7-6 Polynomials

Objectives
Classify polynomials and write polynomials in standard form.

Evaluate polynomial expressions.

Vocabulary
monomial
degree of a monomial
polynomial
degree of a polynomial
standard form of a polynomial
leading coefficient
quadratic
cubic
binomial
trinomial

Who uses this?
Pyrotechnicians can use polynomials to plan complex fireworks displays. (See Example 5.)

A **monomial** is a number, a variable, or a product of numbers and variables with whole-number exponents.

Monomials	Not Monomials
$5 \quad x \quad -7xy \quad 0.5x^4$	$-0.3x^{-2} \quad 4x - y \quad \frac{2}{x^3}$

The **degree of a monomial** is the sum of the exponents of the variables. A constant has degree 0.

EXAMPLE 1 Finding the Degree of a Monomial

Find the degree of each monomial.

A $-2a^2b^4$

The degree is 6. *Add the exponents of the variables: 2 + 4 = 6*

B 4

$4x^0$ *There is no variable, but you can write 4 as $4x^0$.*

The degree is 0.

C $8y$

$8y^1$ *A variable written without an exponent has exponent 1.*

The degree is 1.

Remember!
The *terms* of an expression are the parts being added or subtracted. See Lesson 1-7.

CHECK IT OUT!

Find the degree of each monomial.

1a. $1.5k^2m$ **1b.** $4x$ **1c.** $2c^3$

A **polynomial** is a monomial or a sum or difference of monomials. The **degree of a polynomial** is the degree of the term with the greatest degree.

EXAMPLE 2 Finding the Degree of a Polynomial

Find the degree of each polynomial.

A $4x - 18x^5$

$4x$: degree 1 $-18x^5$: degree 5 *Find the degree of each term.*

The degree of the polynomial is the greatest degree, 5.

Find the degree of each polynomial.

B $0.5x^2y + 0.25xy + 0.75$

$0.5x^2y$: degree 3 $0.25xy$: degree 2 0.75: degree 0

The degree of the polynomial is the greatest degree, 3.

C $6x^4 + 9x^2 - x + 3$

$6x^4$: degree 4 $9x^2$: degree 2 $-x$: degree 1 3: degree 0

The degree of the polynomial is the greatest degree, 4.

Find the degree of each polynomial.

2a. $5x - 6$ **2b.** $x^3y^2 + x^2y^3 - x^4 + 2$

The terms of a polynomial may be written in any order. However, polynomials that contain only one variable are usually written in *standard form*.

The **standard form of a polynomial** that contains one variable is written with the terms in order from greatest degree to least degree. When written in standard form, the coefficient of the first term is called the **leading coefficient**.

EXAMPLE 3 Writing Polynomials in Standard Form

Write each polynomial in standard form. Then give the leading coefficient.

A $20x - 4x^3 + 2 - x^2$

Find the degree of each term. Then arrange them in descending order.

$$\underbrace{20x}_{1}\ \underbrace{-4x^3}_{3}\ \underbrace{+2}_{0}\ \underbrace{-x^2}_{2} \longrightarrow \underbrace{-4x^3}_{3}\ \underbrace{-x^2}_{2}\ \underbrace{+20x}_{1}\ \underbrace{+2}_{0}$$

The standard form is $-4x^3 - x^2 + 20x + 2$. The leading coefficient is -4.

B $y^3 + y^5 + 4y$

Find the degree of each term. Then arrange them in descending order.

$$\underbrace{y^3}_{3}\ \underbrace{+y^5}_{5}\ \underbrace{+4y}_{1} \longrightarrow \underbrace{y^5}_{5}\ \underbrace{+y^3}_{3}\ \underbrace{+4y}_{1}$$

The standard form is $y^5 + y^3 + 4y$. The leading coefficient is 1.

Remember!

A variable written without a coefficient has a coefficient of 1.

$y^5 = 1y^5$

Write each polynomial in standard form. Then give the leading coefficient.

3a. $16 - 4x^2 + x^5 + 9x^3$ **3b.** $18y^5 - 3y^8 + 14y$

Some polynomials have special names based on their degree and the number of terms they have.

