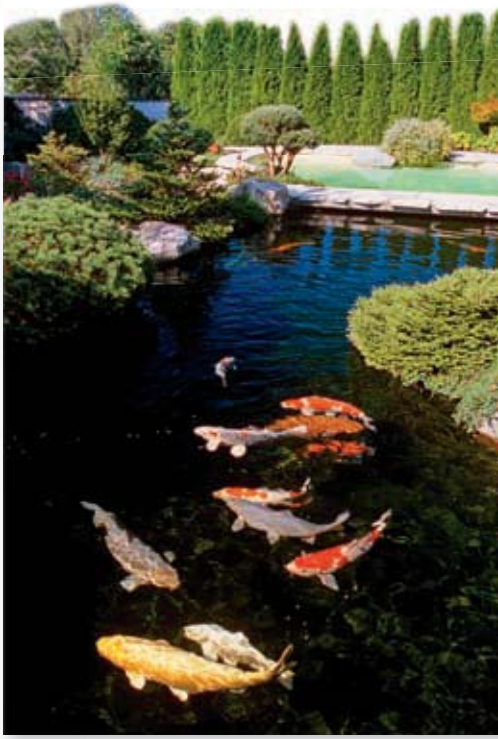
7-9 Special Products of Binomials

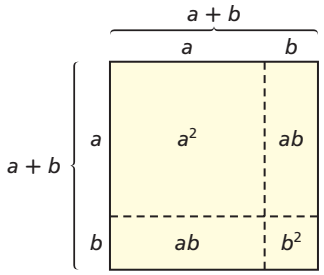


Objective
Find special products of binomials.

Vocabulary
perfect-square trinomial
difference of two squares

Why learn this?
You can use special products to find areas, such as the area of a deck around a pond. (See Example 4.)

Imagine a square with sides of length $(a + b)$:



The area of this square is $(a + b)(a + b)$, or $(a + b)^2$. The area of this square can also be found by adding the areas of the smaller squares and rectangles inside. The sum of the areas inside is $a^2 + ab + ab + b^2$.

This means that $(a + b)^2 = a^2 + 2ab + b^2$.

You can use the FOIL method to verify this:

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$

(F: $a \cdot a$; L: $a \cdot b$; I: $b \cdot a$; O: $b \cdot b$)

$$= a^2 + 2ab + b^2$$

A trinomial of the form $a^2 + 2ab + b^2$ is called a *perfect-square trinomial*. A **perfect-square trinomial** is a trinomial that is the result of squaring a binomial.

EXAMPLE 1 Finding Products in the Form $(a + b)^2$

Multiply.

A $(x + 4)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ — *Use the rule for $(a + b)^2$.*

$(x + 4)^2 = x^2 + 2(x)(4) + 4^2$ — *Identify a and b: a = x and b = 4.*

$= x^2 + 8x + 16$ — *Simplify.*

B $(3x + 2y)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ — *Use the rule for $(a + b)^2$.*

$(3x + 2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2$ — *Identify a and b: a = 3x and b = 2y.*

$= 9x^2 + 12xy + 4y^2$ — *Simplify.*

Multiply.

C $(4 + s^2)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ *Use the rule for $(a + b)^2$.*

$(4 + s^2)^2 = (4)^2 + 2(4)(s^2) + (s^2)^2$ *Identify a and b: a = 4 and b = s^2.*

$= 16 + 8s^2 + s^4$ *Simplify.*

D $(-m + 3)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ *Use the rule for $(a + b)^2$.*

$(-m + 3)^2 = (-m)^2 + 2(-m)(3) + 3^2$ *Identify a and b: a = –m and b = 3.*

$= m^2 - 6m + 9$ *Simplify.*

Multiply.

1a. $(x + 6)^2$ **1b.** $(5a + b)^2$ **1c.** $(1 + c^3)^2$

You can use the FOIL method to find products in the form $(a - b)^2$:

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2$$

F L I O

$$= a^2 - 2ab + b^2$$

A trinomial of the form $a^2 - 2ab + b^2$ is also a perfect-square trinomial because it is the result of squaring the binomial $(a - b)$.

EXAMPLE 2 Finding Products in the Form $(a - b)^2$

Multiply.

A $(x - 5)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(x - 5)^2 = x^2 - 2(x)(5) + 5^2$ *Identify a and b: a = x and b = 5.*

$= x^2 - 10x + 25$ *Simplify.*

B $(6a - 1)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(6a - 1)^2 = (6a)^2 - 2(6a)(1) + (1)^2$ *Identify a and b: a = 6a and b = 1.*

$= 36a^2 - 12a + 1$ *Simplify.*

C $(4c - 3d)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

$(4c - 3d)^2 = (4c)^2 - 2(4c)(3d) + (3d)^2$ *Identify a and b: a = 4c and b = 3d.*

$= 16c^2 - 24cd + 9d^2$ *Simplify.*

D $(3 - x^2)^2$

$(a - b)^2 = (a)^2 - 2ab + b^2$ *Use the rule for $(a - b)^2$.*

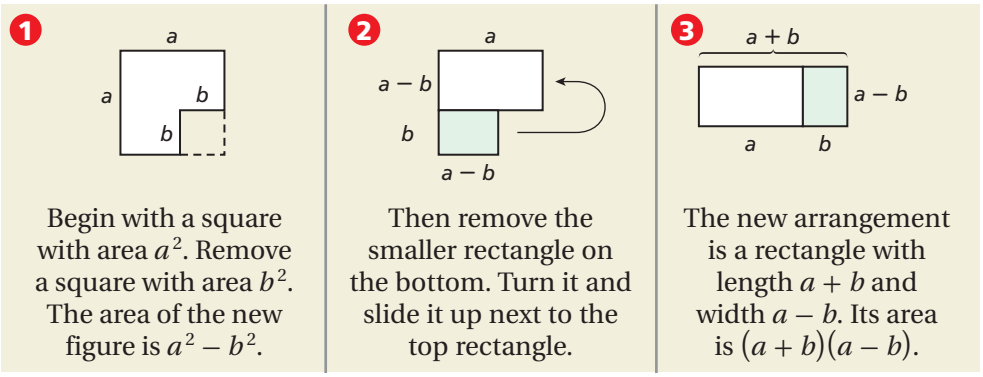
$(3 - x^2)^2 = (3)^2 - 2(3)(x^2) + (x^2)^2$ *Identify a and b: a = 3 and b = x^2.*

$= 9 - 6x^2 + x^4$ *Simplify.*

Multiply.

