7-8 Multiplying Polynomials

Objective
Multiply polynomials.

Why learn this?
You can multiply polynomials to write expressions for areas, such as the area of a dulcimer. (See Example 5.)

To multiply monomials and polynomials, you will use some of the properties of exponents that you learned earlier in this chapter.

EXAMPLE 1 Multiplying Monomials

Multiply.

A $(5x^2)(4x^3)$

$(5x^2)(4x^3)$

$(5 \cdot 4)(x^2 \cdot x^3)$ *Group factors with like bases together.*

$20x^5$ *Multiply.*

B $(-3x^3y^2)(4xy^5)$

$(-3x^3y^2)(4xy^5)$

$(-3 \cdot 4)(x^3 \cdot x)(y^2 \cdot y^5)$ *Group factors with like bases together.*

$-12x^4y^7$ *Multiply.*

C $\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2} \cdot 6\right)(a^3 \cdot a^2)(b \cdot b^2)(c^2)$ *Group factors with like bases together.*

$3a^5b^3c^2$ *Multiply.*

Remember!
When multiplying powers with the same base, keep the base and add the exponents.
$x^2 \cdot x^3 = x^{2+3} = x^5$

CHECK IT OUT!

Multiply.

1a. $(3x^3)(6x^2)$ **1b.** $(2r^2t)(5t^3)$ **1c.** $\left(\frac{1}{3}x^2y\right)(12x^3z^2)(y^4z^5)$

To multiply a polynomial by a monomial, use the Distributive Property.

EXAMPLE 2 Multiplying a Polynomial by a Monomial

Multiply.

A $5(2x^2 + x + 4)$

$5(2x^2 + x + 4)$

$(5)2x^2 + (5)x + (5)4$ *Distribute 5.*

$10x^2 + 5x + 20$ *Multiply.*

Multiply.

B $2x^2y(3x - y)$

$(2x^2y)(3x - y)$

$(2x^2y)3x + (2x^2y)(-y)$ *Distribute $2x^2y$.*

$(2 \cdot 3)(x^2 \cdot x)y + 2(-1)(x^2)(y \cdot y)$ *Group like bases together.*

$6x^3y - 2x^2y^2$ *Multiply.*

C $4a(a^2b + 2b^2)$

$4a(a^2b + 2b^2)$

$(4a)a^2b + (4a)2b^2$ *Distribute 4a.*

$(4)(a \cdot a^2)(b) + (4 \cdot 2)(a)(b^2)$ *Group like bases together.*

$4a^3b + 8ab^2$ *Multiply.*

CHECK IT OUT!

Multiply.

2a. $2(4x^2 + x + 3)$ **2b.** $3ab(5a^2 + b)$ **2c.** $5r^2s^2(r - 3s)$

To multiply a binomial by a binomial, you can apply the Distributive Property more than once:

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$ *Distribute.*

$= x(x + 2) + 3(x + 2)$

$= x(x) + x(2) + 3(x) + 3(2)$ *Distribute again.*

$= x^2 + 2x + 3x + 6$ *Multiply.*

$= x^2 + 5x + 6$ *Combine like terms.*

Another method for multiplying binomials is called the FOIL method.

1. Multiply the **First** terms. $(x+3)(x+2) \rightarrow x \cdot x = x^2$
2. Multiply the **Outer** terms. $(x+3)(x+2) \rightarrow x \cdot 2 = 2x$
3. Multiply the **Inner** terms. $(x+3)(x+2) \rightarrow 3 \cdot x = 3x$
4. Multiply the **Last** terms. $(x+3)(x+2) \rightarrow 3 \cdot 2 = 6$

$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

F O I L

EXAMPLE 3 Multiplying Binomials

Multiply.

