7-4 Division Properties of Exponents

Objective
Use division properties of exponents to evaluate and simplify expressions.

Who uses this?
Economists can use expressions with exponents to calculate national debt statistics. (See Example 3.)

A quotient of powers with the same base can be found by writing the powers in factored form and dividing out common factors.

$$\frac{3^5}{3^3} = \frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot 3\cdot 3}{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}} = 3\cdot 3 = 3^2$$

Notice the relationship between the exponents in the original quotient and the exponent in the final answer: $5 - 3 = 2$.

Know it! Note

Quotient of Powers Property

WORDS	NUMBERS	ALGEBRA
The quotient of two nonzero powers with the same base equals the base raised to the difference of the exponents.	$\frac{6^7}{6^4} = 6^{7-4} = 6^3$	If a is a nonzero real number and m and n are integers, then $\frac{a^m}{a^n} = a^{m-n}$.

EXAMPLE 1 **Finding Quotients of Powers**

Simplify.

A $\frac{3^8}{3^2}$

$\frac{3^8}{3^2} = 3^{8-2}$

$= 3^6 = 729$

B $\frac{x^5}{x^5}$

$\frac{x^5}{x^5} = x^{5-5}$

$= x^0 = 1$

C $\frac{a^5b^9}{(ab)^4}$

$\frac{a^5b^9}{(ab)^4} = \frac{a^5b^9}{a^4b^4}$

$= a^{5-4} \cdot b^{9-4}$

$= a^1 \cdot b^5$

$= ab^5$

D $\frac{2^3 \cdot 3^2 \cdot 5^7}{2 \cdot 3^4 \cdot 5^5}$

$\frac{2^3 \cdot 3^2 \cdot 5^7}{2 \cdot 3^4 \cdot 5^5} = 2^{3-1} \cdot 3^{2-4} \cdot 5^{7-5}$

$= 2^2 \cdot 3^{-2} \cdot 5^2$

$= \frac{2^2 \cdot 5^2}{3^2}$

$= \frac{4 \cdot 25}{9} = \frac{100}{9}$

Helpful Hint
$3^6 = 729$
Both 3^6 and 729 are considered to be simplified.

CHECK IT OUT! Simplify.

1a. $\frac{2^9}{2^7}$ **1b.** $\frac{y}{y^4}$ **1c.** $\frac{m^5n^4}{(m^5)^2n}$ **1d.** $\frac{3^5 \cdot 2^4 \cdot 4^3}{3^4 \cdot 2^2 \cdot 4^6}$

EXAMPLE 2

Dividing Numbers in Scientific Notation

Simplify $(2 \times 10^8) \div (8 \times 10^5)$ and write the answer in scientific notation.

$$(2 \times 10^8) \div (8 \times 10^5) = \frac{2 \times 10^8}{8 \times 10^5}$$

$$= \frac{2}{8} \times \frac{10^8}{10^5}$$ *Write as a product of quotients.*

$$= 0.25 \times 10^{8-5}$$ *Simplify each quotient.*

$$= 0.25 \times 10^3$$ *Simplify the exponent.*

$$= 2.5 \times 10^{-1} \times 10^3$$ *Write 0.25 in scientific notation as 2.5×10^{-1}.*

$$= 2.5 \times 10^{-1+3}$$ *The second two terms have the same base, so add the exponents.*

$$= 2.5 \times 10^2$$ *Simplify the exponent.*

Writing Math

You can "split up" a quotient of products into a product of quotients:

$$\frac{a \times c}{b \times d} = \frac{a}{b} \times \frac{c}{d}$$

Example:

$$\frac{3 \times 4}{5 \times 7} = \frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$$

CHECK IT OUT!

2. Simplify $(3.3 \times 10^6) \div (3 \times 10^8)$ and write the answer in scientific notation.

EXAMPLE 3

Economics Application

In the year 2000, the United States public debt was about 5.6×10^{12} dollars. The population of the United States in that year was about 2.8×10^8 people. What was the average debt per person? Give your answer in standard form.

To find the average debt per person, divide the total debt by the number of people.

$$\frac{\text{total debt}}{\text{number of people}} = \frac{5.6 \times 10^{12}}{2.8 \times 10^8}$$

$$= \frac{5.6}{2.8} \times \frac{10^{12}}{10^8}$$ *Write as a product of quotients.*

$$= 2 \times 10^{12-8}$$ *Simplify each quotient.*

$$= 2 \times 10^4$$ *Simplify the exponent.*

$$= 20{,}000$$ *Write in standard form.*

The average debt per person was about \$20,000.

CHECK IT OUT!

3. In 1990, the United States public debt was about 3.2×10^{12} dollars. The population of the United States in 1990 was about 2.5×10^8 people. What was the average debt per person? Write your answer in standard form.