2a. $(x - 7)^2$ **2b.** $(3b - 2c)^2$ **2c.** $(a^2 - 4)^2$

You can use an area model to see that $(a + b)(a - b) = a^2 - b^2$.



Begin with a square with area a^2. Remove a square with area b^2. The area of the new figure is $a^2 - b^2$.

Then remove the smaller rectangle on the bottom. Turn it and slide it up next to the top rectangle.

The new arrangement is a rectangle with length $a + b$ and width $a - b$. Its area is $(a + b)(a - b)$.

So $(a + b)(a - b) = a^2 - b^2$. A binomial of the form $a^2 - b^2$ is called a **difference of two squares**.

EXAMPLE 3 Finding Products in the Form $(a + b)(a - b)$

Multiply.

A $(x + 6)(x - 6)$

$(a + b)(a - b) = a^2 - b^2$ *Use the rule for $(a + b)(a - b)$.*

$(x + 6)(x - 6) = x^2 - 6^2$ *Identify a and b: $a = x$ and $b = 6$.*

$= x^2 - 36$ *Simplify.*

B $(x^2 + 2y)(x^2 - 2y)$

$(a + b)(a - b) = a^2 - b^2$ *Use the rule for $(a + b)(a - b)$.*

$(x^2 + 2y)(x^2 - 2y) = (x^2)^2 - (2y)^2$ *Identify a and b: $a = x^2$ and $b = 2y$.*

$= x^4 - 4y^2$ *Simplify.*

C $(7 + n)(7 - n)$

$(a + b)(a - b) = a^2 - b^2$ *Use the rule for $(a + b)(a - b)$.*

$(7 + n)(7 - n) = 7^2 - n^2$ *Identify a and b: $a = 7$ and $b = n$.*

$= 49 - n^2$ *Simplify.*

CHECK IT OUT!

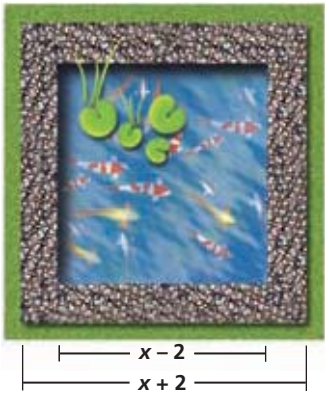
Multiply.

3a. $(x + 8)(x - 8)$ **3b.** $(3 + 2y^2)(3 - 2y^2)$ **3c.** $(9 + r)(9 - r)$

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

A square koi pond is surrounded by a gravel path. Write an expression that represents the area of the path.



1 Understand the Problem

The **answer** will be an expression that represents the area of the path.

List the important information:

- The pond is a square with a side length of $x - 2$.
- The path has a side length of $x + 2$.

2 Make a Plan

The area of the pond is $(x - 2)^2$. The total area of the path plus the pond is $(x + 2)^2$. You can subtract the area of the pond from the total area to find the area of the path.

3 Solve

Step 1 Find the total area.

$$(x + 2)^2 = x^2 + 2(x)(2) + 2^2 \qquad \textit{Use the rule for } (a + b)^2\textit{: } a = x \textit{ and } b = 2.$$
$$= x^2 + 4x + 4$$

Step 2 Find the area of the pond.

$$(x - 2)^2 = x^2 - 2(x)(2) + 2^2 \qquad \textit{Use the rule for } (a - b)^2\textit{: } a = x \textit{ and } b = 2.$$
$$= x^2 - 4x + 4$$

Remember!

To subtract a polynomial, add the opposite of *each* term.

Step 3 Find the area of the path.

$$\text{area of path} = \text{total area} - \text{area of pond}$$

$$a = x^2 + 4x + 4 - (x^2 - 4x + 4)$$
$$= x^2 + 4x + 4 - x^2 + 4x - 4 \qquad \textit{Identify like terms.}$$
$$= (x^2 - x^2) + (4x + 4x) + (4 - 4) \qquad \textit{Group like terms together.}$$
$$= 8x \qquad \textit{Combine like terms.}$$

The area of the path is $8x$.

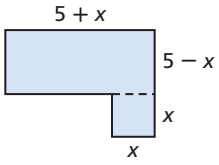
4 Look Back

Suppose that $x = 10$. Then one side of the path is 12, and the total area is 12^2, or 144. Also, if $x = 10$, one side of the pond is 8, and the area of the pond is 8^2, or 64. This means the area of the path is $144 - 64 = 80$.

According to the solution above, the area of the path is $8x$. If $x = 10$, then $8x = 8(10) = 80$. ✓

CHECK IT OUT!

4. Write an expression that represents the area of the swimming pool at right.



Know it! Note

Special Products of Binomials

Perfect-Square Trinomials

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$
$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Difference of Two Squares

$$(a + b)(a - b) = a^2 - b^2$$

THINK AND DISCUSS

1. Use the FOIL method to verify that $(a + b)(a - b) = a^2 - b^2$.
2. When a binomial is squared, the middle term of the resulting trinomial is twice the _____?_____ of the first and last terms.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. Complete the special product rules and give an example of each.

Special Products of Binomials		
Perfect-Square Trinomials		Difference of Two Squares
$(a + b)^2 = ?$	$(a - b)^2 = ?$	$(a + b)(a - b) = ?$

7-9 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA11 7-9
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** In your own words, describe a *perfect-square trinomial*.

Multiply.

SEE EXAMPLE 1 p. 521

2. $(x + 7)^2$
3. $(2 + x)^2$
4. $(x + 1)^2$
5. $(2x + 6)^2$
6. $(5x + 9)^2$
7. $(2a + 7b)^2$

SEE EXAMPLE 2 p. 522

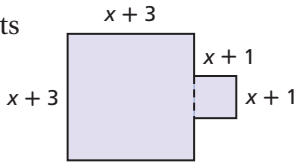
8. $(x - 6)^2$
9. $(x - 2)^2$
10. $(2x - 1)^2$
11. $(8 - x)^2$
12. $(6p - q)^2$
13. $(7a - 2b)^2$

SEE EXAMPLE 3 p. 523

14. $(x + 5)(x - 5)$
15. $(x + 6)(x - 6)$
16. $(5x + 1)(5x - 1)$
17. $(2x^2 + 3)(2x^2 - 3)$
18. $(9 - x^3)(9 + x^3)$
19. $(2x - 5y)(2x + 5y)$

SEE EXAMPLE 4 p. 523

20. **Geometry** Write a polynomial that represents the area of the figure.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21–26	1
27–32	2
33–38	3
39	4

Extra Practice
Skills Practice p. S17
Application Practice p. S34

Multiply.

