7-2 Powers of 10 and Scientific Notation

Nucleus of a silicon atom

Objectives
Evaluate and multiply by powers of 10.

Convert between standard notation and scientific notation.

Vocabulary
scientific notation

Why learn this?
Powers of 10 can be used to read and write very large and very small numbers, such as the masses of atomic particles. (See Exercise 44.)

The table shows relationships between several powers of 10.

	÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	
Power	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value	1000	100	10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$
	× 10	× 10	× 10	× 10	× 10	× 10	

- Each time you **divide by 10**, the exponent decreases by 1 and the decimal point moves one place to the left.
- Each time you **multiply by 10**, the exponent increases by 1 and the decimal point moves one place to the right.

Know it! Note

Powers of 10

WORDS	NUMBERS
Positive Integer Exponent	
If n is a positive integer, find the value of 10^n by starting with 1 and moving the decimal point n places to the right.	$10^4 = 1\,0,\,0\,0\,0$ (4 places)
Negative Integer Exponent	
If n is a positive integer, find the value of 10^{-n} by starting with 1 and moving the decimal point n places to the left.	$10^{-6} = \frac{1}{10^6} = 0.0\,0\,0\,0\,0\,1$ (6 places)

EXAMPLE 1 **Evaluating Powers of 10**

Find the value of each power of 10.

A 10^{-3}

Start with 1 and move the decimal point three places to the left.

$0.\,0\,0\,1$

0.001

B 10^2

Start with 1 and move the decimal point two places to the right.

$1\,0\,0$

100

C 10^0

Start with 1 and move the decimal point zero places.

1

Writing Math

You may need to add zeros to the right or left of a number in order to move the decimal point in that direction.

CHECK IT OUT! Find the value of each power of 10.

1a. 10^{-2} **1b.** 10^{5} **1c.** 10^{10}

EXAMPLE 2 Writing Powers of 10

Reading Math

If you do not see a decimal point in a number, it is understood to be at the end of the number.

Write each number as a power of 10.

A 10,000,000

The decimal point is seven places to the right of 1, so the exponent is 7.

10^{7}

B 0.001

The decimal point is three places to the left of 1, so the exponent is −3.

10^{-3}

C 10

The decimal point is one place to the right of 1, so the exponent is 1.

10^{1}

CHECK IT OUT! Write each number as a power of 10.

2a. 100,000,000 **2b.** 0.0001 **2c.** 0.1

You can also move the decimal point to find the product of any number and a power of 10. You start with the number instead of starting with 1.

Know it! Note

Multiplying by Powers of 10	
If the exponent is a positive integer, move the decimal point to the right.	$125 \times 10^{5} = 12{,}5\,0\,0,\,0\,0\,0$ (5 places)
If the exponent is a negative integer, move the decimal point to the left.	$36.2 \times 10^{-3} = 0.0\,3\,6\,2$ (3 places)

EXAMPLE 3 Multiplying by Powers of 10

Find the value of each expression.

A 97.86×10^{6}

$97.8\,6\,0\,0\,0\,0$ *Move the decimal point 6 places to the right.*

97,860,000

B 19.5×10^{-4}

$0\,0\,1\,9.5$ *Move the decimal point 4 places to the left.*

0.00195

CHECK IT OUT! Find the value of each expression.

3a. 853.4×10^{5} **3b.** 0.163×10^{-2}

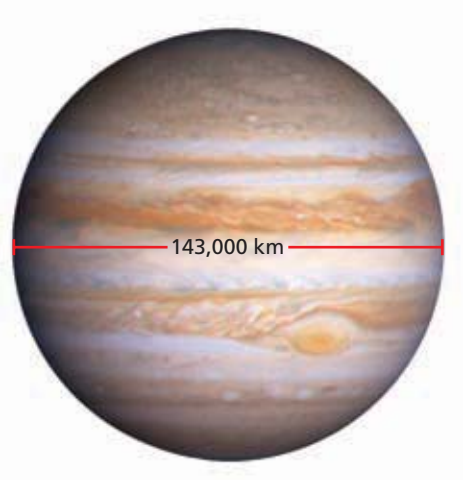
Scientific notation is a method of writing numbers that are very large or very small. A number written in scientific notation has two parts that are multiplied.

