7-7 Adding and Subtracting Polynomials

Objective
Add and subtract polynomials.

Who uses this?
Business owners can add and subtract polynomials that model profit. (See Example 4.)

Just as you can perform operations on numbers, you can perform operations on polynomials. To add or subtract polynomials, combine like terms.

www.cartoonstock.com

EXAMPLE 1 Adding and Subtracting Monomials

Add or subtract.

A $15m^3 + 6m^2 + 2m^3$

$15m^3 + 6m^2 + 2m^3$ — *Identify like terms.*

$15m^3 + 2m^3 + 6m^2$ — *Rearrange terms so that like terms are together.*

$17m^3 + 6m^2$ — *Combine like terms.*

B $3x^2 + 5 - 7x^2 + 12$

$3x^2 + 5 - 7x^2 + 12$ — *Identify like terms.*

$3x^2 - 7x^2 + 5 + 12$ — *Rearrange terms so that like terms are together.*

$-4x^2 + 17$ — *Combine like terms.*

C $0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$

$0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$ — *Identify like terms.*

$0.9y^5 - 0.4y^5 + y^5 + 0.5x^5$ — *Rearrange terms so that like terms are together.*

$1.5y^5 + 0.5x^5$ — *Combine like terms.*

D $2x^2y - x^2y - x^2y$

$2x^2y - x^2y - x^2y$ — *All terms are like terms.*

0 — *Combine.*

Remember!
Like terms are constants or terms with the same variable(s) raised to the same power(s). To review combining like terms, see Lesson 1-7.

CHECK IT OUT! **Add or subtract.**

1a. $2s^2 + 3s^2 + s$

1b. $4z^4 - 8 + 16z^4 + 2$

1c. $2x^8 + 7y^8 - x^8 - y^8$

1d. $9b^3c^2 + 5b^3c^2 - 13b^3c^2$

Polynomials can be added in either vertical or horizontal form.

In vertical form, align the like terms and add:

$$\begin{array}{r} 5x^2 + 4x + 1 \\ +\ 2x^2 + 5x + 2 \\ \hline 7x^2 + 9x + 3 \end{array}$$

In horizontal form, use the Associative and Commutative Properties to regroup and combine like terms:

$$\begin{aligned} &(5x^2 + 4x + 1) + (2x^2 + 5x + 2) \\ &= (5x^2 + 2x^2) + (4x + 5x) + (1 + 2) \\ &= 7x^2 + 9x + 3 \end{aligned}$$

EXAMPLE 2 Adding Polynomials

Add.

A $(2x^2 - x) + (x^2 + 3x - 1)$

$(2x^2 - x) + (x^2 + 3x - 1)$ *Identify like terms.*

$(2x^2 + x^2) + (-x + 3x) + (-1)$ *Group like terms together.*

$3x^2 + 2x - 1$ *Combine like terms.*

B $(-2ab + b) + (2ab + a)$

$(-2ab + b) + (2ab + a)$ *Identify like terms.*

$(-2ab + 2ab) + b + a$ *Group like terms together.*

$0 + b + a$ *Combine like terms.*

$b + a$ *Simplify.*

C $(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$

$(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$ *Identify like terms.*

$(4b^5 + 8b) + (-4b^5 + 7b)$ *Combine like terms in the second polynomial.*

$4b^5 + 8b$

$+ \underline{-4b^5 + 7b}$ *Use the vertical method.*

$0 \quad + 15b$ *Combine like terms.*

$15b$ *Simplify.*

D $(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$

$(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$ *Identify like terms.*

$20.2y^2 + 6y + 5$ *Use the vertical method.*

$+ \underline{1.7y^2 + 0y - 8}$ *Write 0y as a placeholder in the second polynomial.*

$21.9y^2 + 6y - 3$ *Combine like terms.*

Writing Math

When you use the Associative and Commutative Properties to rearrange the terms, the sign in front of each term must stay with that term.

CHECK IT OUT!