Degree	Name
0	Constant
1	Linear
2	**Quadratic**
3	**Cubic**
4	Quartic
5	Quintic
6 or more	6th degree, 7th degree, and so on

Terms	Name
1	Monomial
2	**Binomial**
3	**Trinomial**
4 or more	Polynomial

EXAMPLE 4 Classifying Polynomials

Classify each polynomial according to its degree and number of terms.

A $5x - 6$

Degree: 1 Terms: 2 $5x - 6$ is a linear binomial.

B $y^2 + y + 4$

Degree: 2 Terms: 3 $y^2 + y + 4$ is a quadratic trinomial.

C $6x^7 + 9x^2 - x + 3$

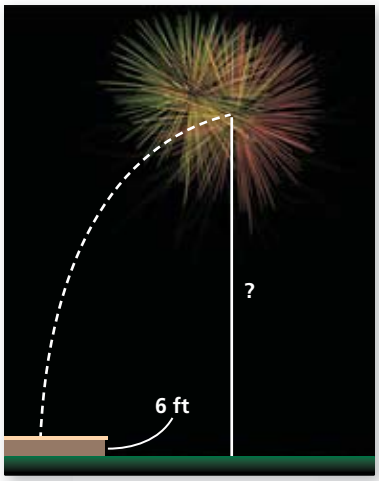
Degree: 7 Terms: 4 $6x^7 + 9x^2 - x + 3$ is a 7th-degree polynomial.

CHECK IT OUT! **Classify each polynomial according to its degree and number of terms.**

4a. $x^3 + x^2 - x + 2$ **4b.** 6 **4c.** $-3y^8 + 18y^5 + 14y$

EXAMPLE 5 *Physics Application*

A firework is launched from a platform 6 feet above the ground at a speed of 200 feet per second. The firework has a 5-second fuse. The height of the firework in feet is given by the polynomial $-16t^2 + 200t + 6$, where t is the time in seconds. How high will the firework be when it explodes?



Substitute the time for t to find the firework's height.

$-16t^2 + 200t + 6$

$-16(5)^2 + 200(5) + 6$ *The time is 5 seconds.*

$-16(25) + 200(5) + 6$

$-400 + 1000 + 6$ *Evaluate the polynomial by using the order of operations.*

606

When the firework explodes, it will be 606 feet above the ground.

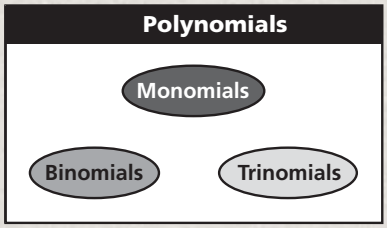
CHECK IT OUT! **5. What if...?** Another firework with a 5-second fuse is launched from the same platform at a speed of 400 feet per second. Its height is given by $-16t^2 + 400t + 6$. How high will this firework be when it explodes?

THINK AND DISCUSS

1. Explain why each expression is not a polynomial: $2x^2 + 3x^{-3}$; $1 - \frac{a}{b}$.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each oval, write an example of the given type of polynomial.



7-6 Exercises

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GUIDED PRACTICE

Vocabulary Match each polynomial on the left with its classification on the right.

1. $2x^3 + 6$
2. $3x^3 + 4x^2 - 7$
3. $5x^2 - 2x + 3x^4 - 6$

a. quartic polynomial
b. quadratic polynomial
c. cubic trinomial
d. cubic binomial

SEE EXAMPLE 1 p. 496

Find the degree of each monomial.

4. 10^6
5. $-7xy^2$
6. $0.4n^8$
7. 2

SEE EXAMPLE 2 p. 496

Find the degree of each polynomial.

8. $x^2 - 2x + 1$
9. $0.75a^2b - 2a^3b^5$
10. $15y - 84y^3 + 100 - 3y^2$
11. $r^3 + r^2 - 5$
12. $a^3 + a^2 - 2a$
13. $3k^4 + k^3 - 2k^2 + k$

SEE EXAMPLE 3 p. 497

Write each polynomial in standard form. Then give the leading coefficient.