21. $(x + 3)^2$
22. $(4 + z)^2$
23. $(x^2 + y^2)^2$
24. $(p + 2q^3)^2$
25. $(2 + 3x)^2$
26. $(r^2 + 5t)^2$
27. $(s^2 - 7)^2$
28. $(2c - d^3)^2$
29. $(a - 8)^2$
30. $(5 - w)^2$
31. $(3x - 4)^2$
32. $(1 - x^2)^2$
33. $(a - 10)(a + 10)$
34. $(y + 4)(y - 4)$
35. $(7x + 3)(7x - 3)$
36. $(x^2 - 2)(x^2 + 2)$
37. $(5a^2 + 9)(5a^2 - 9)$
38. $(x^3 + y^2)(x^3 - y^2)$

39. Entertainment Write a polynomial that represents the area of the circular puzzle. Remember that the formula for area of a circle is $A = \pi r^2$, where r is the radius of the circle. Leave the symbol π in your answer.

40. Multi-Step A square has sides that are $(x - 1)$ units long and a rectangle has a length of x units and a width of $(x - 2)$ units.

a. What are the possible values of x? Explain.

b. Which has the greater area, the square or the rectangle?

c. What is the difference in the areas?

Multiply.

41. $(x + y)^2$ **42.** $(x - y)^2$ **43.** $(x^2 + 4)(x^2 - 4)$

44. $(x^2 + 4)^2$ **45.** $(x^2 - 4)^2$ **46.** $(1 - x)^2$

47. $(1 + x)^2$ **48.** $(1 - x)(1 + x)$ **49.** $(x^3 - a^3)(x^3 - a^3)$

50. $(5 + n)(5 + n)$ **51.** $(6a - 5b)(6a + 5b)$ **52.** $(r - 4t^4)(r - 4t^4)$

Copy and complete the tables to verify the special products of binomials.

	a	b	$(a - b)^2$	$a^2 - 2ab + b^2$
	1	4	$(1 - 4)^2 = 9$	$1^2 - 2(1)(4) + 4^2 = 9$
53.	2	4		
54.	3	2		

	a	b	$(a + b)^2$	$a^2 + 2ab + b^2$
55.	1	4		
56.	2	5		
57.	3	0		

	a	b	$(a + b)(a - b)$	$a^2 - b^2$
58.	1	4		
59.	2	3		
60.	3	2		

LINK Math History

Beginning about 3000 B.C.E., the Babylonians lived in what is now Iraq and Turkey. Around 575 B.C.E., they built the Ishtar Gate to serve as one of eight main entrances into the city of Babylon. The image above is a relief sculpture from a restoration of the Ishtar Gate.

61. Math History The Babylonians used tables of squares and the formula $ab = \frac{(a + b)^2 - (a - b)^2}{4}$ to multiply two numbers. Use this formula to find the product $35 \cdot 24$.

62. Critical Thinking Find a value of c that makes $16x^2 - 24x + c$ a perfect-square trinomial.

63. ERROR ANALYSIS Explain the error below. What is the correct product?
$(a - b)^2 = a^2 - b^2$

64. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. Michael is fencing part of his yard. He started with a square of length x on each side. He then added 3 feet to the length and subtracted 3 feet from the width. Make a sketch to show the fenced area with the length and width labeled.

b. Write a polynomial that represents the area of the fenced region.

c. Michael bought a total of 48 feet of fencing. What is the area of his fenced region?

65. Critical Thinking The polynomial $ax^2 - 49$ is a difference of two squares. Find all possible values of a between 1 and 100 inclusive.

66. Write About It When is the product of two binomials also a binomial? Explain and give an example.

TEST PREP

67. What is $(5x - 6y)(5x - 6y)$?

Ⓐ $25x^2 - 22xy + 36y^2$ Ⓒ $25x^2 + 22xy + 36y^2$

Ⓑ $25x^2 - 60xy + 36y^2$ Ⓓ $25x^2 + 60xy + 36y^2$

68. Which product is represented by the model?

$25x^2$	$10x$
$10x$	4

Ⓕ $(2x + 5)(2x + 5)$ Ⓗ $(5x + 2)(5x - 2)$

Ⓖ $(5x - 2)(5x - 2)$ Ⓙ $(5x + 2)(5x + 2)$

69. If $a + b = 12$ and $a^2 - b^2 = 96$ what is the value of a?

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 10

70. If $rs = 15$ and $(r + s)^2 = 64$, what is the value of $r^2 + s^2$?

Ⓕ 25 Ⓖ 30 Ⓗ 34 Ⓙ 49

CHALLENGE AND EXTEND

71. Multiply $(x + 4)(x + 4)(x - 4)$.

72. Multiply $(x + 4)(x - 4)(x - 4)$.

73. If $x^2 + bx + c$ is a perfect-square trinomial, what is the relationship between b and c?

74. You can multiply two numbers by rewriting the numbers as the difference of two squares. For example:

$$36 \cdot 24 = (30 + 6)(30 - 6) = 30^2 - 6^2 = 900 - 36 = 864$$

Use this method to multiply $27 \cdot 19$. Explain how you rewrote the numbers.

SPIRAL REVIEW

75. The square paper that Yuki is using to make an origami frog has an area of 165 cm^2. Find the side length of the paper to the nearest centimeter. *(Lesson 1-5)*

Use intercepts to graph the line described by each equation. *(Lesson 5-2)*

76. $2x + 3y = 6$

77. $y = -3x + 9$

78. $\frac{1}{2}x + y = 4$

Add or subtract. *(Lesson 7-7)*

79. $3x^2 + 8x - 2x + 9x^2$

80. $(8m^4 + 2n - 3m^3 + 6) + (9m^3 + 5 - 4m^4)$

81. $(2p^3 + p) - (5p^3 + 9p)$

82. $(12t - 3t^2 + 10) - (-5t^2 - 7 - 4t)$

CHAPTER 7

MULTI-STEP TEST PREP

SECTION 7B

Polynomials

Don't Fence Me In James has 500 feet of fencing to enclose a rectangular region on his farm for some sheep.

1. Make a sketch of three possible regions that James could enclose and give the corresponding areas.
2. If the length of the region is x, find an expression for the width.
3. Use your answer to Problem 2 to write an equation for the area of the region.
4. Graph your equation from Problem 3 on your calculator. Sketch the graph.
5. James wants his fenced region to have the largest area possible using 500 feet of fencing. Find this area using the graph or a table of values.
6. What are the length and width of the region with the area from Problem 5? Describe this region.