2. Add $(5a^3 + 3a^2 - 6a + 12a^2) + (7a^3 - 10a)$.

To subtract polynomials, remember that subtracting is the same as adding the opposite. To find the opposite of a polynomial, you must write the opposite of *each* term in the polynomial:

$$-(2x^3 - 3x + 7) = -2x^3 + 3x - 7$$

EXAMPLE 3 Subtracting Polynomials

Subtract.

A $(2x^2 + 6) - (4x^2)$

$(2x^2 + 6) + (-4x^2)$ *Rewrite subtraction as addition of the opposite.*

$(2x^2 + 6) + (-4x^2)$ *Identify like terms.*

$(2x^2 - 4x^2) + 6$ *Group like terms together.*

$-2x^2 + 6$ *Combine like terms.*

B $(a^4 - 2a) - (3a^4 - 3a + 1)$

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Rewrite subtraction as addition of the opposite.*

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Identify like terms.*

$(a^4 - 3a^4) + (-2a + 3a) - 1$ *Group like terms together.*

$-2a^4 + a - 1$ *Combine like terms.*

Subtract.

C $(3x^2 - 2x + 8) - (x^2 - 4)$

$(3x^2 - 2x + 8) + (-x^2 + 4)$ *Rewrite subtraction as addition of the opposite.*

$(3x^2 - 2x + 8) + (-x^2 + 4)$ *Identify like terms.*

$3x^2 - 2x + 8$ *Use the vertical method.*

$+ -x^2 + 0x + 4$ *Write 0x as a placeholder.*

$2x^2 - 2x + 12$ *Combine like terms.*

D $(11z^3 - 2z) - (z^3 - 5)$

$(11z^3 - 2z) + (-z^3 + 5)$ *Rewrite subtraction as addition of the opposite.*

$(11z^3 - 2z) + (-z^3 + 5)$ *Identify like terms.*

$11z^3 - 2z + 0$ *Use the vertical method.*

$+ -z^3 + 0z + 5$ *Write 0 and 0z as placeholders.*

$10z^3 - 2z + 5$ *Combine like terms.*

CHECK IT OUT! **3.** Subtract $(2x^2 - 3x^2 + 1) - (x^2 + x + 1)$.

EXAMPLE 4 *Business Application*

The profits of two different manufacturing plants can be modeled as shown, where x is the number of units produced at each plant.

Eastern:
$-0.03x^2 + 25x - 1500$

Southern:
$-0.02x^2 + 21x - 1700$

Write a polynomial that represents the difference of the profits at the eastern plant and the profits at the southern plant.

$(-0.03x^2 + 25x - 1500)$ *Eastern plant profits*

$-(-0.02x^2 + 21x - 1700)$ *Southern plant profits*

$-0.03x^2 + 25x - 1500$

$+(+0.02x^2 - 21x + 1700)$ *Write subtraction as addition of the opposite.*

$-0.01x^2 + 4x + 200$ *Combine like terms.*

CHECK IT OUT! **4.** Use the information above to write a polynomial that represents the total profits from both plants.

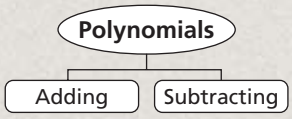
THINK AND DISCUSS

1. Identify the like terms in the following list: $-12x^2$, $-4.7y$, $\frac{1}{5}x^2y$, y, $3xy^2$, $-9x^2$, $5x^2y$, $-12x$

2. Describe how to find the opposite of $9t^2 - 5t + 8$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example that shows how to perform the given operation.



7-7 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA11 7-7
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 504

Add or subtract.

1. $7a^2 - 10a^2 + 9a$
2. $13x^2 + 9y^2 - 6x^2$
3. $0.07r^4 + 0.32r^3 + 0.19r^4$
4. $\frac{1}{4}p^3 + \frac{2}{3}p^3$
5. $5b^3c + b^3c - 3b^3c$
6. $-8m + 5 - 16 + 11m$

SEE EXAMPLE 2 p. 505

Add.