A $(x+2)(x-5)$

$(x+2)(x-5)$

$x(x-5)+2(x-5)$ *Distribute.*

$x(x)+x(-5)+2(x)+2(-5)$ *Distribute again.*

$x^2-5x+2x-10$ *Multiply.*

$x^2-3x-10$ *Combine like terms.*

B $(x+5)^2$

$(x+5)(x+5)$ *Write as a product of two binomials.*

$(x\cdot x)+(x\cdot 5)+(5\cdot x)+(5\cdot 5)$ *Use the FOIL method.*

$x^2+5x+5x+25$ *Multiply.*

$x^2+10x+25$ *Combine like terms.*

C $(3a^2-b)(a^2-2b)$

$3a^2(a^2)+3a^2(-2b)-b(a^2)-b(-2b)$ *Use the FOIL method.*

$3a^4-6a^2b-a^2b+2b^2$ *Multiply.*

$3a^4-7a^2b+2b^2$ *Combine like terms.*

Helpful Hint

In the expression $(x+5)^2$, the base is $(x+5)$.

$(x+5)^2=(x+5)(x+5)$

CHECK IT OUT! **3a.** $(a+3)(a-4)$ **3b.** $(x-3)^2$ **3c.** $(2a-b^2)(a+4b^2)$

To multiply polynomials with more than two terms, you can use the Distributive Property several times. Multiply $(5x+3)$ by $(2x^2+10x-6)$:

$$(5x+3)(2x^2+10x-6)=5x(2x^2+10x-6)+3(2x^2+10x-6)$$
$$=5x(2x^2+10x-6)+3(2x^2+10x-6)$$
$$=5x(2x^2)+5x(10x)+5x(-6)+3(2x^2)+3(10x)+3(-6)$$
$$=10x^3+50x^2-30x+6x^2+30x-18$$
$$=10x^3+56x^2-18$$

You can also use a rectangle model to multiply polynomials with more than two terms. This is similar to finding the area of a rectangle with length $(2x^2+10x-6)$ and width $(5x+3)$:

	$2x^2$	$+10x$	-6
$5x$	$10x^3$	$50x^2$	$-30x$
$+3$	$6x^2$	$30x$	-18

Write the product of the monomials in each row and column.

To find the product, add all of the terms inside the rectangle by combining like terms and simplifying if necessary.

$$10x^3+6x^2+50x^2+30x-30x-18$$
$$10x^3+56x^2-18$$

Another method that can be used to multiply polynomials with more than two terms is the vertical method. This is similar to methods used to multiply whole numbers.

$$\begin{array}{rl} & 2x^2 + 10x - 6 \\ \times & \qquad 5x + 3 \\ \hline & 6x^2 + 30x - 18 \\ + & 10x^3 + 50x^2 - 30x \\ \hline & 10x^3 + 56x^2 + 0x - 18 \\ & 10x^3 + 56x^2 \qquad - 18 \end{array}$$

Multiply each term in the top polynomial by 3.

Multiply each term in the top polynomial by 5x, and align like terms.

Combine like terms by adding vertically.

Simplify.

EXAMPLE 4 Multiplying Polynomials

Multiply.

A $(x + 2)(x^2 - 5x + 4)$

$(x + 2)(x^2 - 5x + 4)$

$x(x^2 - 5x + 4) + 2(x^2 - 5x + 4)$ *Distribute.*

$x(x^2) + x(-5x) + x(4) + 2(x^2) + 2(-5x) + 2(4)$ *Distribute again.*

$x^3 + 2x^2 - 5x^2 - 10x + 4x + 8$ *Simplify.*

$x^3 - 3x^2 - 6x + 8$ *Combine like terms.*

B $(3x - 4)(-2x^3 + 5x - 6)$

$(3x - 4)(-2x^3 + 5x - 6)$

$$\begin{array}{rl} & -2x^3 + 0x^2 + 5x - 6 \\ \times & \qquad 3x - 4 \\ \hline & 8x^3 + 0x^2 - 20x + 24 \\ + & -6x^4 + 0x^3 + 15x^2 - 18x \\ \hline & -6x^4 + 8x^3 + 15x^2 - 38x + 24 \end{array}$$

Add $0x^2$ as a placeholder.

Multiply each term in the top polynomial by –4.

Multiply each term in the top polynomial by 3x, and align like terms.

Combine like terms by adding vertically.

