# Reference

## Properties

### Properties of Equality

**Addition Property of Equality**
If $a = b$, then $a + c = b + c$.

**Subtraction Property of Equality**
If $a = b$, then $a - c = b - c$.

**Multiplication Property of Equality**
If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

**Division Property of Equality**
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

**Reflexive Property of Equality**
$a = a$

**Symmetric Property of Equality**
If $a = b$, then $b = a$.

**Transitive Property of Equality**
If $a = b$ and $b = c$, then $a = c$.

**Substitution Property of Equality**
If $a = b$, then $a$ can be substituted for $b$ (or $b$ for $a$) in any equation or expression.

### Properties of Segment and Angle Congruence

**Reflexive Property of Congruence**
For any segment $AB$, $\overline{AB} \cong \overline{AB}$.

For any angle $A$, $\angle A \cong \angle A$.

**Symmetric Property of Congruence**
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

**Transitive Property of Congruence**
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

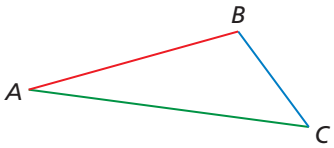
### Other Properties

**Transitive Property of Parallel Lines**
If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

**Distributive Property**
Sum
$a(b + c) = ab + ac$
Difference
$a(b - c) = ab - ac$

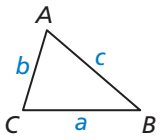
### Triangle Inequalities

**Triangle Inequality Theorem**

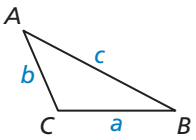



$AB + BC > AC$
$AC + BC > AB$
$AB + AC > BC$

**Pythagorean Inequalities Theorem**




If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.




If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

# Formulas

## Coordinate Geometry

**Slope**

$m = \frac{y_2 - y_1}{x_2 - x_1}$

**Slope-intercept form**

$y = mx + b$

**Point-slope form**

$y - y_1 = m(x - x_1)$

**Midpoint Formula**

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

**Distance Formula**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Partitioning a segment on a number line**

$\frac{ax_1 + bx_2}{a + b}$ partitions the segment in the ratio $b : a$.

**Standard form of a linear equation**

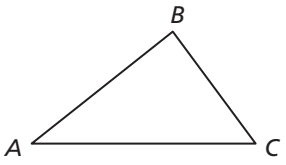
$Ax + By = C$

**Standard equation of a circle**

$(x - h)^2 + (y - k)^2 = r^2$, with center $(h, k)$ and radius $r$

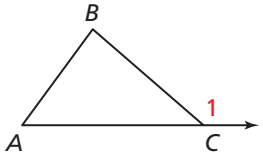
## Polygons

**Triangle Sum Theorem**



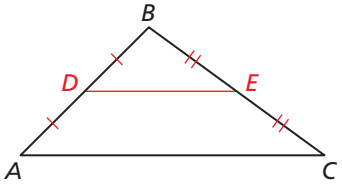

$m\angle A + m\angle B + m\angle C = 180°$

**Exterior Angle Theorem**



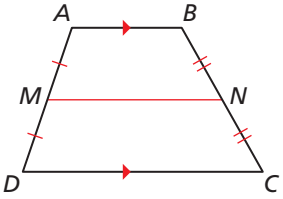

$m\angle 1 = m\angle A + m\angle B$

**Triangle Midsegment Theorem**



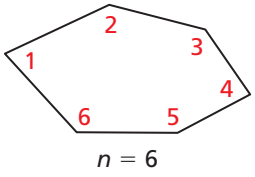

$\overline{DE} \parallel \overline{AC}, DE = \frac{1}{2}AC$

**Trapezoid Midsegment Theorem**




$\overline{MN} \parallel \overline{AB}, \overline{MN} \parallel \overline{DC}, MN = \frac{1}{2}(AB + CD)$

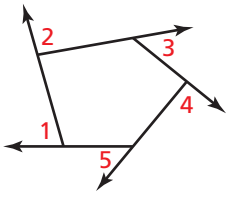
**Polygon Interior Angles Theorem**




$n = 6$

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180°$

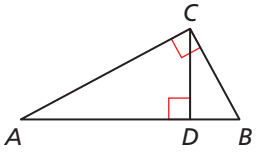
**Polygon Exterior Angles Theorem**




$n = 5$

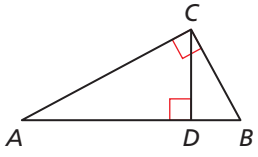
$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