7. $(5n^3 + 3n + 6) + (18n^3 + 9)$
8. $(3.7q^2 - 8q + 3.7) + (4.3q^2 - 2.9q + 1.6)$
9. $(-3x + 12) + (9x^2 + 2x - 18)$
10. $(9x^4 + x^3) + (2x^4 + 6x^3 - 8x^4 + x^3)$

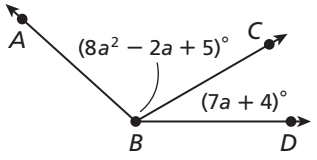
SEE EXAMPLE 3 p. 505

Subtract.

11. $(6c^4 + 8c + 6) - (2c^4)$
12. $(16y^2 - 8y + 9) - (6y^2 - 2y + 7y)$
13. $(2r + 5) - (5r - 6)$
14. $(-7k^2 + 3) - (2k^2 + 5k - 1)$

SEE EXAMPLE 4 p. 506

15. **Geometry** Write a polynomial that represents the measure of angle *ABD*.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–24	1
25–28	2
29–32	3
33–34	4

Extra Practice

Skills Practice p. S17
Application Practice p. S34

Add or subtract.

16. $4k^3 + 6k^2 + 9k^3$
17. $5m + 12n^2 + 6n - 8m$
18. $2.5a^4 - 8.1b^4 - 3.6b^4$
19. $2d^5 + 1 - d^5$
20. $7xy - 4x^2y - 2xy$
21. $-6x^3 + 5x + 2x^3 + 4x^3$
22. $x^2 + x + 3x + 2x^2$
23. $3x^3 - 4 - x^3 - 1$
24. $3b^3 - 2b - 1 - b^3 - b$

Add.

25. $(2t^2 - 8t) + (8t^2 + 9t)$
26. $(-7x^2 - 2x + 3) + (4x^2 - 9x)$
27. $(x^5 - x) + (x^4 + x)$
28. $(-2z^3 + z + 2z^3 + z) + (3z^3 - 5z^2)$

Subtract.

29. $(t^3 + 8t^2) - (3t^3)$
30. $(3x^2 - x) - (x^2 + 3x - x)$
31. $(5m + 3) - (6m^3 - 2m^2)$
32. $(3s^2 + 4s) - (-10s^2 + 6s)$

33. **Photography** The measurements of a photo and its frame are shown in the diagram. Write a polynomial that represents the width of the photo.

34. **Geometry** The length of a rectangle is represented by $4a + 3b$, and its width is represented by $7a - 2b$. Write a polynomial for the perimeter of the rectangle.

Add or subtract.

35. $(2t - 7) + (-t + 2)$

36. $(4m^2 + 3m) + (-2m^2)$

37. $(4n - 2) - 2n$

38. $(-v - 7) - (-2v)$

39. $(4x^2 + 3x - 6) + (2x^2 - 4x + 5)$

40. $(2z^2 - 3z - 3) + (2z^2 - 7z - 1)$

41. $(5u^2 + 3u + 7) - (u^3 + 2u^2 + 1)$

42. $(-7h^2 - 4h + 7) - (7h^2 - 4h + 11)$

43. Geometry The length of a rectangle is represented by $2x + 3$, and its width is represented by $3x + 7$. The perimeter of the rectangle is 35 units. Find the value of x.

44. Write About It If the parentheses are removed from $(3m^2 - 5m) + (12m^2 + 7m - 10)$, is the new expression equivalent to the original? If the parentheses are removed from $(3m^2 - 5m) - (12m^2 + 7m - 10)$, is the new expression equivalent to the original? Explain.

45. ERROR ANALYSIS Two students found the sum of the polynomials $(-3n^4 + 6n^3 + 4n^2)$ and $(8n^4 - 3n^2 + 9n)$. Which is incorrect? Explain the error.

A

$$\begin{array}{r} -3n^4 + 6n^3 + 4n^2 + 0n \\ +\ 8n^4 + 0n^3 - 3n^2 + 9n \\ \hline 5n^4 + 6n^3 + n^2 + 9n \end{array}$$

B

$$\begin{array}{r} -3n^4 + 6n^3 + 4n^2 \\ +\ 8n^4 - 3n^2 + 9n \\ \hline 5n^4 + 3n^3 + 13n^2 \end{array}$$

Copy and complete the table by finding the missing polynomials.

