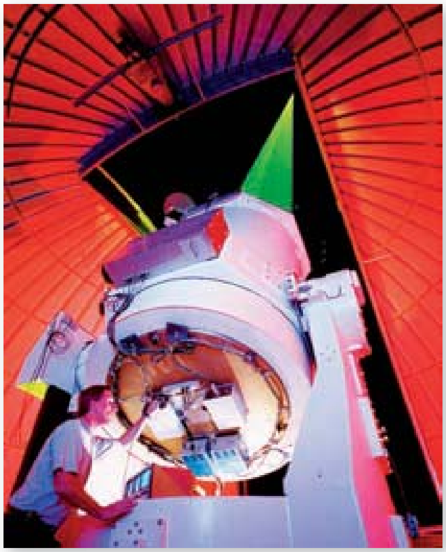
7-3 Multiplication Properties of Exponents



Objective
Use multiplication properties of exponents to evaluate and simplify expressions.

Who uses this?
Astronomers can multiply expressions with exponents to find the distance between objects in space. (See Example 2.)

You have seen that exponential expressions are useful when writing very small or very large numbers. To perform operations on these numbers, you can use properties of exponents. You can also use these properties to simplify your answer.

In this lesson, you will learn some properties that will help you simplify exponential expressions containing multiplication.

Know it! Note

Simplifying Exponential Expressions

An exponential expression is completely simplified if...

- There are no negative exponents.
- The same base does not appear more than once in a product or quotient.
- No powers are raised to powers.
- No products are raised to powers.
- No quotients are raised to powers.
- Numerical coefficients in a quotient do not have any common factor other than 1.

Examples	Nonexamples
$\frac{b}{a}$ x^3 z^{12} a^4b^4 $\frac{s^5}{t^5}$ $\frac{5a^2}{2b}$	$a^{-2}ba$ $x \cdot x^2$ $(z^3)^4$ $(ab)^4$ $\left(\frac{s}{t}\right)^5$ $\frac{10a^2}{4b}$

Products of powers with the same base can be found by writing each power as repeated multiplication.

$$3^5 \cdot 3^2 = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3) = 3^7$$

Notice the relationship between the exponents in the factors and the exponent in the product: $5 + 2 = 7$.

Know it! Note

Product of Powers Property

WORDS	NUMBERS	ALGEBRA
The product of two powers with the same base equals that base raised to the sum of the exponents.	$6^7 \cdot 6^4 = 6^{7+4} = 6^{11}$	If a is any nonzero real number and m and n are integers, then $a^m \cdot a^n = a^{m+n}$.

EXAMPLE 1 Finding Products of Powers

Simplify.

A $2^5 \cdot 2^6$

$2^5 \cdot 2^6$

2^{5+6} — *Since the powers have the same base, keep the base and add the exponents.*

2^{11}

B $4^2 \cdot 3^{-2} \cdot 4^5 \cdot 3^6$

$4^2 \cdot 3^{-2} \cdot 4^5 \cdot 3^6$ — *Group powers with the same base together.*

$(4^2 \cdot 4^5) \cdot (3^{-2} \cdot 3^6)$

$4^{2+5} \cdot 3^{-2+6}$ — *Add the exponents of powers with the same base.*

$4^7 \cdot 3^4$

C $a^4 \cdot b^5 \cdot a^2$

$a^4 \cdot b^5 \cdot a^2$

$(a^4 \cdot a^2) \cdot b^5$ — *Group powers with the same base together.*

$a^6 \cdot b^5$ — *Add the exponents of powers with the same base.*

a^6b^5

D $y^2 \cdot y \cdot y^{-4}$

$(y^2 \cdot y^1) \cdot y^{-4}$ — *Group the first two powers.*

$y^3 \cdot y^{-4}$ — *The first two powers have the same base, so add the exponents.*

y^{-1} — *The two remaining powers have the same base, so add the exponents.*

$\frac{1}{y}$ — *Write with a positive exponent.*

Remember!

A number or variable written without an exponent actually has an exponent of 1.

$10 = 10^1$

$y = y^1$

CHECK IT OUT! Simplify.