14. $-2b + 5 + b^2$
15. $9a^8 - 8a^9$
16. $5s^2 - 3s + 3 - s^7$
17. $2x + 3x^2 - 1$
18. $5g - 7 + g^2$
19. $3c^2 + 5c^4 + 5c^3 - 4$

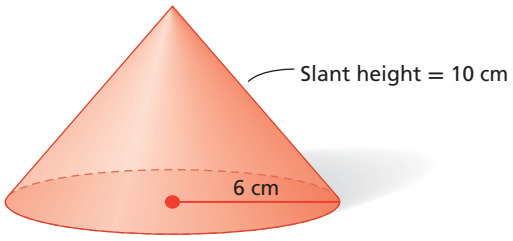
SEE EXAMPLE 4 p. 498

Classify each polynomial according to its degree and number of terms.

20. $x^2 + 2x + 3$
21. $x - 7$
22. $8 + k + 5k^4$
23. $q^2 + 6 - q^3 + 3q^4$
24. $5k^2 + 7k^3$
25. $2a^3 + 4a^2 - a^4$

SEE EXAMPLE 5 p. 498

26. **Geometry** The surface area of a cone is approximated by the polynomial $3.14r^2 + 3.14r\ell$, where r is the radius and ℓ is the slant height. Find the approximate surface area of this cone.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–34	1
35–40	2
41–49	3
50–57	4
58	5

Extra Practice

Skills Practice p. S17
Application Practice p. S34

Find the degree of each monomial.

27. $3y^4$
28. $6k$
29. $2a^3b^2c$
30. 325
31. $2y^4z^3$
32. $9m^5$
33. p
34. 5

Find the degree of each polynomial.

35. $a^2 + a^4 - 6a$
36. $3^2b - 5$
37. $3.5y^2 - 4.1y - 6$
38. $-5f^4 + 2f^6 + 10f^8$
39. $4n^3 - 2n$
40. $4r^3 + 4r^6$

Write each polynomial in standard form. Then give the leading coefficient.

41. $2.5 + 4.9t^3 - 4t^2 + t$
42. $8a - 10a^2 + 2$
43. $x^7 - x + x^3 - x^5 + x^{10}$
44. $-m + 7 - 3m^2$
45. $3x^2 + 5x - 4 + 5x^3$
46. $-2n + 1 - n^2$
47. $4d + 3d^2 - d^3 + 5$
48. $3s^2 + 12s^3 + 6$
49. $4x^2 - x^5 - x^3 + 1$

LINK

Transportation

Hybrid III is the crash test dummy used by the Insurance Institute for Highway Safety. During a crash test, sensors in the dummy's head, neck, chest, legs, and feet measure and record forces. Engineers study this data to help design safer cars.

Classify each polynomial according to its degree and number of terms.

50. 12 **51.** $6k$ **52.** $3.5x^3 - 4.1x - 6$ **53.** $4g + 2g^2 - 3$

54. $2x^2 - 6x$ **55.** $6 - s^3 - 3s^4$ **56.** $c^2 + 7 - 2c^3$ **57.** $-y^2$

58. Transportation The polynomial $3.675v + 0.096v^2$ is used by transportation officials to estimate the stopping distance in feet for a car whose speed is v miles per hour on flat, dry pavement. What is the stopping distance for a car traveling at 30 miles per hour?

Tell whether each statement is sometimes, always, or never true.

59. A monomial is a polynomial.

60. A trinomial is a 3rd-degree polynomial.

61. A binomial is a trinomial.

62. A polynomial has two or more terms.

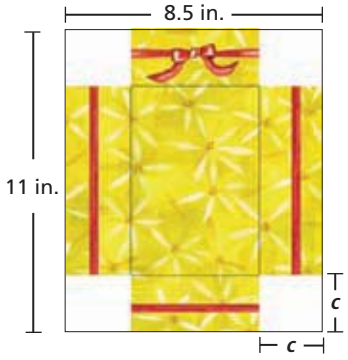
63. Geometry A piece of 8.5-by-11-inch cardboard has identical squares cut from its corners. It is then folded into a box with no lid. The volume of the box in cubic inches is $4c^3 - 39c^2 + 93.5c$, where c is the side length of the missing squares in inches.

a. What is the volume of the box if $c = 1$ in.?

b. What is the volume of the box if $c = 1.5$ in.?

c. What is the volume of the box if $c = 4.25$ in.?

d. Critical Thinking Does your answer to part **c** make sense? Explain why or why not.