**Geometric Mean (Altitude) Theorem**




$CD^2 = AD \cdot BD$

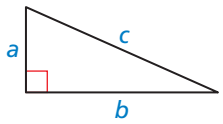
**Geometric Mean (Leg) Theorem**




$CB^2 = DB \cdot AB$ $\quad$ $AC^2 = AD \cdot AB$

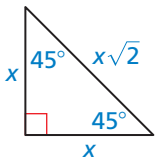
## Right Triangles

**Pythagorean Theorem**



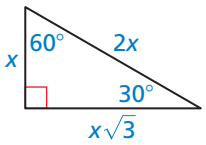

$a^2 + b^2 = c^2$

**45°-45°-90° Triangles**




hypotenuse = leg • $\sqrt{2}$

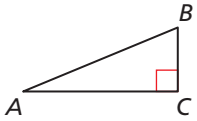
**30°-60°-90° Triangles**




hypotenuse = shorter leg • 2
longer leg = shorter leg • $\sqrt{3}$

## Trigonometry

**Ratios**




$\sin A = \frac{BC}{AB}$ $\quad \cos A = \frac{AC}{AB}$ $\quad \tan A = \frac{BC}{AC}$

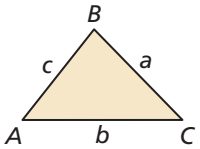
$\sin^{-1}\frac{BC}{AB} = m\angle A$ $\quad \cos^{-1}\frac{AC}{AB} = m\angle A$ $\quad \tan^{-1}\frac{BC}{AC} = m\angle A$

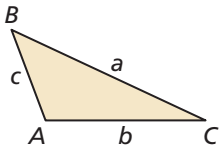
**Sine and cosine of complementary angles**
Let $A$ and $B$ be complementary angles. Then the following statements are true.

$\sin A = \cos(90° - A) = \cos B$
$\sin B = \cos(90° - B) = \cos A$
$\cos A = \sin(90° - A) = \sin B$
$\cos B = \sin(90° - B) = \cos A$

**Conversion between degrees and radians**
$180° = \pi$ radians

**Any Triangle**




**Area**

Area $= \frac{1}{2}bc \sin A$

Area $= \frac{1}{2}ac \sin B$

Area $= \frac{1}{2}ab \sin C$

**Law of Sines**

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Law of Cosines**

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

## Probability and Combinatorics

**Theoretical Probability** $= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

**Experimental Probability** $= \frac{\text{Number of successes}}{\text{Number of trials}}$

**Probability of the complement of an event**
$P(\overline{A}) = 1 - P(A)$

**Probability of independent events**
$P(A \text{ and } B) = P(A) \bullet P(B)$

**Probability of dependent events**
$P(A \text{ and } B) = P(A) \bullet P(B \mid A)$

**Probability of compound events**
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

**Permutations**

${}_nP_r = \frac{n!}{(n-r)!}$

**Combinations**

${}_nC_r = \frac{n!}{(n-r)! \bullet r!}$

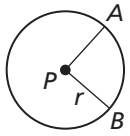
**Binomial experiments**

$P(k \text{ successes}) = {}_nC_k p^k (1-p)^{n-k}$

Reference

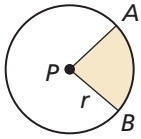
## Circles

**Arc length**



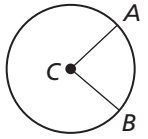

Arc length of $\overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot 2\pi r$

**Area of a sector**



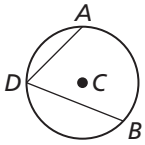

Area of sector $APB = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$

**Central angles**



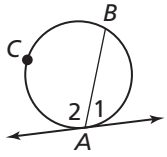

$m\angle ACB = m\overset{\frown}{AB}$

**Inscribed angles**




$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

**Tangent and intersected chord**

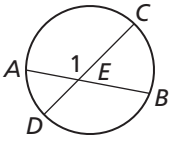



$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$

$m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

## Angles and Segments of Circles

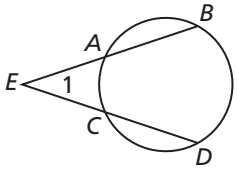
**Two chords**




$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AC} + m\overset{\frown}{DB}\right)$

$EA \cdot EB = EC \cdot ED$

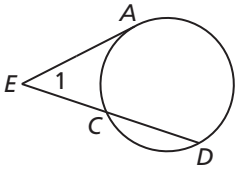
**Two secants**




$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{BD} - m\overset{\frown}{AC}\right)$

$EA \cdot EB = EC \cdot ED$

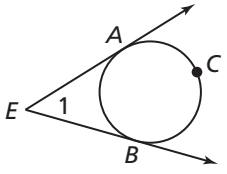
**Tangent and secant**




$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{AC}\right)$

$EA^2 = EC \cdot ED$

**Two tangents**

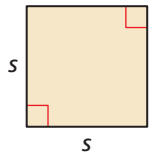



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{ACB} - m\overset{\frown}{AB}\right)$