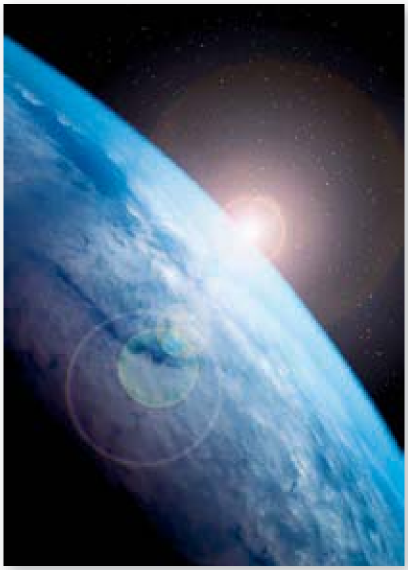
1a. $7^8 \cdot 7^4$

1b. $3^{-3} \cdot 5^8 \cdot 3^4 \cdot 5^2$

1c. $m \cdot n^{-4} \cdot m^4$

1d. $x \cdot x^{-1} \cdot x^{-3} \cdot x^{-4}$

EXAMPLE 2 *Astronomy Application*



Light from the Sun travels at about 1.86×10^5 miles per second. It takes about 500 seconds for the light to reach Earth. Find the approximate distance from the Sun to Earth. Write your answer in scientific notation.

distance = rate × time

$= (1.86 \times 10^5) \times 500$

$= (1.86 \times 10^5) \times (5 \times 10^2)$ — *Write 500 in scientific notation.*

$= (1.86 \times 5) \times (10^5 \times 10^2)$ — *Use the Commutative and Associative Properties to group.*

$= 9.3 \times 10^7$ — *Multiply within each group.*

The Sun is about 9.3×10^7 miles from Earth.

CHECK IT OUT!

2. Light travels at about 1.86×10^5 miles per second. Find the approximate distance that light travels in one hour. Write your answer in scientific notation.

To find a power of a power, you can use the meaning of exponents.

$$(4^3)^2 = 4^3 \cdot 4^3 = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) = 4^6$$

Notice the relationship between the exponents in the original power and the exponent in the final power: $3 \cdot 2 = 6$.

Know it! Note

Power of a Power Property

WORDS	NUMBERS	ALGEBRA
A power raised to another power equals that base raised to the product of the exponents.	$(6^7)^4 = 6^{7 \cdot 4} = 6^{28}$	If a is any nonzero real number and m and n are integers, then $(a^m)^n = a^{mn}$.

EXAMPLE 3 Finding Powers of Powers

Simplify.

A $(7^4)^3$

$7^{4 \cdot 3}$	*Use the Power of a Power Property.*
7^{12}	*Simplify.*

B $(3^6)^0$

$3^{6 \cdot 0}$	*Use the Power of a Power Property.*
3^0	*Zero multiplied by any number is zero.*
1	*Any number raised to the zero power is 1.*

C $(x^2)^{-4} \cdot x^5$

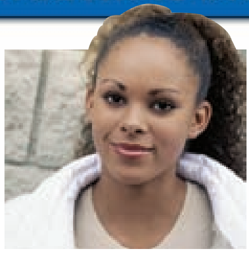
$x^{2 \cdot (-4)} \cdot x^5$	*Use the Power of a Power Property.*
$x^{-8} \cdot x^5$	*Simplify the exponent of the first term.*
x^{-8+5}	*Since the powers have the same base, add the exponents.*
x^{-3}	
$\frac{1}{x^3}$	*Write with a positive exponent.*

CHECK IT OUT! **Simplify.**

3a. $(3^4)^5$ **3b.** $(6^0)^3$ **3c.** $(a^3)^4 \cdot (a^{-2})^{-3}$

Student to Student

Multiplication Properties of Exponents



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Sometimes I can't remember when to add exponents and when to multiply them. When this happens, I write everything in expanded form.

For example, I would write $x^2 \cdot x^3$ as $(x \cdot x)(x \cdot x \cdot x) = x^5$. Then $x^2 \cdot x^3 = x^{2+3} = x^5$.

I would write $(x^2)^3$ as $x^2 \cdot x^2 \cdot x^2$, which is $(x \cdot x)(x \cdot x)(x \cdot x) = x^6$.

Then $(x^2)^3 = x^{2 \cdot 3} = x^6$.

This way I get the right answer even if I forget the properties.