C $(x - 2)^3$

$[(x - 2)(x - 2)](x - 2)$ *Write as the product of three binomials.*

$[x \cdot x + x(-2) + (-2)x + (-2)(-2)](x - 2)$ *Use the FOIL method on the first two factors.*

$(x^2 - 2x - 2x + 4)(x - 2)$ *Multiply.*

$(x^2 - 4x + 4)(x - 2)$ *Combine like terms.*

$(x - 2)(x^2 - 4x + 4)$ *Use the Commutative Property of Multiplication.*

$x(x^2 - 4x + 4) + (-2)(x^2 - 4x + 4)$ *Distribute.*

$x(x^2) + x(-4x) + x(4) + (-2)(x^2) + (-2)(-4x) + (-2)(4)$ *Distribute again.*

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$ *Simplify.*

$x^3 - 6x^2 + 12x - 8$ *Combine like terms.*

Helpful Hint

A polynomial with *m* terms multiplied by a polynomial with *n* terms has a product that, before simplifying, has *mn* terms. In Example 4A, there are 2 • 3, or 6, terms before simplifying.

Multiply.

D $(2x+3)(x^2-6x+5)$

	x^2	$-6x$	$+5$
$2x$	$2x^3$	$-12x^2$	$10x$
$+3$	$3x^2$	$-18x$	15

Write the product of the monomials in each row and column.

$2x^3 + 3x^2 - 12x^2 - 18x + 10x + 15$ *Add all terms inside the rectangle.*

$2x^3 - 9x^2 - 8x + 15$ *Combine like terms.*

CHECK IT OUT!

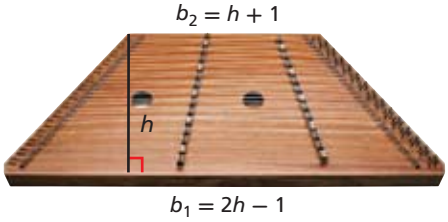
Multiply.

4a. $(x+3)(x^2-4x+6)$

4b. $(3x+2)(x^2-2x+5)$

EXAMPLE 5 *Music Application*

A dulcimer is a musical instrument that is sometimes shaped like a trapezoid.



A **Write a polynomial that represents the area of the dulcimer shown.**

$A = \frac{1}{2}h(b_1 + b_2)$ *Write the formula for area of a trapezoid.*

$= \frac{1}{2}h[(2h-1) + (h+1)]$ *Substitute $2h - 1$ for b_1 and $h + 1$ for b_2.*

$= \frac{1}{2}h(3h)$ *Combine like terms.*

$= \frac{3}{2}h^2$ *Simplify.*

The area is represented by $\frac{3}{2}h^2$.

B **Find the area of the dulcimer when the height is 22 inches.**

$A = \frac{3}{2}h^2$ *Use the polynomial from part **a**.*

$= \frac{3}{2}(22)^2$ *Substitute 22 for h.*

$= \frac{3}{2}(484) = 726$

The area is 726 square inches.

CHECK IT OUT!

5. The length of a rectangle is 4 meters shorter than its width.

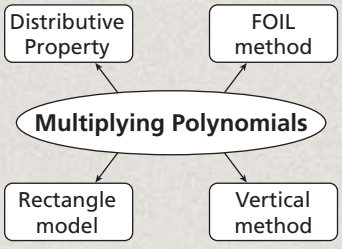
a. Write a polynomial that represents the area of the rectangle.

b. Find the area of the rectangle when the width is 6 meters.

THINK AND DISCUSS

1. Compare the vertical method for multiplying polynomials with the vertical method for multiplying whole numbers.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, multiply two polynomials using the given method.



Know it! Note

7-8 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA11 7-8
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Multiply.

SEE EXAMPLE 1 p. 512

1. $(2x^2)(7x^4)$
2. $(-5mn^3)(4m^2n^2)$
3. $(6rs^2)(s^3t^2)\left(\frac{1}{2}r^4t^3\right)$
4. $\left(\frac{1}{3}a^5\right)(12a)$
5. $(-3x^4y^2)(-7x^3y)$
6. $(-2pq^3)(5p^2q^2)(-3q^4)$

SEE EXAMPLE 2 p. 512

7. $4(x^2 + 2x + 1)$
8. $3ab(2a^2 + 3b^3)$
9. $2a^3b(3a^2b + ab^2)$
10. $-3x(x^2 - 4x + 6)$
11. $5x^2y(2xy^3 - y)$
12. $5m^2n^3 \cdot mn^2(4m - n)$