READY TO GO ON?

Quiz for Lessons 7-6 Through 7-9

7-6 Polynomials

Write each polynomial in standard form and give the leading coefficient.

1. $4r^2 + 2r^6 - 3r$
2. $y^2 + 7 - 8y^3 + 2y$
3. $-12t^3 - 4t + t^4$
4. $n + 3 + 3n^2$
5. $2 + 3x^3$
6. $-3a^2 + 16 + a^7 + a$

Classify each polynomial according to its degree and number of terms.

7. $2x^3 + 5x - 4$
8. $5b^2$
9. $6p^2 + 3p - p^4 + 2p^3$
10. $x^2 + 12 - x$
11. $-2x^3 - 5 + x - 2x^7$
12. $5 - 6b^2 + b - 4b^4$

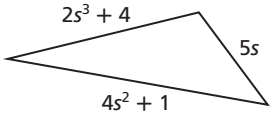
13. **Business** The function $C(x) = x^3 - 15x + 14$ gives the cost to manufacture x units of a product. What is the cost to manufacture 900 units?

7-7 Adding and Subtracting Polynomials

Add or subtract.

14. $(10m^3 + 4m^2) + (7m^2 + 3m)$
15. $(3t^2 - 2t) + (9t^2 + 4t - 6)$
16. $(12d^6 - 3d^2) + (2d^4 + 1)$
17. $(6y^3 + 4y^2) - (2y^2 + 3y)$
18. $(7n^2 - 3n) - (5n^2 + 5n)$
19. $(b^2 - 10) - (-5b^3 + 4b)$

20. **Geometry** The measures of the sides of a triangle are shown as polynomials. Write a simplified polynomial to represent the perimeter of the triangle.



7-8 Multiplying Polynomials

Multiply.

21. $2h^3 \cdot 5h^5$
22. $(s^8t^4)(-6st^3)$
23. $2ab(5a^3 + 3a^2b)$
24. $(3k + 5)^2$
25. $(2x^3 + 3y)(4x^2 + y)$
26. $(p^2 + 3p)(9p^2 - 6p - 5)$

27. **Geometry** Write a simplified polynomial expression for the area of a parallelogram whose base is $(x + 7)$ units and whose height is $(x - 3)$ units.

7-9 Special Products of Binomials

Multiply.

28. $(d + 9)^2$
29. $(3 + 2t)^2$
30. $(2x + 5y)^2$
31. $(m - 4)^2$
32. $(a - b)^2$
33. $(3w - 1)^2$
34. $(c + 2)(c - 2)$
35. $(5r + 6)(5r - 6)$

36. **Sports** A child's basketball has a radius of $(x - 5)$ inches. Write a polynomial that represents the surface area of the basketball. (The formula for the surface area of a sphere is $S = 4\pi r^2$, where r represents the radius of the sphere.) Leave the symbol π in your answer.

CHAPTER 7

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ polynomial is a polynomial of degree 3.
2. When a polynomial is written with the terms in order from highest to lowest degree, it is in ___?___.
3. A(n) ___?___ is a number, a variable, or a product of numbers and variables with whole-number exponents.
4. A(n) ___?___ is a polynomial with three terms.
5. ___?___ is a method of writing numbers that are very large or very small.

7-1 Integer Exponents *(pp. 460–465)*

EXAMPLES

Simplify.

■ -2^{-4}

$-2^{-4} = -\frac{1}{2^4} = -\frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{1}{16}$

■ 3^0

$3^0 = 1$ *Any nonzero number raised to the zero power is 1.*

■ **Evaluate r^3s^{-4} for $r = -3$ and $s = 2$.**

r^3s^{-4}

$(-3)^3(2)^{-4} = \frac{(-3)(-3)(-3)}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{27}{16}$

■ **Simplify $\frac{a^{-3}b^4}{c^{-2}}$.**

$\frac{a^{-3}b^4}{c^{-2}} = \frac{b^4c^2}{a^3}$

EXERCISES

6. The diameter of a certain bearing is 2^{-5} in. Simplify this expression.

Simplify.

7. $(3.6)^0$
8. $(-1)^{-4}$
9. 5^{-3}
10. 10^{-4}

Evaluate each expression for the given value(s) of the variable(s).

11. b^{-4} for $b = 2$
12. $\left(\frac{2}{5}b\right)^{-4}$ for $b = 10$
13. $-2p^3q^{-3}$ for $p = 3$ and $q = -2$

Simplify.

14. m^{-2}
15. bc^0
16. $-\frac{1}{2}x^{-2}y^{-4}$
17. $\frac{2b^6}{c^{-4}}$
18. $\frac{3a^2c^{-2}}{4b^0}$
19. $\frac{q^{-1}r^{-2}}{s^{-3}}$

7-2 Powers of 10 and Scientific Notation *(pp. 466–471)*

EXAMPLES

■ **Write 1,000,000 as a power of 10.**

1,000,000 — *The decimal point is 6 places to the right of 1.*

$1{,}000{,}000 = 10^6$

■ **Find the value of 386.21×10^5.**

$386.2\,1\,0\,0\,0$ — *Move the decimal point 5 places to the right.*

38,621,000

■ **Write 0.000000041 in scientific notation.**

$0.0\,0\,0\,0\,0\,0\,0\,4\,1$ — *Move the decimal point 8 places to the right to get a number between 1 and 10.*

4.1×10^{-8}

EXERCISES

Find the value of each power of 10.

20. 10^7 **21.** 10^{-5}

Write each number as a power of 10.

22. 100 **23.** 0.00000000001

Find the value of each expression.

24. 3.25×10^5 **25.** 0.18×10^4

26. 17×10^{-2} **27.** 299×10^{-6}

28. Order the list of numbers from least to greatest. 6.3×10^{-3}, 1.2×10^4, 5.8×10^{-7}, 2.2×10^2

29. In 2003, the average daily value of shares traded on the New York Stock Exchange was about $\$3.85 \times 10^{10}$. Write this amount in standard form.

7-3 Multiplication Properties of Exponents *(pp. 474–480)*

EXAMPLES

Simplify.