Powers of products can be found by using the meaning of an exponent.

$$(8x)^3 = 8x \cdot 8x \cdot 8x = 8 \cdot 8 \cdot 8 \cdot x \cdot x \cdot x = 8^3x^3 = 512x^3$$

Know it! Note

Power of a Product Property

WORDS	NUMBERS	ALGEBRA
A product raised to a power equals the product of each factor raised to that power.	$(2 \cdot 4)^3 = 2^3 \cdot 4^3$ $= 8 \cdot 64$ $= 512$	If a and b are any nonzero real numbers and n is any integer, then $(ab)^n = a^nb^n$.

EXAMPLE 4 **Finding Powers of Products**

Simplify.

A $(-3x)^2$

$(-3)^2 \cdot x^2$ — *Use the Power of a Product Property.*

$9x^2$ — *Simplify.*

Caution!

In Example 4B, the negative sign is not part of the base.

$-(3x)^2 = -1 \cdot (3x)^2$

B $-(3x)^2$

$-(3^2 \cdot x^2)$ — *Use the Power of a Product Property.*

$-(9 \cdot x^2)$ — *Simplify.*

$-9x^2$

C $(x^{-2} \cdot y^0)^3$

$(x^{-2})^3 \cdot (y^0)^3$ — *Use the Power of a Product Property.*

$x^{-2 \cdot 3} \cdot y^{0 \cdot 3}$ — *Use the Power of a Power Property.*

$x^{-6} \cdot y^0$ — *Simplify.*

$x^{-6} \cdot 1$ — *Write y^0 as 1.*

$\frac{1}{x^6}$ — *Write with a positive exponent.*

CHECK IT OUT! Simplify.

4a. $(4p)^3$ **4b.** $(-5t^2)^2$ **4c.** $(x^2y^3)^4 \cdot (x^2y^4)^{-4}$

THINK AND DISCUSS

1. Explain why $(a^2)^3$ and $a^2 \cdot a^3$ are not equivalent expressions.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, supply the missing exponents. Then give an example for each property.

Multiplication Properties of Exponents		
Product of Powers Property	Power of a Power Property	Power of a Product Property
$a^m \cdot a^n = a^{\blacksquare}$	$(a^m)^n = a^{\blacksquare}$	$(ab)^n = a^{\blacksquare}b^{\blacksquare}$

7-3 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 475

Simplify.

1. $2^2 \cdot 2^3$
2. $5^3 \cdot 5^3$
3. $n^6 \cdot n^2$
4. $x^2 \cdot x^{-3} \cdot x^4$

SEE EXAMPLE 2 p. 475

5. **Science** If you traveled in space at a speed of 1000 miles per hour, how far would you travel in 7.5×10^5 hours? Write your answer in scientific notation.

Simplify.

SEE EXAMPLE 3 p. 476

6. $(x^2)^5$
7. $(y^4)^8$
8. $(p^3)^3$
9. $(3^{-2})^2$
10. $(a^{-3})^4 \cdot (a^7)^2$
11. $xy \cdot (x^2)^3 \cdot (y^3)^4$

SEE EXAMPLE 4 p. 477

12. $(2t)^5$
13. $(6k)^2$
14. $(r^2s)^7$
15. $(-2x^5)^3$
16. $-(2x^5)^3$
17. $(a^2b^2)^5 \cdot (a^{-5})^2$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22	2
23–28	3
29–34	4

Extra Practice
Skills Practice p. S16
Application Practice p. S34

Simplify.

18. $3^3 \cdot 2^3 \cdot 3$
19. $6 \cdot 6^2 \cdot 6^3 \cdot 6^2$
20. $a^5 \cdot a^0 \cdot a^{-5}$
21. $x^7 \cdot x^{-6} \cdot y^{-3}$
22. **Geography** Rhode Island is the smallest state in the United States. Its land area is about 2.9×10^{10} square feet. Alaska, the largest state, is about 5.5×10^2 times as large as Rhode Island. What is the land area of Alaska in square feet? Write your answer in scientific notation.

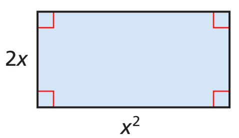
Simplify.

