions for the length and width of a rectangle whose area is expressed as a polynomial, you need to write the polynomial as a product. You can write a polynomial as a product by factoring it.

EXAMPLE 2 *Science Application*

Mandy's calculator is powered by solar energy. The area of the solar panel is $(7x^2 + x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel.

$A = 7x^2 + x$ *The GCF of $7x^2$ and x is x.*

$= 7x(x) + 1(x)$ *Write each term as a product using the GCF as a factor.*

$= x(7x + 1)$ *Use the Distributive Property to factor out the GCF.*

Possible expressions for the dimensions of the solar panel are x cm and $(7x + 1)$ cm.

CHECK IT OUT!

2. What if...? The area of the solar panel on another calculator is $(2x^2 + 4x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel.

Sometimes the GCF of terms is a binomial. This GCF is called a *common binomial factor*. You factor out a common binomial factor the same way you factor out a monomial factor.

EXAMPLE 3 Factoring Out a Common Binomial Factor

Factor each expression.

A $7(x-3)-2x(x-3)$

$7(x-3)-2x(x-3)$ — *$(x-3)$ is a common binomial factor.*

$(x-3)(7-2x)$ — *Factor out $(x-3)$.*

B $-t(t^2+4)+(t^2+4)$

$-t(t^2+4)+(t^2+4)$ — *(t^2+4) is a common binomial factor.*

$-t(t^2+4)+1(t^2+4)$ — *$(t^2+4)=1(t^2+4)$*

$(t^2+4)(-t+1)$ — *Factor out (t^2+4).*

C $9x(x+4)-5(4+x)$

$9x(x+4)-5(4+x)$ — *$(x+4)=(4+x)$, so $(x+4)$ is a common binomial factor.*

$9x(x+4)-5(x+4)$

$(x+4)(9x-5)$ — *Factor out $(x+4)$.*

D $-3x^2(x+2)+4(x-7)$

$-3x^2(x+2)+4(x-7)$ — *There are no common factors.*

The expression cannot be factored.

Factor each expression.

3a. $4s(s+6)-5(s+6)$

3b. $7x(2x+3)+(2x+3)$

3c. $3x(y+4)-2y(x+4)$

3d. $5x(5x-2)-2(5x-2)$

You may be able to factor a polynomial by grouping. When a polynomial has four terms, you can make two groups and factor out the GCF from each group.

EXAMPLE 4 Factoring by Grouping

Factor each polynomial by grouping. Check your answer.

A $12a^3-9a^2+20a-15$

$(12a^3-9a^2)+(20a-15)$ — *Group terms that have a common number or variable as a factor.*

$3a^2(4a-3)+5(4a-3)$ — *Factor out the GCF of each group.*

$3a^2(4a-3)+5(4a-3)$ — *$(4a-3)$ is a common factor.*

$(4a-3)(3a^2+5)$ — *Factor out $(4a-3)$.*

Check $(4a-3)(3a^2+5)$ — *Multiply to check your solution.*

$4a(3a^2)+4a(5)-3(3a^2)-3(5)$

$12a^3+20a-9a^2-15$

$12a^3-9a^2+20a-15$ ✓ — *The product is the original polynomial.*

Factor each polynomial by grouping. Check your answer.

B $9x^3 + 18x^2 + x + 2$

$(9x^3 + 18x^2) + (x + 2)$ *Group terms.*

$9x^2(x + 2) + 1(x + 2)$ *Factor out the GCF of each group.*

$9x^2(x + 2) + 1(x + 2)$ *$(x + 2)$ is a common factor.*

$(x + 2)(9x^2 + 1)$ *Factor out $(x + 2)$.*

Check $(x + 2)(9x^2 + 1)$ *Multiply to check your solution.*

$x(9x^2) + x(1) + 2(9x^2) + 2(1)$

$9x^3 + x + 18x^2 + 2$

$9x^3 + 18x^2 + x + 2$ ✓ *The product is the original polynomial.*

CHECK IT OUT! **Factor each polynomial by grouping. Check your answer.**

4a. $6b^3 + 8b^2 + 9b + 12$ **4b.** $4r^3 + 24r + r^2 + 6$

Helpful Hint

If two quantities are opposites, their sum is 0.