The first part is a number that is greater than or equal to 1 and less than 10.

3.5×10^{11} 9.98×10^{-2}

The second part is a power of 10.

EXAMPLE 4 Astronomy Application



Jupiter has a diameter of about 143,000 km. Its shortest distance from Earth is about 5.91×10^8 km, and its average distance from the Sun is about 778,400,000 km. Jupiter's orbital speed is approximately 1.3×10^4 m/s.

A **Write Jupiter's shortest distance from Earth in standard form.**

5.91×10^8

5.9 1 0 0 0 0 0 0 0

Move the decimal point 8 places to the right.

591,000,000 km

B **Write Jupiter's average distance from the Sun in scientific notation.**

778,400,000

7 7 8, 4 0 0, 0 0 0

8 places

Count the number of places you need to move the decimal point to get a number between 1 and 10.

Use that number as the exponent of 10.

7.784×10^8 km

Reading Math

Standard form refers to the usual way that numbers are written.

CHECK IT OUT!

4a. Use the information above to write Jupiter's diameter in scientific notation.

4b. Use the information above to write Jupiter's orbital speed in standard form.

EXAMPLE 5 Comparing and Ordering Numbers in Scientific Notation

Order the list of numbers from least to greatest.

$1.2 \times 10^{-1}, 8.2 \times 10^4, 6.2 \times 10^5, 2.4 \times 10^5, 1 \times 10^{-1}, 9.9 \times 10^{-4}$

Step 1 List the numbers in order by powers of 10.

$9.9 \times 10^{-4}, 1.2 \times 10^{-1}, 1 \times 10^{-1}, 8.2 \times 10^4, 6.2 \times 10^5, 2.4 \times 10^5$

Step 2 Order the numbers that have the same power of 10.

$9.9 \times 10^{-4}, 1 \times 10^{-1}, 1.2 \times 10^{-1}, 8.2 \times 10^4, 2.4 \times 10^5, 6.2 \times 10^5$

CHECK IT OUT!

5. Order the list of numbers from least to greatest.

$5.2 \times 10^{-3}, 3 \times 10^{14}, 4 \times 10^{-3}, 2 \times 10^{-12}, 4.5 \times 10^{30}, 4.5 \times 10^{14}$

THINK AND DISCUSS

1. Tell why 34.56×10^4 is not correctly written in scientific notation.

2. GET ORGANIZED Copy and complete the graphic organizer.

Know it! Note

Powers of 10 and Scientific Notation

- A negative exponent corresponds to moving the decimal point ____?____.
- A positive exponent corresponds to moving the decimal point ____?____.

7-2 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 7-2
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain how you can tell whether a number is written in *scientific notation*.

SEE EXAMPLE 1 p. 466

Find the value of each power of 10.

2. 10^6 3. 10^{-5} 4. 10^{-4} 5. 10^8

SEE EXAMPLE 2 p. 467

Write each number as a power of 10.

6. 10,000 7. 0.000001 8. 100,000,000,000,000,000

SEE EXAMPLE 3 p. 467

Find the value of each expression.

9. 650.3×10^6 10. 48.3×10^{-4} 11. 92×10^{-3}

SEE EXAMPLE 4 p. 468

12. **Astronomy** A light-year is the distance that light travels in a year and is equivalent to 9.461×10^{12} km. Write this distance in standard form.

SEE EXAMPLE 5 p. 468

13. Order the list of numbers from least to greatest.
8.5×10^{-1}, 3.6×10^8, 5.85×10^{-3}, 2.5×10^{-1}, 8.5×10^8

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–17	1
18–20	2
21–24	3
25–26	4
27	5

Extra Practice
Skills Practice p. S16
Application Practice p. S34

Find the value of each power of 10.

14. 10^3 15. 10^{-9} 16. 10^{-12} 17. 10^{14}

Write each number as a power of 10.