■ $5^3 \cdot 5^{-2}$ — *The powers have the same base.*

$5^3 \cdot 5^{-2}$

$5^{3+(-2)}$ — *Add the exponents.*

5^1

5

■ $a^4 \cdot b^{-3} \cdot b \cdot a^{-2}$

$a^4 \cdot b^{-3} \cdot b \cdot a^{-2}$ — *Use properties to group factors.*

$(a^4 \cdot a^{-2}) \cdot (b^{-3} \cdot b)$

$a^2 \cdot b^{-2}$ — *Add the exponents of powers with the same base.*

$\frac{a^2}{b^2}$ — *Write with a positive exponent.*

■ $(a^{-3}b^2)^{-2}$

$(a^{-3})^{-2} \cdot (b^2)^{-2}$ — *Power of a Product Property*

$a^6 \cdot b^{-4}$ — *Power of a Power Property*

$\frac{a^6}{b^4}$ — *Write with a positive exponent.*

EXERCISES

Simplify.

30. $5^3 \cdot 5^6$ **31.** $2^6 \cdot 3 \cdot 2^{-3} \cdot 3^3$

32. $b^2 \cdot b^8$ **33.** $r^4 \cdot r$

34. $(x^3)^4$ **35.** $(s^3)^0$

36. $(2^3)^{-1}$ **37.** $(5^2)^{-2}$

38. $(4b^3)^{-2}$ **39.** $(g^3h^2)^4$

40. $(-x^2y)^2$ **41.** $-(x^2y)^2$

42. $(x^2y^3)(xy^3)^4$ **43.** $(j^2k^3)(j^4k^6)$

44. $(5^3 \cdot 5^{-2})^{-1}$ **45.** $(mn^3)^5(mn^5)^3$

46. $(4 \times 10^8)(2 \times 10^3)$ **47.** $(3 \times 10^2)(3 \times 10^5)$

48. $(5 \times 10^3)(2 \times 10^6)$ **49.** $(7 \times 10^5)(4 \times 10^9)$

50. $(3 \times 10^{-4})(2 \times 10^5)$ **51.** $(3 \times 10^{-8})(6 \times 10^{-1})$

52. In 2003, Wyoming's population was about 5.0×10^5. California's population was about 7.1 × 10 times as large as Wyoming's. What was the approximate population of California? Write your answer in scientific notation.

7-4 Division Properties of Exponents *(pp. 481–487)*

EXAMPLES

■ **Simplify $\frac{x^9}{x^2}$.**

$\frac{x^9}{x^2} = x^{9-2} = x^7$ *Subtract the exponents.*

EXERCISES

Simplify.

53. $\frac{2^8}{2^2}$ **54.** $\frac{m^6}{m}$ **55.** $\frac{2^6 \cdot 4 \cdot 7^3}{2^5 \cdot 4^4 \cdot 7^2}$

56. $\frac{24b^6}{4b^5}$ **57.** $\frac{t^4v^5}{tv}$ **58.** $\left(\frac{1}{2}\right)^{-4}$

Simplify each quotient and write the answer in scientific notation.

59. $(2.5 \times 10^8) \div (0.5 \times 10^7)$

60. $(2 \times 10^{10}) \div (8 \times 10^2)$

7-5 Rational Exponents *(pp. 488–493)*

EXAMPLES

■ **Simplify $\sqrt[3]{r^6s^{12}}$.**

$\sqrt[3]{r^6s^{12}} = (r^6s^{12})^{\frac{1}{3}}$ *Definition of $b^{\frac{1}{n}}$*

$= (r^6)^{\frac{1}{3}} \cdot (s^{12})^{\frac{1}{3}}$ *Power of a Product Property*

$= \left(r^{6 \cdot \frac{1}{3}}\right) \cdot \left(s^{12 \cdot \frac{1}{3}}\right)$ *Power of a Power Property*

$= (r^2) \cdot (s^4)$ *Simplify exponents.*

$= r^2s^4$

EXERCISES

Simplify each expression.

61. $81^{\frac{1}{2}}$ **62.** $343^{\frac{1}{3}}$

63. $64^{\frac{2}{3}}$ **64.** $(2^6)^{\frac{1}{2}}$

Simplify each expression. All variables represent nonnegative numbers.

65. $\sqrt[5]{z^{10}}$ **66.** $\sqrt[3]{125x^6}$

67. $\sqrt{x^8y^6}$ **68.** $\sqrt[3]{m^6n^{12}}$

7-6 Polynomials *(pp. 496–501)*

EXAMPLES

■ **Find the degree of the polynomial $3x^2 + 8x^5$.**

$3x^2 + 8x^5$ *$8x^5$ has the highest degree.*

The degree is 5.

■ **Classify the polynomial $y^3 - 2y$ according to its degree and number of terms.**

Degree: 3
Terms: 2

The polynomial $y^3 - 2y$ is a **cubic binomial**.

EXERCISES

Find the degree of each polynomial.

69. 5 **70.** $8st^3$

71. $3z^6$ **72.** $6h$

Write each polynomial in standard form. Then give the leading coefficient.

73. $2n - 4 + 3n^2$ **74.** $2a - a^4 - a^6 + 3a^3$

Classify each polynomial according to its degree and number of terms.

75. $2s - 6$ **76.** $-8p^5$

77. $-m^4 - m^2 - 1$ **78.** 2

7-7 Adding and Subtracting Polynomials *(pp. 504–509)*

EXAMPLES

Add.

■ $(h^3 - 2h) + (3h^2 + 4h) - 2h^3$

$(h^3 - 2h) + (3h^2 + 4h) - 2h^3$

$(h^3 - 2h^3) + (3h^2) + (4h - 2h)$

$-h^3 + 3h^2 + 2h$

Subtract.

■ $(n^3 + 5 - 6n^2) - (3n^2 - 7)$

$(n^3 + 5 - 6n^2) + (-3n^2 + 7)$

$(n^3 + 5 - 6n^2) + (-3n^2 + 7)$

$n^3 + (-6n^2 - 3n^2) + (5 + 7)$

$n^3 - 9n^2 + 12$

EXERCISES

Add or subtract.

79. $3t + 5 - 7t - 2$

80. $4x^5 - 6x^6 + 2x^5 - 7x^5$

81. $-h^3 - 2h^2 + 4h^3 - h^2 + 5$

82. $(3m - 7) + (2m^2 - 8m + 6)$

83. $(12 + 6p) - (p - p^2 + 4)$

84. $(3z - 9z^2 + 2) + (2z^2 - 4z + 8)$

85. $(10g - g^2 + 3) - (-4g^2 + 8g - 1)$

92. $(-5x^3 + 2x^2 - x + 5) - (-5x^3 + 3x^2 - 5x - 3)$

7-8 Multiplying Polynomials *(pp. 512–519)*

EXAMPLES

Multiply.