$(5 - x) + (x - 5)$

$5 - x + x - 5$

$(-x + x) + (5 - 5)$

$0 + 0$

0

Recognizing opposite binomials can help you factor polynomials. The binomials $(5 - x)$ and $(x - 5)$ are opposites. Notice $(5 - x)$ can be written as $-1(x - 5)$.

$-1(x - 5) = (-1)(x) + (-1)(-5)$ *Distributive Property*

$= -x + 5$ *Simplify.*

$= 5 - x$ *Commutative Property of Addition*

So, $(5 - x) = -1(x - 5)$.

EXAMPLE 5 Factoring with Opposites

Factor $3x^3 - 15x^2 + 10 - 2x$ by grouping.

$3x^3 - 15x^2 + 10 - 2x$

$(3x^3 - 15x^2) + (10 - 2x)$ *Group terms.*

$3x^2(x - 5) + 2(5 - x)$ *Factor out the GCF of each group.*

$3x^2(x - 5) + 2(-1)(x - 5)$ *Write $(5 - x)$ as $-1(x - 5)$.*

$3x^2(x - 5) - 2(x - 5)$ *Simplify. $(x - 5)$ is a common factor.*

$(x - 5)(3x^2 - 2)$ *Factor out $(x - 5)$.*

CHECK IT OUT! **Factor each polynomial by grouping. Check your answer.**

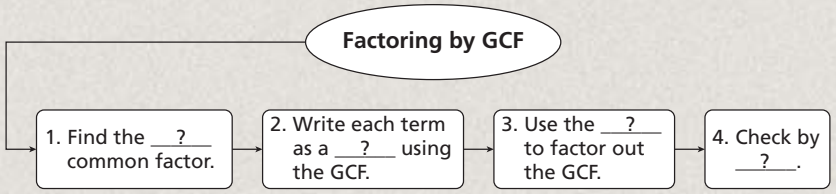
5a. $15x^2 - 10x^3 + 8x - 12$ **5b.** $8y - 8 - x + xy$

THINK AND DISCUSS

1. Explain how finding the GCF of monomials helps you factor a polynomial.

2. GET ORGANIZED Copy and complete the graphic organizer.

Know it! Note



8-2 Exercises

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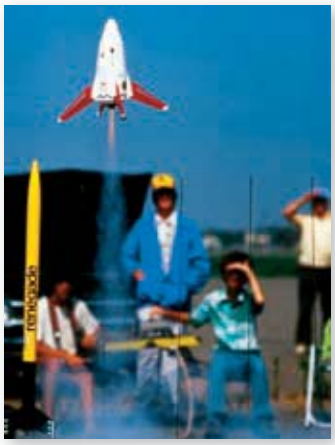
GUIDED PRACTICE

SEE EXAMPLE 1 p. 551

Factor each polynomial. Check your answer.

1. $15a - 5a^2$
2. $10g^3 - 3g$
3. $-35x + 42$
4. $-4x^2 - 6x$
5. $12h^4 + 8h^2 - 6h$
6. $3x^2 - 9x + 3$
7. $9m^2 + m$
8. $14n^3 + 7n + 7n^2$
9. $36f + 18f^2 + 3$
10. $-15b^2 + 7b$

SEE EXAMPLE 2 p. 552



11. **Physics** A model rocket is fired vertically into the air at 320 ft/s. The expression $-16t^2 + 320t$ gives the rocket's height after t seconds. Factor this expression.

SEE EXAMPLE 3 p. 553

Factor each expression.

12. $5(m-2) - m(m-2)$
13. $2b(b+3) + 5(b+3)$
14. $4(x-3) - x(y+2)$

Factor each polynomial by grouping. Check your answer.

SEE EXAMPLE 4 p. 553

15. $x^3 + 4x^2 + 2x + 8$
16. $6x^3 + 4x^2 + 3x + 2$
17. $4b^3 - 6b^2 + 10b - 15$
18. $2m^3 + 4m^2 + 6m + 12$
19. $7r^3 - 35r^2 + 6r - 30$
20. $10a^3 + 4a^2 + 5a + 2$

SEE EXAMPLE 5 p. 554

21. $2r^2 - 6r + 12 - 4r$
22. $6b^2 - 3b + 4 - 8b$
23. $14q^2 - 21q + 6 - 4q$
24. $3r - r^2 + 2r - 6$
25. $2m^3 - 6m^2 + 9 - 3m$
26. $6a^3 - 9a^2 - 12 + 8a$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–35	1
36	2
37–42	3
43–48	4
49–54	5

Extra Practice

Skills Practice p. S18
Application Practice p. S35

Factor each polynomial. Check your answer.