18. 0.01 19. 1,000,000 20. 0.000000000000001

Find the value of each expression.

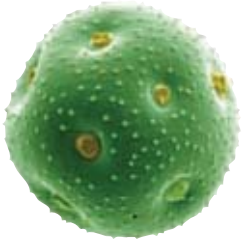
21. 9.2×10^4 22. 1.25×10^{-7} 23. 42×10^{-5} 24. 0.05×10^7

25. **Biology** The human body is made of about 1×10^{13} cells. Write this number in standard form.

26. **Statistics** At the beginning of the twenty-first century, the population of China was about 1,287,000,000. Write this number in scientific notation.

27. Order the list of numbers from least to greatest.
2.13×10^{-1}, 3.12×10^2, 1.23×10^{-3}, 2.13×10^1, 1.32×10^{-3}, 3.12×10^{-3}

28. **Health** Donnell is allergic to pollen. The diameter of a grain of pollen is between 1.2×10^{-5} m and 9×10^{-5} m. Donnell's air conditioner has a filter that removes particles larger than 3×10^{-7} m. Will the filter remove pollen? Explain.


Grain of pollen, enlarged 1300 times

29. **Entertainment** In the United States, a CD is certified platinum if it sells 1,000,000 copies. A CD that has gone 2 times platinum has sold 2,000,000 copies. How many copies has a CD sold if it has gone 27 times platinum? Write your answer in scientific notation.

Write each number in scientific notation.

30. 40,080,000 31. 235,000 32. 170,000,000,000
33. 0.0000006 34. 0.000077 35. 0.0412

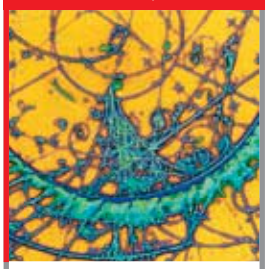
State whether each number is written in scientific notation. If not, write it in scientific notation.

36. 50×10^{-5} **37.** 8.1×10^{-2} **38.** 1,200,000 **39.** 0.25×10^{3}

40. 0.1 **41.** 7×10^{8} **42.** 48,000 **43.** 3.5×10^{-6}

LINK

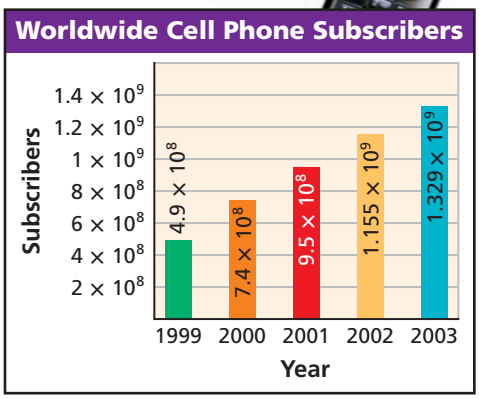
Chemistry



The image above is a colored bubble-chamber photograph. It shows the tracks left by subatomic particles in a particle accelerator.

44. Chemistry Atoms are made of three elementary particles: protons, electrons, and neutrons. The mass of a proton is about 1.67×10^{-27} kg. The mass of an electron is about 0.00000000000000000000000000000911 kg. The mass of a neutron is about 1.68×10^{-27} kg. Which particle has the least mass? (*Hint:* Compare the numbers after they are written in scientific notation.)

45. Communication This bar graph shows the increase of cellular telephone subscribers worldwide.



a. Write the number of subscribers for the following years in standard form: 1999, 2000, and 2003.

b. Zorah looks at the bar graph and says, "It looks like the number of cell phone subscribers nearly doubled from 2000 to 2003." Do you agree with Zorah? Use scientific notation to explain your answer.

46. Measurement In the metric system, the basic unit for measuring length is the meter (m). Other units for measuring length are based on the meter and powers of 10, as shown in the table.