Copy and complete the table by evaluating each polynomial for the given values of x.

	Polynomial	$x = -2$	$x = 0$	$x = 5$
64.	$5x - 6$	$5(-2) - 6 = -16$	$5(0) - 6 = -6$	
65.	$x^5 + x^3 + 4x$			
66.	$-10x^2$			

Give one example of each type of polynomial.

67. quadratic trinomial **68.** linear binomial **69.** constant monomial

70. cubic monomial **71.** quintic binomial **72.** 12th-degree trinomial

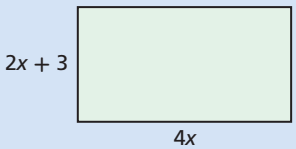
73. Write About It Explain the steps you would follow to write the polynomial $4x^3 - 3 + 5x^2 - 2x^4 - x$ in standard form.

MULTI-STEP TEST PREP

74. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. The perimeter of the rectangle shown is $12x + 6$. What is the degree of this polynomial?

b. The area of the rectangle is $8x^2 + 12x$. What is the degree of this polynomial?



75. **ERROR ANALYSIS** Two students evaluated $4x - 3x^5$ for $x = -2$. Which is incorrect? Explain the error.

A

$4(-2) - 3(-2)^5$
$-8 + 6^5$
$-8 + 7776$
7768

B

$4(-2) - 3(-2)^5$
$-8 - 3(-32)$
$-8 + 96$
88

76. Which polynomial has the highest degree?

Ⓐ $3x^8 - 2x^7 + x^6$ Ⓑ $5x - 100$ Ⓒ $25x^{10} + 3x^5 - 15$ Ⓓ $134x^2$

77. What is the value of $-3x^3 + 4x^2 - 5x + 7$ when $x = -1$?

Ⓕ 3 Ⓖ 13 Ⓗ 9 Ⓙ 19

78. **Short Response** A toy rocket is launched from the ground at 75 feet per second. The polynomial $-16t^2 + 75t$ gives the rocket's height in feet after t seconds. Make a table showing the rocket's height after 1 second, 2 seconds, 3 seconds, and 4 seconds. At which of these times will the rocket be the highest?

CHALLENGE AND EXTEND

79. **Medicine** Doctors and nurses use growth charts and formulas to tell whether a baby is developing normally. The polynomial $0.016m^3 - 0.390m^2 + 4.562m + 50.310$ gives the average length in centimeters of a baby boy between 0 and 10 months of age, where m is the baby's age in months.

a. What is the average length of a 2-month-old baby boy? a 5-month-old baby boy? Round your answers to the nearest centimeter.

b. What is the average length of a newborn (0-month-old) baby boy?

c. How could you find the answer to part **b** without doing any calculations?

80. Consider the binomials $4x^5 + x$, $4x^4 + x$, and $4x^3 + x$.

a. Without calculating, which binomial has the greatest value for $x = 5$?

b. Are there any values of x for $4x^3 + x$ which will have the greatest value? Explain.

SPIRAL REVIEW

81. Jordan is allowed 90 minutes of screen time per day. Today, he has already used m minutes. Write an expression for the remaining number of minutes Jordan has today. *(Lesson 1-1)*

82. Pens cost \$0.50 each. Giselle bought p pens. Write an expression for the total cost of Giselle's pens. *(Lesson 1-1)*

Classify each system. Give the number of solutions. *(Lesson 6-4)*

83. $\begin{cases} y = -4x + 5 \\ 4x + y = 2 \end{cases}$ **84.** $\begin{cases} 2x + 8y = 10 \\ 4y = -x + 5 \end{cases}$ **85.** $\begin{cases} y = 3x + 2 \\ y = -5x - 6 \end{cases}$

Simplify. *(Lesson 7-4)*

86. $\frac{4^7}{4^4}$ **87.** $\frac{x^6y^4}{x^4y^9}$ **88.** $\left(\frac{2v^4}{vw^5}\right)^2$ **89.** $\left(\frac{2p}{p^3}\right)^{-4}$

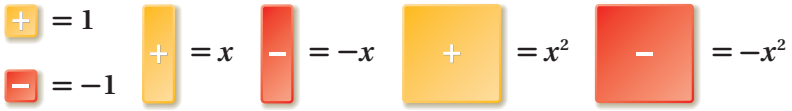
7-7 Algebra LAB

Model Polynomial Addition and Subtraction

You can use algebra tiles to model polynomial addition and subtraction.