27. $9y^2 + 45y$
28. $36d^3 + 24$
29. $-14x^4 + 5x^2$
30. $-15f - 10f^2$
31. $-4d^4 + d^3 - 3d^2$
32. $14x^3 + 63x^2 - 7x$
33. $21c^2 + 14c$
34. $33d^3 + 22d + 11$
35. $-5g^3 - 15g^2$

36. **Finance** After t years, the amount of money in a savings account that earns simple interest is $P + Prt$, where P is the starting amount and r is the yearly interest rate. Factor this expression.

Factor each expression.

37. $6a(a-2) - 5b(b+4)$
38. $-4x(x+2) + 9(x+2)$
39. $6y(y-7) + (y-7)$
40. $a(x-3) + 2b(x-3)$
41. $-3(2+b) + 4b(b+2)$
42. $5(3x-2) + x(3x-2)$

Factor each polynomial by grouping. Check your answer.

43. $2a^3 - 8a^2 + 3a - 12$
44. $x^3 + 3x^2 + 5x + 15$
45. $6x^3 + 18x^2 + x + 3$
46. $7x^3 + 2x^2 + 28x + 8$
47. $n^3 - 2n^2 + 5n - 10$
48. $10b^3 - 16b^2 + 25b - 40$
49. $2m^3 - 2m^2 + 3 - 3m$
50. $2d^3 - d^2 - 3 + 6d$
51. $6f^3 - 8f^2 + 20 - 15f$
52. $5k^2 - k^3 + 3k - 15$
53. $b^3 - 2b - 8 + 4b^2$
54. $20 - 15x - 6x^2 + 8x$

Fill in the missing part of each factorization.

55. $16v + 12v^2 = 4v(4 + \square)$

56. $15x - 25x^2 = 5x(3 - \square)$

57. $-16k^3 - 24k^2 = -8k^2(\square + 3)$

58. $-x - 10 = -1(\square + 10)$

Copy and complete the table.

	Polynomial	Number of Terms	Name	Completely Factored Form
	$3y + 3x + 9$	3	trinomial	$3(y + x + 3)$
59.	$x^2 + 5x$			
60.	$28c^2 - 49c$			
61.	$a^4 + a^3 + a^2$			
62.	$36 + 99r - 40r^2 - 110r^3$			

63. Personal Finance The final amount of money earned by a certificate of deposit (CD) after n years is $P(1 + r)^n$, where P is the original amount contributed and r is the interest rate as a decimal.

Year	Amount of CD
2004	\$100.00
2005	\$200.00
2006	\$400.00

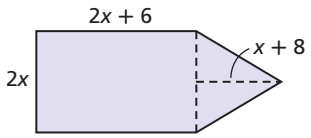
Justin's aunt purchased three CDs with the same interest rate to help him pay for college. The table shows the amount of the CD she purchased each year. In 2007, she paid \$800.00 directly to the college.

a. Let $x = 1 + r$. Write expressions in terms of x for the value of the CDs purchased in 2004, 2005, and 2006 when Justin started college in 2007.

b. Write a polynomial in terms of x to represent the total value of the CDs purchased in 2004, 2005, and 2006 plus the amount paid to the college in 2007.

c. Factor the polynomial in part **b** by grouping. Evaluate the factored form of the polynomial when the interest rate is 9%. (*Hint*: Remember that $x = 1 +$ the interest rate expressed as a decimal.)

64. Write About It Describe how to find the area of the figure shown. Show each step and write your answer in factored form.



65. Critical Thinking Show two methods of factoring the expression $3a - 3b - 4a + 4b$.

66. Geometry The area of a triangle is $\frac{1}{2}(x^3 - 2x + 2x^2 - 4)$. The height h is $x + 2$. Write an expression for the base b of the triangle. (*Hint:* Area of a triangle $= \frac{1}{2}bh$)

67. Write About It Explain how you know when two binomials are opposites.

68. This problem will prepare you for the Multi-Step Test Prep on page 576.

a. The Multiplication Property of Zero states that the product of any number and 0 is 0. What must be true about either a or b to make $ab = 0$?

b. A toy car's distance in feet from the starting point is given by the equation $d = t(3 - t)$. Explain why $t(3 - t) = 0$ means that either $t = 0$ or $3 - t = 0$.

c. When $d = 0$, the car is at the starting point. Use the fact that $t = 0$ or $3 - t = 0$ when $d = 0$ to find the two times when the car is at the starting point.