■ $(2x - 4)(3x + 5)$

$2x(3x) + 2x(5) - 4(3x) - 4(5)$

$6x^2 + 10x - 12x - 20$

$6x^2 - 2x - 20$

■ $(b - 2)(b^2 + 4b - 5)$

$b(b^2) + b(4b) - b(5) - 2(b^2) - 2(4b) - 2(-5)$

$b^3 + 4b^2 - 5b - 2b^2 - 8b + 10$

$b^3 + 2b^2 - 13b + 10$

EXERCISES

Multiply.

87. $(2r)(4r)$

88. $(3a^5)(2ab)$

89. $(-3xy)(-6x^2y)$

90. $(3s^3t^2)(2st^4)\left(\frac{1}{2}s^2t^8\right)$

91. $2(x^2 - 4x + 6)$

92. $-3ab(ab - 2a^2b + 5a)$

93. $(a + 3)(a - 6)$

94. $(b - 9)(b + 3)$

95. $(x - 10)(x - 2)$

96. $(t - 1)(t + 1)$

97. $(2q + 6)(4q + 5)$

98. $(5g - 8)(4g - 1)$

7-9 Special Products of Binomials *(pp. 521–527)*

EXAMPLES

Multiply.

■ $(2h - 6)^2$

$(2h - 6)^2 = (2h)^2 + 2(2h)(-6) + (-6)^2$

$4h^2 - 24h + 36$

■ $(4x - 3)(4x + 3)$

$(4x - 3)(4x + 3) = (4x)^2 - 3^2$

$16x^2 - 9$

EXERCISES

Multiply.

99. $(p - 4)^2$

100. $(x + 12)^2$

101. $(m + 6)^2$

102. $(3c + 7)^2$

103. $(2r - 1)^2$

104. $(3a - b)^2$

105. $(2n - 5)^2$

106. $(h - 13)^2$

107. $(x - 1)(x + 1)$

108. $(z + 15)(z - 15)$

109. $(c^2 - d)(c^2 + d)$

110. $(3k^2 + 7)(3k^2 - 7)$

CHAPTER 7

CHAPTER TEST

Evaluate each expression for the given value(s) of the variable(s).

1. $\left(\frac{1}{3}b\right)^{-2}$ for $b = 12$

2. $\left(14 - a^0b^2\right)^{-3}$ for $a = -2$ and $b = 4$

Simplify.

3. $2r^{-3}$

4. $-3f^0g^{-1}$

5. m^2n^{-3}

6. $\frac{1}{2}s^{-5}t^3$

Write each number as a power of 10.

7. 0.0000001

8. 10,000,000,000,000

9. 1

Find the value of each expression.

10. 1.25×10^{-5}

11. $10^8 \times 10^{-11}$

12. 325×10^{-2}

13. **Technology** In 2002, there were approximately 544,000,000 Internet users worldwide. Write this number in scientific notation.

Simplify.

14. $\left(f^4\right)^3$

15. $\left(4b^2\right)^0$

16. $\left(a^3b^6\right)^6$

17. $-\left(x^3\right)^5 \cdot \left(x^2\right)^6$

Simplify each quotient and write the answer in scientific notation.

18. $\left(3.6 \times 10^9\right) \div \left(6 \times 10^4\right)$

19. $\left(3 \times 10^{12}\right) \div \left(9.6 \times 10^{16}\right)$

Simplify.

20. $\frac{y^4}{y}$

21. $\frac{d^2f^5}{\left(d^3\right)^2f^{-4}}$

22. $\frac{2^5 \cdot 3^3 \cdot 5^4}{2^8 \cdot 3^2 \cdot 5^4}$

23. $\left(\frac{4s}{3t}\right)^{-2} \cdot \left(\frac{2s}{6t}\right)^2$

24. **Geometry** The surface area of a cone is approximated by the polynomial $3.14r^2 + 3.14r\ell$, where r is the radius and ℓ is the slant height. Find the approximate surface area of a cone when $\ell = 5$ cm and $r = 3$ cm.

Simplify each expression. All variables represent nonnegative numbers.

25. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$

26. $\sqrt[3]{43^3}$

27. $\sqrt{25y^8}$

28. $\sqrt[5]{3^5t^{10}}$

Add or subtract.

29. $3a - 4b + 2a$

30. $\left(2b^2 - 4b^3\right) - \left(6b^3 + 8b^2\right)$

31. $-9g^2 + 3g - 4g^3 - 2g + 3g^2 - 4$

Multiply.

32. $-5\left(r^2s - 6\right)$

33. $(2t - 7)(t + 4)$

34. $\left(4g - 1\right)\left(4g^2 - 5g - 3\right)$

35. $(m + 6)^2$

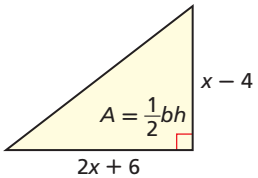
36. $(3t - 7)(3t + 7)$

37. $\left(3x^2 - 7\right)^2$

38. **Carpentry** Carpenters use a tool called a *speed square* to help them mark right angles. A speed square is a right triangle.

a. Write a polynomial that represents the area of the speed square shown.

b. Find the area when $x = 4.5$ in.



College Entrance Exam Practice

CHAPTER 7

FOCUS ON SAT

When you receive your SAT scores, you will find a percentile for each score. The percentile tells you what percent of students scored lower than you on the same test. Your percentile at the national and state levels may differ because of the different groups being compared.

You may want to time yourself as you take this practice test. It should take you about 7 minutes to complete.

HOT TIP!

You may use some types of calculators on the math section of the SAT. For about 40% of the test items, a graphing calculator is recommended. Bring a calculator that you are comfortable using. You won't have time to figure out how a new calculator works.