Use with Lesson 7-7

KEY



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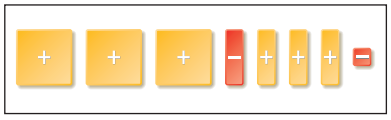
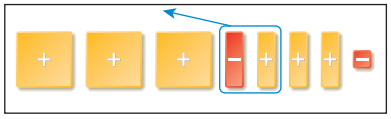
Activity 1

Use algebra tiles to find $(2x^2 - x) + (x^2 + 3x - 1)$.

MODEL		ALGEBRA
	*Use tiles to represent all terms from both expressions.*	$(2x^2 - x) + (x^2 + 3x - 1)$
	*Rearrange tiles so that like tiles are together. Like tiles are the same size and shape.*	$(2x^2 + x^2) + (-x + 3x) - 1$
	*Remove any zero pairs.*	$3x^2 - x + x + 2x - 1$
	*The remaining tiles represent the sum.*	$3x^2 + 2x - 1$

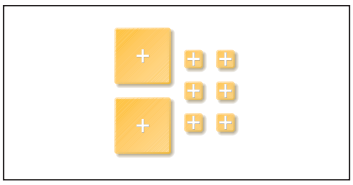
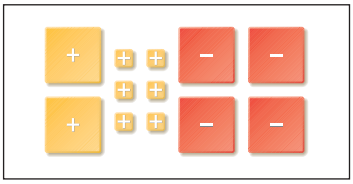
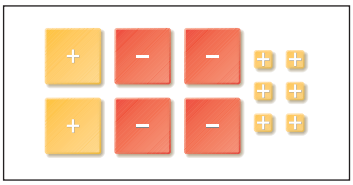
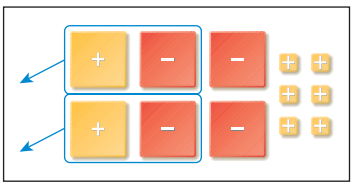
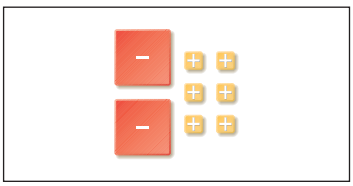
Try This

Use algebra tiles to find each sum.

1. $(-2x^2 + 1) + (-x^2)$
2. $(3x^2 + 2x + 5) + (x^2 - x - 4)$
3. $(x - 3) + (2x - 2)$
4. $(5x^2 - 3x - 6) + (x^2 + 3x + 6)$
5. $-5x^2 + (2x^2 + 5x)$
6. $(x^2 - x - 1) + (6x - 3)$

Activity 2

Use algebra tiles to find $(2x^2 + 6) - 4x^2$.

MODEL		ALGEBRA
	*Use tiles to represent the terms in the first expression.*	$2x^2 + 6$
To subtract $4x^2$, you would remove 4 yellow x^2-tiles, but there are not enough to do this. Remember that subtraction is the same as adding the opposite, so rewrite $(2x^2 + 6) - 4x^2$ as $(2x^2 + 6) + (-4x^2)$. 	*Add 4 red x^2-tiles.*	$2x^2 + 6 + (-4x^2)$
	*Rearrange tiles so that like tiles are together.*	$2x^2 + (-4x^2) + 6$
	*Remove zero pairs.*	$2x^2 + (-2x^2) + (-2x^2) + 6$
	*The remaining tiles represent the difference.*	$-2x^2 + 6$

Try This

Use algebra tiles to find each difference.

7. $(6x^2 + 4x) - 3x^2$

8. $(2x^2 + x - 7) - 5x$

9. $(3x + 6) - 6$

10. $(8x + 5) - (-2x)$

11. $(x^2 + 2x) - (-4x^2 + x)$

12. $(3x^2 - 4) - (x^2 + 6x)$

13.  represents a zero pair. Use algebra tiles to model two other zero pairs.

14. When is it not necessary to "add the opposite" for polynomial subtraction using algebra tiles?