$EA = EB$

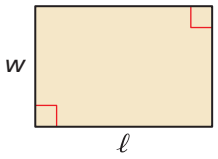
# Perimeter, Area, and Volume Formulas

### Square



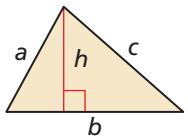

$P = 4s$
$A = s^2$

### Rectangle



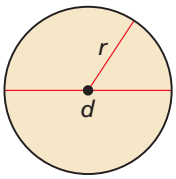

$P = 2\ell + 2w$
$A = \ell w$

### Triangle



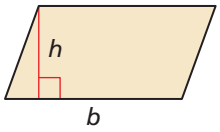

$P = a + b + c$
$A = \frac{1}{2}bh$

### Circle



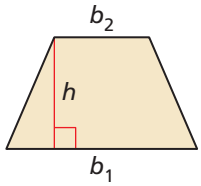

$C = \pi d$ or $C = 2\pi r$
$A = \pi r^2$

### Parallelogram



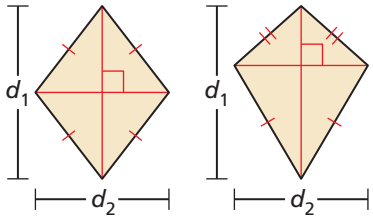

$A = bh$

### Trapezoid



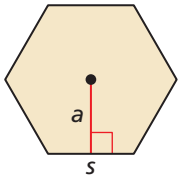

$A = \frac{1}{2}h(b_1 + b_2)$

### Rhombus/Kite



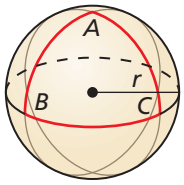

$A = \frac{1}{2}d_1d_2$

### Regular *n*-gon



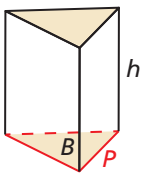

$A = \frac{1}{2}aP$ or $A = \frac{1}{2}a \cdot ns$

### Spherical triangle



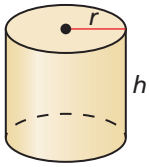

$A = \frac{\pi r^2}{180°}(m\angle A + m\angle B + m\angle C - 180°)$

### Prism



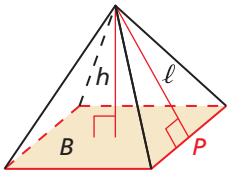

$L = Ph$
$S = 2B + Ph$
$V = Bh$

### Cylinder



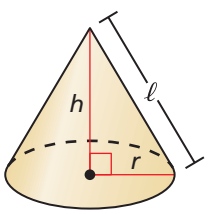

$L = 2\pi rh$
$S = 2\pi r^2 + 2\pi rh$
$V = \pi r^2h$

### Pyramid



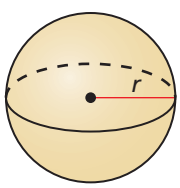

$L = \frac{1}{2}P\ell$
$S = B + \frac{1}{2}P\ell$
$V = \frac{1}{3}Bh$

### Cone




$L = \pi r\ell$
$S = \pi r^2 + \pi r\ell$
$V = \frac{1}{3}\pi r^2h$

### Sphere




$S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Reference

# Other Formulas

**Geometric mean**

$x = \sqrt{a \cdot b}$

**Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$

**Similar polygons or similar solids with scale factor $a : b$**

Ratio of perimeters $= a : b$
Ratio of areas $= a^2 : b^2$
Ratio of volumes $= a^3 : b^3$

# Conversions

**U.S. Customary**

1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet
1 mile = 1760 yards
1 acre = 43,560 square feet
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 pound = 16 ounces
1 ton = 2000 pounds

**U.S. Customary to Metric**

1 inch = 2.54 centimeters
1 foot ≈ 0.3 meter
1 mile ≈ 1.61 kilometers
1 quart ≈ 0.95 liter
1 gallon ≈ 3.79 liters
1 cup ≈ 237 milliliters
1 pound ≈ 0.45 kilogram
1 ounce ≈ 28.3 grams
1 gallon ≈ 3785 cubic centimeters

**Time**

1 minute = 60 seconds
1 hour = 60 minutes
1 hour = 3600 seconds
1 year = 52 weeks

**Temperature**

$C = \frac{5}{9}(F - 32)$

$F = \frac{9}{5}C + 32$

**Metric**

1 centimeter = 10 millimeters
1 meter = 100 centimeters
1 kilometer = 1000 meters
1 liter = 1000 milliliters
1 kiloliter = 1000 liters
1 milliliter = 1 cubic centimeter
1 liter = 1000 cubic centimeters
1 cubic millimeter = 0.001 milliliter
1 gram = 1000 milligrams
1 kilogram = 1000 grams

**Metric to U.S. Customary**

1 centimeter ≈ 0.39 inch
1 meter ≈ 3.28 feet
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile
1 liter ≈ 1.06 quarts
1 liter ≈ 0.26 gallon
1 kilogram ≈ 2.2 pounds
1 gram ≈ 0.035 ounce
1 cubic meter ≈ 264 gallons