SEE EXAMPLE 3 p. 514

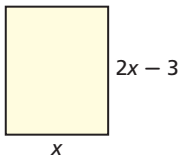
13. $(x + 1)(x - 2)$
14. $(x + 1)^2$
15. $(x - 2)^2$
16. $(y - 3)(y - 5)$
17. $(4a^3 - 2b)(a - 3b^2)$
18. $(m^2 - 2mn)(3mn + n^2)$

SEE EXAMPLE 4 p. 515

19. $(x + 5)(x^2 - 2x + 3)$
20. $(3x + 4)(x^2 - 5x + 2)$
21. $(2x - 4)(-3x^3 + 2x - 5)$
22. $(-4x + 6)(2x^3 - x^2 + 1)$
23. $(x - 5)(x^2 + x + 1)$
24. $(a + b)(a - b)(b - a)$

SEE EXAMPLE 5 p. 516

25. **Photography** The length of a rectangular photograph is 3 inches less than twice the width.
    a. Write a polynomial that represents the area of the photograph.
    b. Find the area of the photograph when the width is 4 inches.



PRACTICE AND PROBLEM SOLVING

Independent Practice

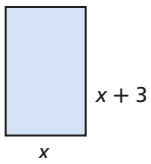
For Exercises	See Example
26–34	1
35–43	2
44–52	3
53–61	4
62	5

Extra Practice
Skills Practice p. S17
Application Practice p. S34

Multiply.

26. $(3x^2)(8x^5)$
27. $(-2r^3s^4)(6r^2s)$
28. $(15xy^2)\left(\frac{1}{3}x^2z^3\right)(y^3z^4)$
29. $(-2a^3)(-5a)$
30. $(6x^3y^2)(-2x^2y)$
31. $(-3a^2b)(-2b^3)(-a^3b^2)$
32. $(7x^2)(xy^5)(2x^3y^2)$
33. $(-4a^3bc^2)(a^3b^2c)(3ab^4c^5)$
34. $(12mn^2)(2m^2n)(mn)$
35. $9s(s + 6)$
36. $9(2x^2 - 5x)$
37. $3x(9x^2 - 4x)$
38. $3(2x^2 + 5x + 4)$
39. $5s^2t^3(2s - 3t^2)$
40. $x^2y^3 \cdot 5x^2y(6x + y^2)$
41. $-5x(2x^2 - 3x - 1)$
42. $-2a^2b^3(3ab^2 - a^2b)$
43. $-7x^3y \cdot x^2y^2(2x - y)$
44. $(x + 5)(x - 3)$
45. $(x + 4)^2$
46. $(m - 5)^2$
47. $(5x - 2)(x + 3)$
48. $(3x - 4)^2$
49. $(5x + 2)(2x - 1)$
50. $(x - 1)(x - 2)$
51. $(x - 8)(7x + 4)$
52. $(2x + 7)(3x + 7)$
53. $(x + 2)(x^2 - 3x + 5)$
54. $(2x + 5)(x^2 - 4x + 3)$
55. $(5x - 1)(-2x^3 + 4x - 3)$
56. $(x - 3)(x^2 - 5x + 6)$
57. $(2x^2 - 3)(4x^3 - x^2 + 7)$
58. $(x - 4)^3$
59. $(x - 2)(x^2 + 2x + 1)$
60. $(2x + 10)(4 - x + 6x^3)$
61. $(1 - x)^3$

62. **Geometry** The length of the rectangle at right is 3 feet longer than its width.
    a. Write a polynomial that represents the area of the rectangle.
    b. Find the area of the rectangle when the width is 5 feet.



63. A square tabletop has side lengths of $(4x - 6)$ units. Write a polynomial that represents the area of the tabletop.

MULTI-STEP TEST PREP

64. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. Marie is creating a garden. She designs a rectangular garden with a length of $(x + 4)$ feet and a width of $(x + 1)$ feet. Draw a diagram of Marie's garden with the length and width labeled.

b. Write a polynomial that represents the area of Marie's garden.

c. What is the area when $x = 4$?