A power of a quotient can be found by first writing factors and then writing the numerator and denominator as powers.

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{2^3}{3^3}$$

Notice that the exponents in the final answer are the same as the exponent in the original expression.

Know it! Note

Positive Power of a Quotient Property

WORDS	NUMBERS	ALGEBRA
A quotient raised to a positive power equals the quotient of each base raised to that power.	$\left(\frac{3}{5}\right)^4 = \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{3^4}{5^4}$	If a and b are nonzero real numbers and n is a positive integer, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

EXAMPLE 4

Finding Positive Powers of Quotients

Simplify.

A $\left(\frac{3}{4}\right)^3$

$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3}$ *Use the Power of a Quotient Property.*

$= \frac{27}{64}$ *Simplify.*

B $\left(\frac{2x^3}{yz}\right)^3$

$\left(\frac{2x^3}{yz}\right)^3 = \frac{(2x^3)^3}{(yz)^3}$ *Use the Power of a Quotient Property.*

$= \frac{2^3(x^3)^3}{y^3z^3}$ *Use the Power of a Product Property: $(2x^3)^3 = 2^3(x^3)^3$ and $(yz)^3 = y^3z^3$.*

$= \frac{8x^9}{y^3z^3}$ *Simplify 2^3 and use the Power of a Power Property: $(x^3)^3 = x^{3 \cdot 3} = x^9$.*

CHECK IT OUT! **Simplify.**

4a. $\left(\frac{2^3}{3^2}\right)^2$ **4b.** $\left(\frac{ab^4}{c^2d^3}\right)^5$ **4c.** $\left(\frac{a^3b}{a^2b^2}\right)^3$

Remember that $x^{-n} = \frac{1}{x^n}$. What if x is a fraction?

$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = 1 \div \left(\frac{a}{b}\right)^n$ *Write the fraction as division.*

$= 1 \div \frac{a^n}{b^n}$ *Use the Power of a Quotient Property.*

$= 1 \cdot \frac{b^n}{a^n}$ *Multiply by the reciprocal.*

$= \frac{b^n}{a^n}$ *Simplify.*

$= \left(\frac{b}{a}\right)^n$ *Use the Power of a Quotient Property.*

Therefore, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Know it! Note

Negative Power of a Quotient Property

WORDS	NUMBERS	ALGEBRA
A quotient raised to a negative power equals the reciprocal of the quotient raised to the opposite (positive) power.	$\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^{4} = \frac{3^4}{2^4}$	If a and b are nonzero real numbers and n is a positive integer, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$.

EXAMPLE 5 **Finding Negative Powers of Quotients**

Simplify.

A $\left(\frac{2}{5}\right)^{-3}$

$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$ — *Rewrite with a positive exponent.*

$= \frac{5^3}{2^3}$ — *Use the Power of a Quotient Property.*

$= \frac{125}{8}$ — *$5^3 = 125$ and $2^3 = 8$.*

B $\left(\frac{3x}{y^2}\right)^{-3}$

$\left(\frac{3x}{y^2}\right)^{-3} = \left(\frac{y^2}{3x}\right)^{3}$ — *Rewrite with a positive exponent.*

$= \frac{(y^2)^3}{(3x)^3}$ — *Use the Power of a Quotient Property.*

$= \frac{y^6}{3^3x^3}$ — *Use the Power of a Power Property: $(y^2)^3 = y^{2\cdot 3} = y^6$. Use the Power of a Product Property: $(3x)^3 = 3^3x^3$.*

$= \frac{y^6}{27x^3}$ — *Simplify the denominator.*

C $\left(\frac{3}{4}\right)^{-1}\left(\frac{2x}{3y}\right)^{-2}$

$\left(\frac{3}{4}\right)^{-1}\left(\frac{2x}{3y}\right)^{-2} = \left(\frac{4}{3}\right)^{1}\left(\frac{3y}{2x}\right)^{2}$ — *Rewrite each fraction with a positive exponent.*

$= \frac{4}{3} \cdot \frac{(3y)^2}{(2x)^2}$ — *Use the Power of a Quotient Property.*

$= \frac{4}{3} \cdot \frac{3^2y^2}{2^2x^2}$ — *Use the Power of a Product Property: $(3y)^2 = 3^2y^2$ and $(2x)^2 = 2^2x^2$.*

$= \frac{{}^{1}\cancel{4}}{{}_{1}\cancel{3}} \cdot \frac{\cancel{9}^{3}y^2}{{}_{1}\cancel{4}x^2}$ — *Divide out common factors.*

$= \frac{3y^2}{x^2}$

Helpful Hint

Whenever all of the factors in the numerator or the denominator divide out, replace them with 1.