Selected Metric Lengths	
1 millimeter (mm) = 10^{-3} m	1 dekameter (dam) = 10^{1} m
1 centimeter (cm) = 10^{-2} m	1 hectometer (hm) = 10^{2} m
1 decimeter (dm) = 10^{-1} m	1 kilometer (km) = 10^{3} m

a. Which lengths in the table are longer than a meter? Which are shorter than a meter? How do you know?

b. Evaluate each power of 10 in the table to check your answers to part **a.**

47. Critical Thinking Recall that $\frac{1}{10^3} = 10^{-3}$. Based on this information, complete the following statement: Dividing a number by 10^3 is equivalent to multiplying by ▢.

48. Write About It When you change a number from scientific notation to standard form, explain how you know which way to move the decimal point and how many places to move it.

MULTI-STEP TEST PREP

49. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. The speed of light is approximately 3×10^{8} m/s. Write this number in standard form.

b. Why do you think it would be better to express this number in scientific notation rather than standard form?

c. The wavelength of a shade of red light is 0.00000068 meters. Write this number in scientific notation.

TEST PREP

50. There are about 3.2×10^7 seconds in one year. What is this number in standard form?

Ⓐ 0.000000032
Ⓑ 0.00000032
Ⓒ 32,000,000
Ⓓ 320,000,000

51. Which expression is the scientific notation for 82.35?

Ⓕ 8.235×10^1 Ⓖ 823.5×10^{-1} Ⓗ 8.235×10^{-1} Ⓙ 0.8235×10^2

52. Which statement is correct for the list of numbers below?
2.35×10^{-8}, 0.000000029, 1.82×10^8, 1,290,000,000, 1.05×10^9

Ⓐ The list is in increasing order.
Ⓑ If 0.000000029 is removed, the list will be in increasing order.
Ⓒ If 1,290,000,000 is removed, the list will be in increasing order.
Ⓓ The list is in decreasing order.

CHALLENGE AND EXTEND

53. **Technology** The table shows estimates of computer storage. A CD-ROM holds 700 MB. A DVD-ROM holds 4.7 GB. Estimate how many times more storage a DVD has than a CD. Explain how you found your answer.

Computer Storage
1 kilobyte (KB) ≈ 1000 bytes
1 megabyte (MB) ≈ 1 million bytes
1 gigabyte (GB) ≈ 1 billion bytes

54. For parts **a–d,** use what you know about multiplying by powers of 10 and the Commutative and Associative Properties of Multiplication to find each product. Write each answer in scientific notation.

a. $(3 \times 10^2)(2 \times 10^3)$ **b.** $(5 \times 10^8)(1.5 \times 10^{-6})$
c. $(2.2 \times 10^{-8})(4 \times 10^{-3})$ **d.** $(2.5 \times 10^{-12})(2 \times 10^6)$
e. Based on your answers to parts **a–d,** write a rule for multiplying numbers in scientific notation.
f. Does your rule work when you multiply $(6 \times 10^3)(8 \times 10^5)$? Explain.

SPIRAL REVIEW

Define a variable and write an inequality for each situation. Graph the solutions. *(Lesson 3-1)*

55. Melanie must wait at least 45 minutes for the results of her test.

56. Ulee's dog can lose no more than 8 pounds to stay within a healthy weight range.

57. Charlene must spend more than $50 to get the advertised discount.

Solve each system by elimination. *(Lesson 6-3)*

58. $\begin{cases} x + y = 8 \\ x - y = 2 \end{cases}$ **59.** $\begin{cases} 2x + y = -3 \\ 2x + 3y = -1 \end{cases}$ **60.** $\begin{cases} x - 6y = -3 \\ 3x + 4y = 13 \end{cases}$

Evaluate each expression for the given value(s) of the variable(s). *(Lesson 7-1)*

61. t^{-4} for $t = 2$ **62.** $(-8m)^0$ for $m = -5$ **63.** $3a^{-3}b^0$ for $a = 5$ and $b = 6$

7-3 Algebra LAB

Explore Properties of Exponents

You can use patterns to find some properties of exponents.