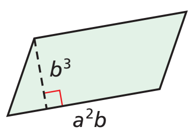
23. $(2^3)^3$
24. $(3^6)^0$
25. $(x^2)^{-1}$
26. $(b^4)^6 \cdot b$
27. $b \cdot (a^3)^4 \cdot (b^{-2})^3$
28. $(x^4)^2 \cdot (x^{-1})^{-4}$
29. $(3x)^3$
30. $(5w^8)^2$
31. $(p^4q^2)^7$
32. $(-4x^3)^4$
33. $-(4x^3)^4$
34. $(x^3y^4)^3 \cdot (xy^3)^{-2}$

Find the missing exponent in each expression.

35. $a^{\square}a^4 = a^{10}$
36. $(a^{\square})^4 = a^{12}$
37. $(a^2b^{\square})^4 = a^8b^{12}$
38. $(a^3b^6)^{\square} = \dfrac{1}{a^9b^{18}}$
39. $(b^2)^{-4} = \dfrac{1}{b^{\square}}$
40. $a^{\square} \cdot a^6 = a^6$

Geometry Write an expression for the area of each figure.

41.


42.


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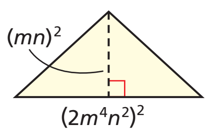
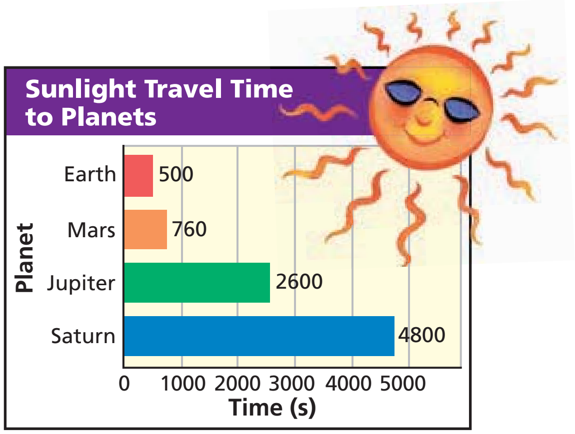


Simplify, if possible.

44. x^6y^5
45. $(2x^2)^2 \cdot (3x^3)^3$
46. $x^2 \cdot y^{-3} \cdot x^{-2} \cdot y^{-3}$
47. $(5x^2)(5x^2)^2$
48. $-(x^2)^4(-x^2)^4$
49. $a^3 \cdot a^0 \cdot 3a^3$
50. $(ab)^3(ab)^{-2}$
51. $10^2 \cdot 10^{-4} \cdot 10^5$
52. $(x^2y^2)^2(x^2y)^{-2}$

53. **Astronomy** The graph shows the approximate time it takes light from the Sun, which travels at a speed of 1.86×10^5 miles per second, to reach several planets. Find the approximate distance from the Sun to each planet in the graph. Write your answers in scientific notation. (*Hint:* Remember $d = rt$.)



54. **Geometry** The volume of a rectangular prism can be found by using the formula $V = \ell wh$ where ℓ, w, and h represent the length, width, and height of the prism. Find the volume of a rectangular prism whose dimensions are $3a^2$, $4a^5$, and $4a^2b^2$.

55. **ERROR ANALYSIS** Explain the error in each simplification below. What is the correct answer in each case?

a. $x^2 \cdot x^4 = x^8$ **b.** $(x^4)^5 = x^9$ **c.** $(x^2)^3 = x^{2^3} = x^8$

Simplify.

56. $(-3x^2)(5x^{-3})$ 57. $(a^4b)(a^3b^{-6})$ 58. $(6w^5)(2v^2)(w^6)$

59. $(3m^7)(m^2n)(5m^3n^8)$ 60. $(b^2)^{-2}(b^4)^5$ 61. $(3st)^2t^5$

62. $(2^2)^2(x^5y)^3$ 63. $(-t)(-t)^2(-t^4)$ 64. $(2m^2)(4m^4)(8n)^2$

65. **Estimation** Estimate the value of each expression. Explain how you estimated.

a. $\left[(-3.031)^2\right]^3$ **b.** $(6.2085 \times 10^2) \times (3.819 \times 10^{-5})$

66. **Physical Science** The speed of sound at sea level is about 344 meters per second. The speed of light is about 8.7×10^5 times faster than the speed of sound. What is the speed of light in meters per second? Write your answer in scientific notation and in standard form.

67. **Write About It** Is $(x^2)^3$ equal to $(x^3)^2$? Explain.