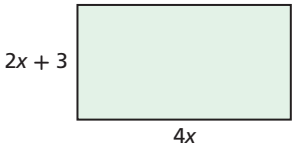
65. Copy and complete the table below.

	A	Degree of A	B	Degree of B	A • B	Degree of A • B
	$2x^2$	2	$3x^5$	5	$6x^7$	7
a.	$5x^3$		$2x^2 + 1$			
b.	$x^2 + 2$		$x^2 - x$			
c.	$x - 3$		$x^3 - 2x^2 + 1$			

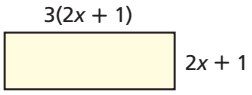
d. Use the results from the table to complete the following: The product of a polynomial of degree m and a polynomial of degree n has a degree of ____.

Geometry **Write a polynomial that represents the area of each rectangle.**

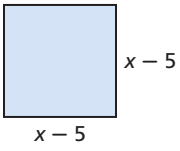
66.



67.



68.

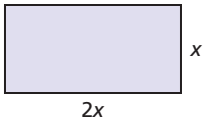


LINK

Sports

Team handball is a game with elements of soccer and basketball. It originated in Europe in the 1900s and was first played at the Olympics in 1936 with teams of 11 players. Today, a handball team consists of seven players—six court players and one goalie.

69. Sports The length of a regulation team handball court is twice its width.



a. Write a polynomial that represents the area of the court.

b. The width of a team handball court is 20 meters. Find the area of the court.

Multiply.

70. $(1.5a^3)(4a^6)$

71. $(2x + 5)(x - 6)$

72. $(3g - 1)(g + 5)$

73. $(4x - 2y)(2x - 3y)$

74. $(x + 3)(x - 3)$

75. $(1.5x - 3)(4x + 2)$

76. $(x - 10)(x + 4)$

77. $x^2(x + 3)$

78. $(x + 1)(x^2 + 2x)$

79. $(x - 4)(2x^2 + x - 6)$

80. $(a + b)(a - b)^2$

81. $(2p - 3q)^3$

82. Multi-Step A rectangular swimming pool is 25 feet long and 10 feet wide. It is surrounded by a fence that is x feet from each side of the pool.

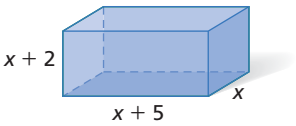
a. Draw a diagram of this situation.

b. Write expressions for the length and width of the fenced region. (*Hint:* How much longer is one side of the fenced region than the corresponding side of the pool?)

c. Write an expression for the area of the fenced region.

83. Write About It Explain why the FOIL method can be used to multiply only two binomials at a time.

84. Geometry Write a polynomial that represents the volume of the rectangular prism.



85. Critical Thinking Is there any value for x that would make the statement $(x + 3)^3 = x^3 + 3^3$ true? Give an example to justify your answer.

86. Estimation The length of a rectangle is 1 foot more than its width. Write a polynomial that represents the area of the rectangle. Estimate the width of the rectangle if its area is 25 square feet.

TEST PREP

87. Which of the following products is equal to $a^2 - 5a - 6$?

Ⓐ $(a - 1)(a - 5)$ Ⓑ $(a - 2)(a - 3)$ Ⓒ $(a + 1)(a - 6)$ Ⓓ $(a + 2)(a - 3)$

88. Which of the following is equal to $2a(a^2 - 1)$?

Ⓕ $2a^2 - 2a$ Ⓖ $2a^3 - 1$ Ⓗ $2a^3 - 2a$ Ⓙ $2a^2 - 1$

89. What is the degree of the product of $3x^3y^2z$ and x^2yz?

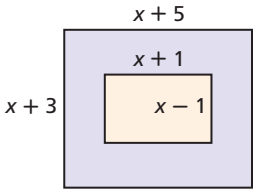
Ⓐ 5 Ⓑ 6 Ⓒ 7 Ⓓ 10

CHALLENGE AND EXTEND

Simplify.

90. $6x^2 - 2(3x^2 - 2x + 4)$ **91.** $x^2 - 2x(x + 3)$ **92.** $x(4x - 2) + 3x(x + 1)$

93. The diagram shows a sandbox and the frame that surrounds it.



a. Write a polynomial that represents the area of the sandbox.

b. Write a polynomial that represents the area of the frame that surrounds the sandbox.