CHECK IT OUT! **Simplify.**

5a. $\left(\frac{4}{3^2}\right)^{-3}$ **5b.** $\left(\frac{2a}{b^2c^3}\right)^{-4}$ **5c.** $\left(\frac{s}{3}\right)^{-2}\left(\frac{9s^2}{t}\right)^{-1}$

THINK AND DISCUSS

1. Compare the Quotient of Powers Property and the Product of Powers Property. Then compare the Power of a Quotient Property and the Power of a Product Property.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each cell, supply the missing information. Then give an example for each property.

If *a* and *b* are nonzero real numbers and *m* and *n* are integers, then…		
$\frac{a^m}{a^n} = \square$	$\left(\frac{a}{b}\right)^n = \frac{\square}{\square}$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{\square}{\square}\right)^{\square}$

7-4 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 481

Simplify.

1. $\frac{5^8}{5^6}$ **2.** $\frac{2^2 \cdot 3^4 \cdot 4^4}{2^9 \cdot 3^5}$ **3.** $\frac{15x^6}{5x^6}$ **4.** $\frac{a^5b^6}{a^3b^7}$

SEE EXAMPLE 2 p. 482

Simplify each quotient and write the answer in scientific notation.

5. $(2.8 \times 10^{11}) \div (4 \times 10^8)$ **6.** $(5.5 \times 10^3) \div (5 \times 10^8)$ **7.** $(1.9 \times 10^4) \div (1.9 \times 10^4)$

SEE EXAMPLE 3 p. 482

8. Sports A star baseball player earns an annual salary of $\$8.1 \times 10^6$. There are 162 games in a baseball season. How much does this player earn per game? Write your answer in standard form.

Simplify.

SEE EXAMPLE 4 p. 483

9. $\left(\frac{2}{5}\right)^2$ **10.** $\left(\frac{x^2}{xy^3}\right)^3$ **11.** $\left(\frac{a^3}{(a^3b)^2}\right)^2$ **12.** $\frac{y^{10}}{y}$

SEE EXAMPLE 5 p. 484

13. $\left(\frac{3}{4}\right)^{-2}$ **14.** $\left(\frac{2x}{y^3}\right)^{-4}$ **15.** $\left(\frac{2}{3}\right)^{-1}\left(\frac{3a}{2b}\right)^{-2}$ **16.** $\left(\frac{x^3}{y^2}\right)^{-4}$

PRACTICE AND PROBLEM SOLVING

Simplify.

17. $\frac{3^9}{3^6}$ **18.** $\frac{5^4 \cdot 3^3}{5^2 \cdot 3^2}$ **19.** $\frac{x^8y^3}{x^3y^3}$ **20.** $\frac{x^8y^4}{x^9yz}$

Simplify each quotient and write the answer in scientific notation.

21. $(4.7 \times 10^{-3}) \div (9.4 \times 10^3)$ **22.** $(8.4 \times 10^9) \div (4 \times 10^{-5})$

23. $(4.2 \times 10^{-5}) \div (6 \times 10^{-3})$ **24.** $(2.1 \times 10^2) \div (8.4 \times 10^5)$

Independent Practice

For Exercises	See Example
17–20	1
21–24	2
25	3
26–29	4
30–33	5

Extra Practice

Skills Practice p. S16
Application Practice p. S34

25. **Astronomy** The mass of Earth is about 3×10^{-3} times the mass of Jupiter. The mass of Earth is about 6×10^{24} kg. What is the mass of Jupiter? Give your answer in scientific notation.

Simplify.

26. $\left(\frac{2}{3}\right)^4$ 27. $\left(\frac{a^4}{b^2}\right)^3$ 28. $\left(\frac{a^3b^2}{ab^3}\right)^6$ 29. $\left(\frac{xy^2}{x^3y}\right)^3$

30. $\left(\frac{1}{7}\right)^{-3}$ 31. $\left(\frac{x^2}{y^5}\right)^{-5}$ 32. $\left(\frac{8w^7}{16}\right)^{-1}$ 33. $\left(\frac{1}{4}\right)^{-2}\left(\frac{6x}{7}\right)^{-2}$

Simplify, if possible.