Fill in each blank with a property or definition that justifies the step.

69. $7x^3 + 2x + 21x^2 + 6 = 7x^3 + 21x^2 + 2x + 6$ **a.** ?

$= (7x^3 + 21x^2) + (2x + 6)$ **b.** ?

$= 7x^2(x + 3) + 2(x + 3)$ **c.** ?

$= (x + 3)(7x^2 + 2)$ **d.** ?

70. ERROR ANALYSIS Which factorization of $3n^3 - n^2$ is incorrect? Explain.

A

$3n^3 - n^2$

$n^2(3n) - n^2(0)$

$n^2(3n - 0)$

B

$3n^3 - n^2$

$n^2(3n) - n^2(1)$

$n^2(3n - 1)$

TEST PREP

71. Which is the complete factorization of $24x^3 - 12x^2$?

Ⓐ $6(4x^3 - 2x^2)$ Ⓑ $12(2x^3 - x^2)$ Ⓒ $12x(2x^2 - x)$ Ⓓ $12x^2(2x - 1)$

72. Which is NOT a factor of $18x^2 + 36x$?

Ⓕ 1 Ⓖ $4x$ Ⓗ $x + 2$ Ⓙ $18x$

73. The area of a rectangle is represented by the polynomial $x^2 + 3x - 6x - 18$. Which of the following could represent the length and width of the rectangle?

Ⓐ Length: $x + 3$; width: $x + 6$

Ⓑ Length: $x - 3$; width: $x - 6$

Ⓒ Length: $x + 3$; width: $x - 6$

Ⓓ Length: $x - 3$; width: $x + 6$

CHALLENGE AND EXTEND

Factor each polynomial.

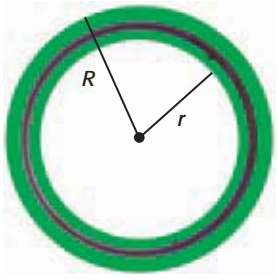
74. $6ab^2 - 24a^2$ **75.** $-72a^2b^2 - 45ab$ **76.** $-18a^2b^2 + 21ab$

77. $ab + bc + ad + cd$ **78.** $4y^2 + 8ay - y - 2a$ **79.** $x^3 - 4x^2 + 3x - 12$

80. **Geometry** The area between two concentric circles is called an *annulus*. The formula for area of an annulus is $A = \pi R^2 - \pi r^2$, where R is the radius of the larger circle and r is the radius of the smaller circle.

a. Factor the GCF from the formula for area of an annulus.

b. Use your answer from part **a** to find the area of an annulus with $R = 12$ cm and $r = 5$ cm.



SPIRAL REVIEW

Solve each equation. *(Lesson 2-4)*

81. $4(x + 1) = 3(2x - 6)$ **82.** $-20 + 8n = n + 29$ **83.** $8\left(n + \frac{3}{4}\right) = 10n - 4$

84. The coordinates of the vertices of a quadrilateral are $A(-2, 5)$, $B(6, 5)$, $C(4, -3)$, and $D(-4, -3)$. Use slope to show that $ABCD$ is a parallelogram. *(Lesson 5-9)*

85. Graph the data in the table and show the rates of change. *(Lesson 5-3)*

Time (yr)	1998	1999	2002	2004	2005
Profit (million $)	0.6	0.8	1.3	1.9	2.4

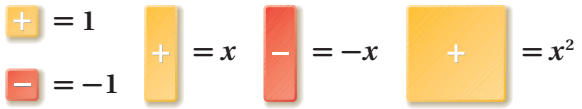
8-3 Algebra LAB

Model Factorization of Trinomials

You can use algebra tiles to write a trinomial as a product of two binomials. This is called *factoring a trinomial.*