94. Geometry The side length of a square is $(8 + 2x)$ units. The area of this square is the same as the perimeter of another square with a side length of $(x^2 + 48)$ units. Find the value of x.

95. Write a polynomial that represents the product of three consecutive integers. Let x represent the first integer.

96. Find m and n so that $x^m(x^n + x^{n-2}) = x^5 + x^3$.

97. Find a so that $2x^a(5x^{2a-3} + 2x^{2a+2}) = 10x^3 + 4x^8$

SPIRAL REVIEW

98. A stop sign is 2.5 meters tall and casts a shadow that is 3.5 meters long. At the same time, a flagpole casts a shadow that is 28 meters long. How tall is the flagpole? *(Lesson 2-8)*

Find the distance, to the nearest hundredth, between each pair of points. *(Lesson 5-5)*

99. (2, 3) and (4, 6) **100.** (−1, 4) and (0, 8) **101.** (−3, 7) and (−6, −2)

Graph the solutions of each linear inequality. *(Lesson 6-5)*

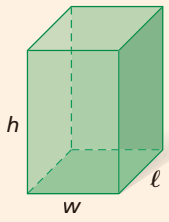
102. $y \leq x - 2$ **103.** $4x - 2y < 10$ **104.** $-y \geq -3x + 1$

Connecting Algebra to Geometry

Volume and Surface Area

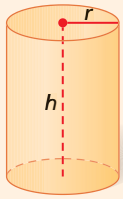
The volume V of a three-dimensional figure is the amount of space it occupies. The surface area S is the total area of the two-dimensional surfaces that make up the figure.

Rectangular Prism



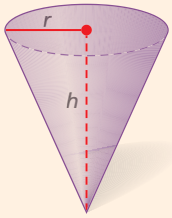
$V = \ell wh$
$S = 2(\ell w + \ell h + wh)$

Cylinder



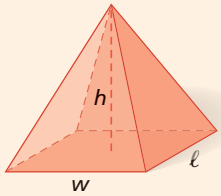
$V = \pi r^2 h$
$S = 2\pi r^2 + 2\pi rh$

Cone



$V = \frac{1}{3}\pi r^2 h$

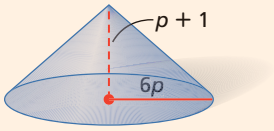
Pyramid



$V = \frac{1}{3}\ell wh$

Example

Write and simplify a polynomial expression for the volume of the cone. Leave the symbol π in your answer.



$V = \frac{1}{3}\pi r^2 h$ — *Choose the correct formula.*

$= \frac{1}{3}\pi(6p)^2(p+1)$ — *Substitute 6p for r and p + 1 for h.*

$= \frac{1}{3}\pi(36p^2)(p+1)$ — *Use the Power of a Product Property.*

$= \frac{1}{3}(36)\pi[p^2(p+1)]$ — *Use the Associative and Commutative Properties of Multiplication.*

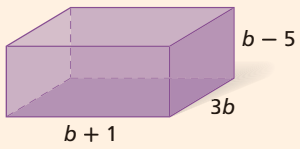
$= 12\pi p^2(p+1)$ — *Distribute $12\pi p^2$.*

$= 12\pi p^3 + 12\pi p^2$

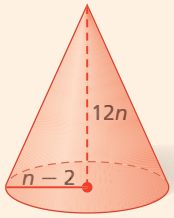
Try This

Write and simplify a polynomial expression for the volume of each figure.

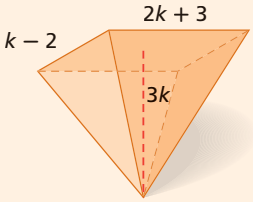
1.



2.

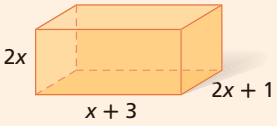


3.

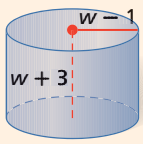


Write and simplify a polynomial expression for the surface area of each figure.

4.



5.



6.