34. $\frac{x^6}{x^5}$ 35. $\frac{8d^5}{4d^3}$ 36. $\frac{x^2y^3}{a^2b^3}$ 37. $\frac{(3x^3)^3}{(6x^2)^2}$

38. $\frac{(5x^2)^3}{5x^2}$ 39. $\left(\frac{c^2a^3}{a^5}\right)^2$ 40. $\left(\frac{3a}{a^3 \cdot a^0}\right)^3$ 41. $\left(\frac{-p^4}{-5p^3}\right)^{-2}$

42. $\left(\frac{b^{-2}}{b^3}\right)^2$ 43. $\left(\frac{10^2}{10^{-5} \cdot 10^5}\right)^{-1}$ 44. $\left(\frac{x^2y^2}{x^2y}\right)^{-3}$ 45. $\frac{(-x^2)^4}{-(x^2)^4}$

46. **Critical Thinking** How can you use the Quotient of a Power Property to explain the definition of x^{-n}? $\left(\textit{Hint:}\text{ Think of } \frac{1}{x^n} \text{ as } \frac{x^0}{x^n}.\right)$

47. **Geography** *Population density* is the number of people per unit of area. The area of the United States is approximately 9.37×10^6 square kilometers. The table shows population data from the U. S. Census Bureau.

United States Population

Year	Population (to nearest million)
2000	2.81×10^8
1995	2.66×10^8
1990	2.48×10^8

Write the approximate population density (people per square kilometer) for each of the given years in scientific notation. Round decimals to the nearest hundredth.

48. **Chemistry** The pH of a solution is a number that describes the concentration of hydrogen ions in that solution. For example, if the concentration of hydrogen ions in a solution is 10^{-4}, that solution has a pH of 4.

Lemon juice
pH 2

Apples
pH 3

Water
pH 7

Ammonia
pH 11

a. What is the concentration of hydrogen ions in lemon juice?
b. What is the concentration of hydrogen ions in water?
c. How many times more concentrated are the hydrogen ions in lemon juice than in water?

49. **Write About It** Explain how to simplify $\frac{4^5}{4^2}$. How is it different from simplifying $\frac{4^2}{4^5}$?

Find the missing exponent(s).

50. $\frac{x^{\square}}{x^4} = x^2$ 51. $\frac{x^7}{x^{\square}} = x^4$ 52. $\left(\frac{a^2}{b^{\square}}\right)^4 = \frac{a^8}{b^{12}}$ 53. $\left(\frac{x^4}{y^{\square}}\right)^{-1} = \frac{y^3}{x^{\square}}$

MULTI-STEP TEST PREP

54. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. Yellow light has a wavelength of 589 nm. A nanometer (nm) is 10^{-9} m. What is 589 nm in meters? Write your answer in scientific notation.

b. The speed of light in air, v, is 3×10^8 m/s, and $v = fw$, where f represents the frequency in hertz (Hz) and w represents the wavelength in meters. What is the frequency of yellow light?

TEST PREP

55. Which of the following is equivalent to $(8 \times 10^6) \div (4 \times 10^2)$?

Ⓐ 2×10^3 Ⓑ 2×10^4 Ⓒ 4×10^3 Ⓓ 4×10^4

56. Which of the following is equivalent to $\left(\frac{x^{12}}{3xy^4}\right)^{-2}$?

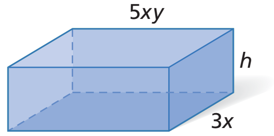
Ⓕ $\frac{9y^8}{x^{22}}$ Ⓖ $\frac{3y^8}{x^{22}}$ Ⓗ $\frac{3y^6}{x^{12}}$ Ⓙ $\frac{6y^8}{x^{26}}$

57. Which of the following is equivalent to $\frac{(-3x)^4}{-(3x)^4}$?

Ⓐ -1 Ⓑ 1 Ⓒ $-81x^4$ Ⓓ $\frac{1}{81x^4}$

CHALLENGE AND EXTEND

58. Geometry The volume of the prism at right is $V = 30x^4y^3$. Write and simplify an expression for the prism's height in terms of x and y.



59. Simplify $\frac{3^{2x}}{3^{2x-1}}$.

60. Simplify $\frac{(x+1)^2}{(x+1)^3}$.

61. Copy and complete the table below to show how the Quotient of Powers Property can be found by using the Product of Powers Property.

Statements	Reasons
1. $a^{m-n} = a^{\square + \square}$	**1.** Subtraction is addition of the opposite.
2. $= a^{\square} \cdot a^{\square}$	**2.** Product of Powers Property
3. $= a^m \cdot \frac{1}{a^n}$	**3.** ______?______
4. $= \frac{a^m}{\square}$	**4.** Multiplication can be written as division.

SPIRAL REVIEW

Find each square root. *(Lesson 1-5)*

62. $\sqrt{36}$ **63.** $\sqrt{1}$ **64.** $-\sqrt{49}$ **65.** $\sqrt{144}$

Solve each equation. *(Lesson 2-4)*

66. $-2(x-1) + 4x = 5x + 3$ **67.** $x - 1 - (4x + 3) = 5x$

Simplify. *(Lesson 7-3)*

68. $3^2 \cdot 3^3$ **69.** $k^5 \cdot k^{-2} \cdot k^{-3}$ **70.** $(4t^5)^2$ **71.** $-(5x^4)^3$