Use with Lesson 7-3

Activity 1

1. Copy and complete the table below.

$3^2 \cdot 3^3 = (3 \cdot 3)(3 \cdot 3 \cdot 3) = 3^{\square}$
$5^4 \cdot 5^2 = (\square \cdot \square \cdot \square \cdot \square)(\square \cdot \square) = 5^{\square}$
$4^3 \cdot 4^3 = (\square \cdot \square \cdot \square)(\square \cdot \square \cdot \square) = \square^{\square}$
$2^3 \cdot 2^2 = (\square \cdot \square \cdot \square)(\square \cdot \square) = \square^{\square}$
$6^3 \cdot 6^4 = (\quad)(\quad) =$

2. Examine your completed table. Look at the two exponents in each factor and the exponent in the final answer. What pattern do you notice?

3. Use your pattern to make a conjecture: $a^m \cdot a^n = a^{\square}$.

Try This

Use your conjecture to write each product below as a single power.

1. $5^3 \cdot 5^5$ **2.** $7^2 \cdot 7^2$ **3.** $10^8 \cdot 10^4$ **4.** $8^7 \cdot 8^3$

5. Make a table similar to the one above to explore what happens when you multiply more than two powers that have the same base. Then write a conjecture in words to summarize what you find.

Activity 2

1. Copy and complete the table below.

$(2^3)^2 = 2^3 \cdot 2^3 = (\square \cdot \square \cdot \square)(\square \cdot \square \cdot \square) = 2^{\square}$
$(2^2)^3 = \square \cdot \square \cdot \square = (\square \cdot \square)(\square \cdot \square)(\square \cdot \square) = \square^{\square}$
$(4^2)^4 = \square \cdot \square \cdot \square \cdot \square = (\square \cdot \square)(\square \cdot \square)(\square \cdot \square)(\square \cdot \square) = \square^{\square}$
$(3^4)^2 = \square \cdot \square = (\square \cdot \square \cdot \square \cdot \square)(\square \cdot \square \cdot \square \cdot \square) = \square^{\square}$
$(6^3)^4 =$

2. Examine your completed table. Look at the two exponents in the original expression and the exponent in the final answer. What pattern do you notice?

3. Use your pattern to make a conjecture: $(a^m)^n = a^{\square}$.

Try This

Use your conjecture to write each product below as a single power.

6. $(5^3)^2$ **7.** $(7^2)^2$ **8.** $(3^3)^4$ **9.** $(9^7)^3$

10. Make a table similar to the one in Activity 2 to explore what happens when you raise a power to two powers, for example, $\left[(4^2)^3\right]^3$. Then write a conjecture in words to summarize what you find.

Activity 3

1 Copy and complete the table below.

$(ab)^3 = (ab)(ab)(ab) = (a \cdot a \cdot a)(b \cdot b \cdot b) = a^{\square} b^{\square}$
$(mn)^4 = (\square)(\square)(\square)(\square) = (\square \cdot \square \cdot \square \cdot \square)(\square \cdot \square \cdot \square \cdot \square) = \square^{\square} \square^{\square}$
$(xy)^2 = (\square)(\square) = (\square \cdot \square)(\square \cdot \square) = \square^{\square} \square^{\square}$
$(cd)^5 = (\square)(\square)(\square)(\square)(\square) = (\square \cdot \square \cdot \square \cdot \square \cdot \square)(\square \cdot \square \cdot \square \cdot \square \cdot \square) = \square^{\square} \square^{\square}$
$(pq)^6 =$

2 Examine your completed table. Look at the original expression and the final answer. What pattern do you notice?

3 Use your pattern to make a conjecture: $(ab)^n = a^{\square} b^{\square}$.

Try This

Use your conjecture to write each power below as a product.

11. $(rs)^8$ **12.** $(yz)^9$ **13.** $(ab)^7$ **14.** $(xz)^{12}$

15. Look at the first row of your table. What property or properties allow you to write $(ab)(ab)(ab)$ as $(a \cdot a \cdot a)(b \cdot b \cdot b)$?

16. Make a table similar to the one above to explore what happens when you raise a product containing more than two factors to a power, for example, $(xyz)^7$. Then write a conjecture in words to summarize what you find.