---

1. If $(x + 1)(x + 4) - (x - 1)(x - 2) = 0$, what is the value of x?

(A) -1

(B) $-\frac{1}{4}$

(C) 0

(D) $\frac{1}{4}$

(E) 1

---

2. Which of the following is equal to 4^5?

I. $3^5 \times 1^5$

II. 2^{10}

III. $4^0 \times 4^5$

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

---

3. If $x^{-4} = 81$, then $x =$

(A) -3

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) 3

(E) 9

---

4. What is the value of $2x^3 - 4x^2 + 3x + 1$ when $x = -2$?

(A) -37

(B) -25

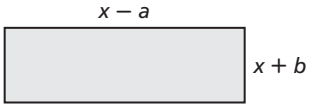
(C) -5

(D) 7

(E) 27

---

5. What is the area of a rectangle with a length of $x - a$ and a width of $x + b$?



(A) $x^2 - a^2$

(B) $x^2 + b^2$

(C) $x^2 - abx + ab$

(D) $x^2 - ax - bx - ab$

(E) $x^2 + bx - ax - ab$

---

6. For integers greater than 0, define the following operations.

$a \square b = 2a^2 + 3b$

$a \triangle b = 5a^2 - 2b$

What is $(a \square b) + (a \triangle b)$?

(A) $7a^2 + b$

(B) $-3a^2 + 5b$

(C) $7a^2 - b$

(D) $3a^2 - 5b$

(E) $-3a^2 - b$

CHAPTER 7

TEST TACKLER

Standardized Test Strategies

Any Question Type: Use a Diagram

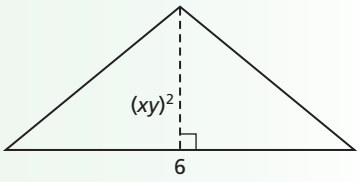
When a test item includes a diagram, use it to help solve the problem. Gather as much information from the drawing as possible. However, keep in mind that diagrams are not always drawn to scale and can be misleading.

EXAMPLE 1

Multiple Choice What is the height of the triangle when $x = 4$ and $y = 1$?

(A) 2 (C) 8

(B) 4 (D) 16



In the diagram, the height appears to be less than 6, so you might eliminate choices C and D. However, doing the math shows that the height is actually greater than 6. Do not rely solely on visual information. Always use the numbers given in the problem.

The height of the triangle is $(xy)^2$.

When $x = 4$ and $y = 1$, $(xy)^2 = (4 \cdot 1)^2 = (4)^2 = 16$.

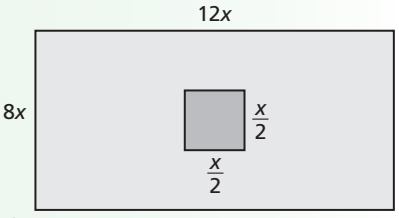
Choice D is the correct answer.

If a test item does not have a diagram, draw a quick sketch of the problem situation. Label your diagram with the data given in the problem.

EXAMPLE 2

Short Response A square placemat is lying in the middle of a rectangular table. The side length of the placemat is $\left(\frac{x}{2}\right)$. The length of the table is $12x$, and the width is $8x$. Write a polynomial to represent the area of the placemat. Then write a polynomial to represent the area of the table that surrounds the placemat.

Use the information in the problem to draw and label a diagram. Then write the polynomials.



$$\text{area of placemat} = s^2 = \left(\frac{x}{2}\right)^2 = \left(\frac{x}{2}\right)\left(\frac{x}{2}\right) = \frac{x^2}{4}$$

$$\text{area of table} = \ell w = (12x)(8x) = 96x^2$$

$$\text{area of table} - \text{area of placemat} = 96x^2 - \frac{x^2}{4} = \frac{384x^2 - x^2}{4} = \frac{383x^2}{4}$$

The area of the placemat is $\frac{x^2}{4}$.

The area of the table that surrounds the placemat is $\frac{383x^2}{4}$.

If a given diagram does not reflect the problem, draw a sketch that is more accurate. If a test item does not have a diagram, use the given information to sketch your own. Try to make your sketch as accurate as possible.

Read each test item and answer the questions that follow.

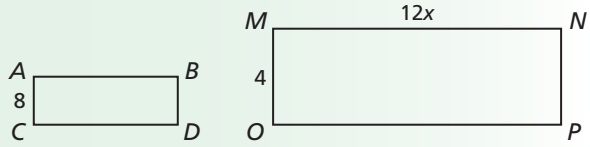
Item A

Short Response The width of a rectangle is 1.5 feet more than 4 times its length. Write a polynomial expression for the area of the rectangle. What is the area when the length is 16.75 feet?

1. What is the unknown measure in this problem?
2. How will drawing a diagram help you solve the problem?
3. Draw and label a sketch of the situation.

Item B

Multiple Choice Rectangle *ABDC* is similar to rectangle *MNPO*. If the width of rectangle *ABDC* is 8, what is its length?

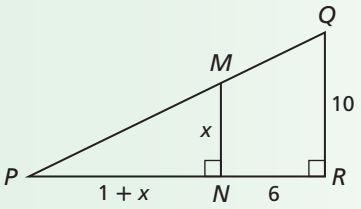


(A) 2
(B) $2x$
(C) $24x$
(D) 24

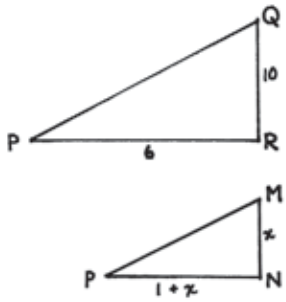
4. Look at the dimensions in the diagram. Do you think that the length of rectangle *ABDC* is greater or less than the length of rectangle *MNPO*?
5. Do you think the drawings reflect the information in the problem accurately? Why or why not?
6. Draw your own sketch to match the information in the problem.

Item C

Short Response Write a polynomial expression for the area of triangle *QRP*. Write a polynomial expression for the area of triangle *MNP*. Then use these expressions to write a polynomial expression for the area of *QRNM*.



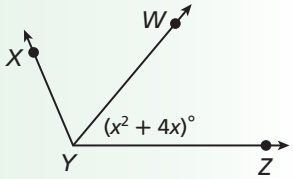
7. Describe how redrawing the figure can help you better understand the information in the problem.
8. After reading this test item, a student redrew the figure as shown below. Is this a correct interpretation of the original figure? Explain.



Item D

Multiple Choice The measure of angle *XYZ* is $(x^2 + 10x + 15)^\circ$. What is the measure of angle *XYW*?

(F) $(6x + 15)^\circ$
(G) $(2x^2 + 14x + 15)^\circ$
(H) $(14x + 15)^\circ$
(J) $(6x^2 + 15)^\circ$



9. What information does the diagram provide that the problem does not?
10. Will the measure of angle *XYW* be less than or greater than the measure of angle *XYZ*? Explain.

CHAPTER 7

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–7

Multiple Choice

1. A negative number is raised to a power. The result is a negative number. What do you know about the power?

Ⓐ It is an even number.
Ⓑ It is an odd number.
Ⓒ It is zero.
Ⓓ It is a whole number.