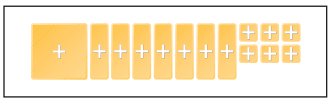
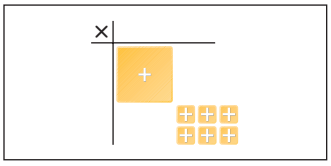
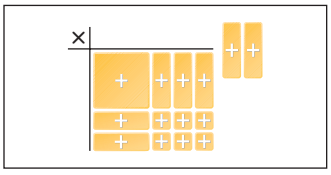
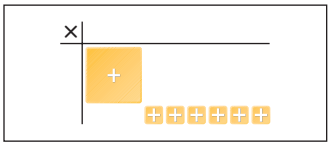
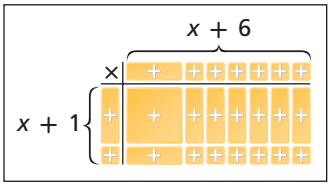
Use with Lesson 8-3

KEY



Activity 1

Use algebra tiles to factor $x^2 + 7x + 6$.

MODEL		ALGEBRA
	*Model $x^2 + 7x + 6$.*	$x^2 + 7x + 6$
	*Try to arrange all of the tiles in a rectangle. Start by placing the x^2-tile in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.*	$x^2 + 7x + 6 \neq (x + 2)(x + 3)$
	*Arrange the x-tiles so that all the tiles together make one large rectangle.* *This arrangement does not work because two x-tiles are left over.*	
	*Rearrange the unit tiles to form another rectangle.*	$x^2 + 7x + 6 = (x + 1)(x + 6)$
	*Fill in the empty spaces with x-tiles. All 7 x-tiles fit. This is the correct arrangement.* *The total area represents the trinomial. The length and width represent the factors.*	

The rectangle has width $x + 1$ and length $x + 6$. So $x^2 + 7x + 6 = (x + 1)(x + 6)$.

Try This

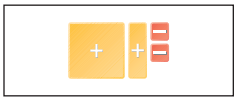
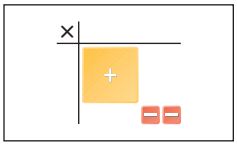
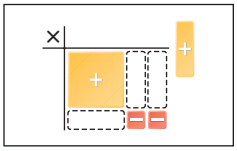
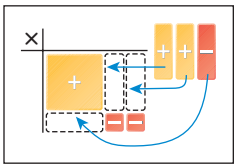
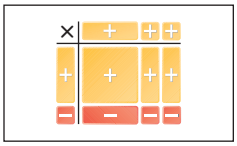
Use algebra tiles to factor each trinomial.

1. $x^2 + 2x + 1$ **2.** $x^2 + 3x + 2$ **3.** $x^2 + 6x + 5$ **4.** $x^2 + 6x + 9$

5. $x^2 + 5x + 4$ **6.** $x^2 + 6x + 8$ **7.** $x^2 + 5x + 6$ **8.** $x^2 + 8x + 12$

Activity 2

Use algebra tiles to factor $x^2 + x - 2$.

MODEL		ALGEBRA
	*Model $x^2 + x - 2$.*	$x^2 + x - 2$
 	*Start by placing the x^2-tile in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.* *To make a rectangle, you need to fill in the empty spaces, but there aren't enough x-tiles to fill in the empty spaces.*	
	*Add a zero pair. Arrange the x-tiles to complete the rectangle.* *Remember that the product of two positive values is positive and the product of a positive and a negative value is negative.*	
	*The total area represents the trinomial. The length and width represent the factors.*	$x^2 + x - 2 = (x - 1)(x + 2)$

The rectangle has width $x - 1$ and length $x + 2$. So, $x^2 + x - 2 = (x - 1)(x + 2)$.

Try This

9. Why can you add one red $-x$-tile and one yellow x-tile?

Use algebra tiles to factor each trinomial.

10. $x^2 - x - 2$ **11.** $x^2 - 2x - 3$ **12.** $x^2 - 5x + 4$ **13.** $x^2 - 7x + 10$

14. $x^2 - 2x + 1$ **15.** $x^2 - 6x + 5$ **16.** $x^2 + 5x - 6$ **17.** $x^2 + 3x - 4$

18. $x^2 - x - 6$ **19.** $x^2 + 3x - 10$ **20.** $x^2 - 2x - 8$ **21.** $x^2 + x - 12$