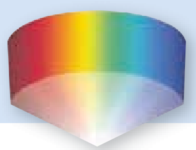
68. **Biology** A newborn baby has about 26,000,000,000 cells. An adult has about 1.9×10^3 times as many cells as a baby. About how many cells does an adult have? Write your answer in scientific notation.

Simplify.

69. $(-4k)^2 + k^2$ 70. $-3z^3 + (-3z)^3$ 71. $(2x^2)^2 + 2(x^2)^2$

72. $(2r)^2s^2 + 6(rs)^2 + 1$ 73. $(3a)^2b^3 + 3(ab)^2(2b)$ 74. $(x^2)(x^2)(x^2) + 3x^2$

MULTI-STEP TEST PREP



75. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. The speed of light v is the product of the frequency f and the wavelength w $(v = fw)$. Wavelengths are often measured in *nanometers*. *Nano* means 10^{-9}, so 1 nanometer $= 10^{-9}$ meters. What is 600 nanometers in meters? Write your answer in scientific notation.

b. Use your answer from part ***a*** to find the speed of light in meters per second if $f = 5 \times 10^{14}$ Hz.

c. Explain why you can rewrite $(6 \times 10^{-7})(5 \times 10^{14})$ as $(6 \times 5)(10^{-7})(10^{14})$.

Critical Thinking Rewrite each expression so that it has only one exponent. (*Hint:* You may use parentheses.)

76. c^3d^3 **77.** $36a^2b^2$ **78.** $\frac{8a^3}{b^3}$ **79.** $\frac{k^{-2}}{4m^2n^2}$

TEST PREP

80. Which of the following is equivalent to $x^2 \cdot x^0$?

Ⓐ 0 Ⓑ 1 Ⓒ x^2 Ⓓ x^{20}

81. Which of the following is equivalent to $(3 \times 10^5)^2$?

Ⓕ 9×10^7 Ⓖ 9×10^{10} Ⓗ 6×10^7 Ⓙ 6×10^{10}

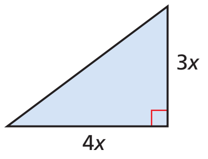
82. What is the value of n^3 when $n = 4 \times 10^5$?

Ⓐ 1.2×10^9 Ⓑ 1.2×10^{16} Ⓒ 6.4×10^9 Ⓓ 6.4×10^{16}

83. Which represents the area of the triangle?

Ⓕ $6x^2$ Ⓗ $7x^2$

Ⓖ $12x^2$ Ⓙ $24x^2$



CHALLENGE AND EXTEND

Simplify.

84. $3^2 \cdot 3^x$ **85.** $(3^2)^x$ **86.** $(x^yz)^2$

87. $(x+1)^{-2}(x+1)^3$ **88.** $(x+1)^2(x+1)^{-3}$ **89.** $(x^y \cdot x^z)^3$

90. $(4^x)^x$ **91.** $(x^x)^x$ **92.** $(3x)^{2y}$

Find the value of *x*.

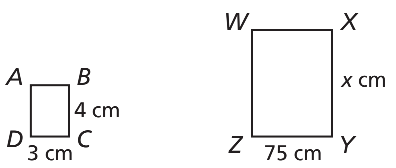
93. $5^x \cdot 5^4 = 5^8$ **94.** $7^3 \cdot 7^x = 7^{12}$ **95.** $(4^x)^3 = 4^{12}$ **96.** $(6^2)^x = 6^{16}$

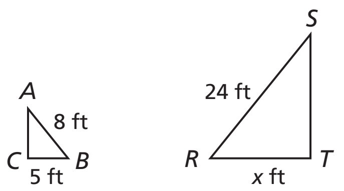
97. **Multi-Step** The edge of a cube measures 1.2×10^{-2} m. What is the volume of the cube in cubic centimeters?

SPIRAL REVIEW

Find the value of *x* in each diagram. *(Lesson 2-8)*

98. $\square ABCD \sim \square WXYZ$ **99.** $\triangle ABC \sim \triangle RST$





Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms. *(Lesson 4-6)*

100. 5, 1, −3, −7, … **101.** −3, −2, 0, 3, … **102.** 0.4, 1.0, 1.6, 2.2, …

Write each number in standard form. *(Lesson 7-2)*

103. 7.8×10^6 **104.** 4.95×10^{-4} **105.** 983×10^{-1} **106.** 0.06×10^8