	Polynomial 1	Polynomial 2	Sum
46.	$x^2 - 6$	$3x^2 - 10x + 2$	
47.	$12x + 5$		$15x + 11$
48.		$5x^4 + 8$	$6x^4 - 3x^2 - 1$
49.	$7x^3 - 6x - 3$		$7x^3 + 11$
50.	$2x^3 + 5x^2$	$7x^3 - 5x^2 + 1$	
51.		$x + x^2 + 6$	$3x^2 + 2x + 1$

52. Critical Thinking Does the order in which you add polynomials affect the sum? Does the order in which you subtract polynomials affect the difference? Explain.

MULTI-STEP TEST PREP

53. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. Ian plans to build a fenced dog pen. At first, he planned for the pen to be a square of length x feet on each side, but then he decided that a square may not be best. He added 4 feet to the length and subtracted 3 feet from the width. Draw a diagram to show the dimensions of the new pen.

b. Write a polynomial that represents the amount of fencing that Ian will need for the new dog pen.

c. How much fencing will Ian need if $x = 15$?

TEST PREP

54. What is the missing term?

$(-14y^2 + 9y^2 - 12y + 3) + (2y^2 + \square - 6y - 2) = (-3y^2 - 15y + 1)$

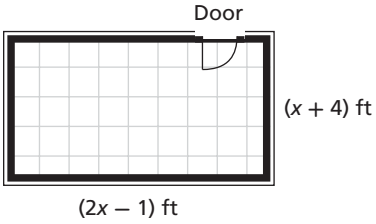
Ⓐ $-6y$ Ⓑ $-3y$ Ⓒ $3y$ Ⓓ $6y$

55. Which is NOT equivalent to $-5t^3 - t$?

Ⓕ $-(5t^3 + t)$ Ⓗ $(t^3 + 6t) - (6t^3 + 7t)$

Ⓖ $(2t^3 - 4t) - (-7t - 3t)$ Ⓙ $(2t^3 - 3t^2 + t) - (7t^3 - 3t^2 + 2t)$

56. Extended Response Tammy plans to put a wallpaper border around the perimeter of her room. She will not put the border across the doorway, which is 3 feet wide.



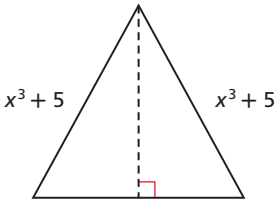
a. Write a polynomial that represents the number of feet of wallpaper border that Tammy will need.

b. A local store has 50 feet of the border that Tammy has chosen. What is the greatest whole-number value of x for which this amount would be enough for Tammy's room? Justify your answer.

c. Determine the dimensions of Tammy's room for the value of x that you found in part **b.**

CHALLENGE AND EXTEND

57. Geometry The legs of the isosceles triangle at right measure $(x^3 + 5)$ units. The perimeter of the triangle is $(2x^3 + 3x^2 + 8)$ units. Write a polynomial that represents the measure of the base of the triangle.



58. Write two polynomials whose sum is $4m^3 + 3m$.

59. Write two polynomials whose difference is $4m^3 + 3m$.

60. Write three polynomials whose sum is $4m^3 + 3m$.

61. Write two monomials whose sum is $4m^3 + 3m$.

62. Write three trinomials whose sum is $4m^3 + 3m$.

SPIRAL REVIEW

Solve each inequality and graph the solutions. *(Lesson 3-2)*

63. $d + 5 \geq -2$ **64.** $15 < m - 11$ **65.** $-6 + t < -6$

Write each equation in slope-intercept form. Then graph the line described by each equation. *(Lesson 5-7)*

66. $3x + y = 8$ **67.** $2y = \frac{1}{2}x + 6$ **68.** $y = 4(-x + 1)$

Simplify. *(Lesson 7-3)*

69. $b^4 \cdot b^7$ **70.** $cd^4 \cdot (c^{-5})^3$ **71.** $(-3z^6)^2$ **72.** $(j^3k^{-5})^3 \cdot (k^2)^4$

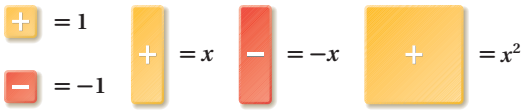
7-8 Algebra LAB

Model Polynomial Multiplication

Use with Lesson 7-8

You can use algebra tiles to multiply polynomials. Use the length and width of a rectangle to represent the factors. The area of the rectangle represents the product.