2. Which expression represents the phrase *eight less than the product of a number and two*?

Ⓕ $2 - 8x$
Ⓖ $8 - 2x$
Ⓗ $2x - 8$
Ⓙ $\frac{x}{2} - 8$

3. An Internet service provider charges a $20 set-up fee plus $12 per month. A competitor charges $15 per month. Which equation can you use to find x, the number of months when the total charge will be the same for both companies?

Ⓐ $15 = 20 + 12x$
Ⓑ $20 + 12x = 15x$
Ⓒ $20x + 12 = 15x$
Ⓓ $20 = 15x + 12x$

4. Which is a solution of the inequality $7 - 3(x - 3) > 2(x + 3)$?

Ⓕ 0
Ⓖ 2
Ⓗ 5
Ⓙ 12

5. One dose of Ted's medication contains 0.625 milligram, or $\frac{5}{8}$ milligram, of a drug. Which expression is equivalent to 0.625?

Ⓐ $5(4)^{-2}$
Ⓑ $5(2)^{-4}$
Ⓒ $5(-2)^{3}$
Ⓓ $5(2)^{-3}$

6. A restaurant claims to have served 352×10^6 hamburgers. What is this number in scientific notation?

Ⓕ 3.52×10^6
Ⓖ 3.52×10^8
Ⓗ 3.52×10^4
Ⓙ 352×10^6

7. Janet is ordering game cartridges from an online retailer. The retailer's prices, including shipping and handling, are given in the table below.

Game Cartridges	Total Cost ($)
1	54.95
2	104.95
3	154.95
4	204.95

Which equation best describes the relationship between the total cost c and the number of game cartridges g?

Ⓐ $c = 54.95g$
Ⓑ $c = 51g + 0.95$
Ⓒ $c = 50g + 4.95$
Ⓓ $c = 51.65g$

8. Which equation describes a line parallel to $y = 5 - 2x$?

Ⓕ $y = -2x + 8$
Ⓖ $y = 2x - 5$
Ⓗ $y = 5 + \frac{1}{2}x$
Ⓙ $y = 5 - \frac{1}{2}x$

9. A square has sides of length $x - 4$. A rectangle has a length of $x + 2$ and a width of $2x - 1$. What is the total combined area of the square and the rectangle?

Ⓐ $10x - 14$
Ⓑ $4x - 3$
Ⓒ $3x^2 - 5x + 14$
Ⓓ $3x^2 + 3x - 18$

Test writers develop multiple-choice test options with distracters. Distracters are incorrect options that are based on common student errors. Be cautious! Even if the answer you calculated is one of the options, it may not be the correct answer. Always check your work carefully.

10. Jennifer has a pocketful of change, all in nickels and quarters. There are 11 coins with a total value of \$1.15. Which system of equations can you use to find the number of each type of coin?

Ⓕ $\begin{cases} n + q = 11 \\ n + q = 1.15 \end{cases}$

Ⓖ $\begin{cases} n + q = 11 \\ 5n + 25q = 1.15 \end{cases}$

Ⓗ $\begin{cases} 5n + 25q = 11 \\ n + q = 1.15 \end{cases}$

Ⓙ $\begin{cases} n + q = 11 \\ 0.05n + 0.25q = 1.15 \end{cases}$

11. Which of the following is a true statement?

Ⓐ $\left[(a^m)^n\right]^p = a^{m+n+p}$

Ⓑ $\left[(a^m)^n\right]^p = a^{mn+p}$

Ⓒ $\left[(a^m)^n\right]^p = a^{mnp}$

Ⓓ $\left[(a^m)^n\right]^p = (a^{m+n})^p$

12. In 1867, the United States purchased the Alaska Territory from Russia for $\$7.2 \times 10^6$. The total area was about 6×10^5 square miles. What was the price per square mile?

Ⓕ About \$0.12 per square mile

Ⓖ About \$1.20 per square mile

Ⓗ About \$12.00 per square mile

Ⓙ About \$120.00 per square mile

Gridded Response

13. Evaluate the expression $3b^{-2}c^0$ for $b = 2$ and $c = -3$.

14. What is the slope of the line described by $-3y = -6x - 12$?

15. The quotient $(5.6 \times 10^8) \div (8 \times 10^3)$ is written in scientific notation as (7×10^n). What is the value of n?

16. The volume of a plastic cylinder is 64 cubic centimeters. A glass cylinder has the same height and a radius that is half that of the plastic cylinder. What is the volume in cubic centimeters of the glass cylinder?

Short Response

17. A sweater that normally sells for \$35 was marked down 20% and placed on the sale rack. Later, the sweater was marked down an additional 30% and placed on the clearance rack.

a. Find the price of the sweater while on the sale rack. Show your work.

b. Find the price of the sweater while on the clearance rack. Show your work.

18. A set of positive integers (a, b, c) is called a *Pythagorean triple* if $a^2 + b^2 = c^2$.

a. Find a^2, b^2, and c^2 when $a = 2x$, $b = x^2 - 1$, and $c = x^2 + 1$. Show your work.

b. Is $(2x, x^2 - 1, x^2 + 1)$ a Pythagorean triple? Explain your reasoning.

19. Ron is making an ice sculpture. The block of ice is in the shape of a rectangular prism with a length of $(x + 2)$ inches, a width of $(x - 2)$ inches, and a height of $2x$ inches.

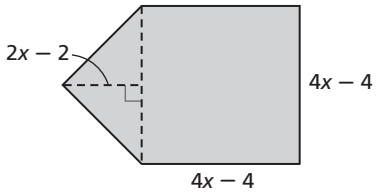
a. Write and simplify a polynomial expression for the volume of the block of ice. Show your work.

b. The final volume of the ice sculpture is $(x^3 + 4x^2 - 10x + 1)$ cubic inches. Write an expression for the volume of ice that Ron carved away. Show your work.

20. Simplify the expression $(3 \cdot a^2 \cdot b^{-4} \cdot a \cdot b^{-3})^{-3}$ using two different methods. Show that the results are the same.

Extended Response

21. Look at the pentagon below.



a. Write and simplify an expression that represents the area of the pentagon. Show your work or explain your answer.

b. Show one method of checking that your expression in part **a** is correct.

c. The triangular part of the pentagon can be rearranged to form a square. Write the area of this square as the square of a binomial.

d. Expand the product that you wrote in part **c**. What type of polynomial is this?

e. Is the square of a binomial ever a binomial? Explain your reasoning.