KEY

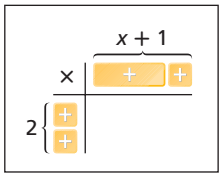
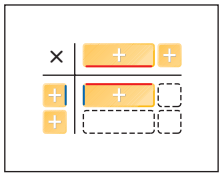
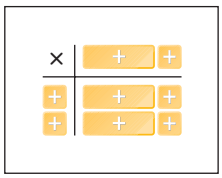


REMEMBER

- The product of two values with the same sign is positive.
- The product of two values with different signs is negative.

Activity 1

Use algebra tiles to find $2(x + 1)$.

MODEL		ALGEBRA
	*Place the first factor in a column along the left side of the grid. This will be the width of the rectangle.* *Place the second factor across the top of the grid. This will be the length of the rectangle.*	$2(x + 1)$
	*Fill in the grid with tiles that have the same width as the tiles in the left column and the same length as the tiles in the top row.*	
	*The area of the rectangle inside the grid represents the product.*	$x + x + 1 + 1$ $2x + 2$

The rectangle has an area of $2x + 2$, so $2(x + 1) = 2x + 2$. Notice that this is the same product you would get by using the Distributive Property to multiply $2(x + 1)$.

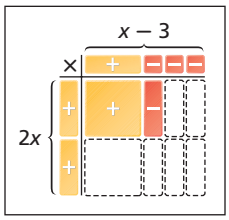
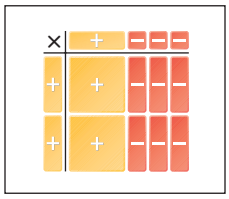
Try This

Use algebra tiles to find each product.

1. $3(x + 2)$ **2.** $2(2x + 1)$ **3.** $3(x + 1)$ **4.** $3(2x + 2)$

Activity 2

Use algebra tiles to find $2x(x-3)$.

MODEL		ALGEBRA
	*Place tiles to form the length and width of a rectangle and fill in the rectangle.* *The product of two values with the same sign (same color) is positive (yellow).* *The product of two values with different signs (different colors) is negative (red).*	$2x(x-3)$
	*The area of the rectangle inside the grid represents the product.* *The rectangle has an area of $2x^2 - 6x$, so $2x(x-3) = 2x^2 - 6x$.*	$x^2 + x^2 - x - x - x - x - x - x$ $2x^2 - 6x$

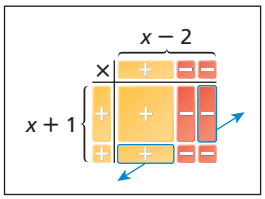
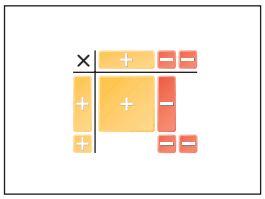
Try This

Use algebra tiles to find each product.

5. $3x(x-2)$ **6.** $x(2x-1)$ **7.** $x(x+1)$ **8.** $(8x+5)(-2x)$

Activity 3

Use algebra tiles to find $(x+1)(x-2)$.

MODEL		ALGEBRA
	*Place tiles for each factor to form the length and width of a rectangle.* *Fill in the grid and remove any zero pairs.*	$(x+1)(x-2)$ $x^2 - x - x + x - 1 - 1$
	*The area inside the grid represents the product.* *The remaining area is $x^2 - x - 2$, so $(x+1)(x-2) = x^2 - x - 2$.*	$x^2 - x - 1 - 1$ $x^2 - x - 2$

Try This

Use algebra tiles to find each product.

9. $(x+2)(x-3)$ **10.** $(x-1)(x+3)$ **11.** $(x-2)(x-3)$ **12.** $(x+1)(x+2)$